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Sensitivity Analysis and Investment Decision Making Under Uncertainty Based on NPV Method

Qingyang Liu[1]

[1] *School of Business and Management, the Hong Kong University of Science and Technology, Hong Kong S.A.R.*
[*]*Corresponding author. Email: qliubb@connect.ust.hk*

ABSTRACT

Sensitivity analysis is often used in investment decision-making to evaluate the risk under uncertainty. However, sensitivity analysis on the NPV method is seldom studied in the existing literature. This paper studied how sensitivity analysis could complement the NPV rules in investment decisions when uncertain. The study is based on a case study of an imaginary company and used the partial derivative method to conduct sensitivity analysis on NPV. The study showed that the NPV of investment projects has a different sensitivity to the discount rate change when the project cashflow distribution differs. Moreover, when there is an expected discount rate change due to exogenous factors such as interest rate, the sensitivity of NPV will also vary. The study provides a more rigorous way of examining the sensitivity compared to previous methods.

Keywords: *Sensitivity Analysis, Investment Decision Making, Net Present Value Rule.*

1. INTRODUCTION

Throughout the long history of financial studies, investors have created many tools to help them make investment decisions. Among all the investment decision-making methods, the Net Present Value (NPV) rule is one of the most common techniques investors use, and one of the best methods, especially when it comes to rejecting certain projects[1].

However, financial investment study always revolves around another important topic - risk. Analysts have many tools to help them understand and mitigate risks when making investment decisions, and one of them is sensitivity analysis (SA). SA has long been used in the field of finance, both in real-world investment decision making and in research, to understand the decision-making under uncertainty and the influence of market risk to the projects. This could help investors to come up with a mitigation plan for the risk. In Jovanovic's paper in 1999, he compared different analysis methods for decision-making under uncertainty and demonstrated that SA is the most effective one [2].

A past paper has studied the sensitivity analysis of different investment decision-making models, such as the NPV, IRR, and pay-back period rules. For example, Borgonovo and Peccati's study of global sensitivity of investment decisions [3] demonstrated the expression of global importance (GI) of cash flows for the NPV method. Further, they showed the impact SA has on gathering information during investment decision-making.

In a 2009 study, Borgonvo further proposed a new way of Sensitivity analysis and analyzed different criteria of investment that investors and lenders have. They found that when investors are making decisions, different groups of exogenous factors could influence the result and affect the final decision in different ways.

Another example is Mangiero and Kraten's study, where they used the real-life case of the Blue Frog Company to conduct the sensitivity analysis on NPV [4]. They adopted a dynamic approach using excel data spreadsheet and demonstrated how analysts identify the thresholds of different variables and when to make different investment decisions.

Sensitivity Analysis could also be used to compare different investment decision-making tools. For example, Marchioni and Magni used the Sensitivity Analysis method to study the coherence between the NPV method and rates of return [5]. They found that the Return on Investment method is highly consistent with the NPV rule, under the analysis of several SA methods.

However, after studying the existing body of literature on SA and investment decision rule, we found that most of the sensitivity analyses are based on numerical methods, without a more holistic or rigorous view on the degree of impact the change of independent variables have on the dependent variable. Therefore, this paper proposes using the partial derivative method to conduct sensitivity analysis of investment projects.

This paper uses the case of an imaginary firm Mahjong Inc. to demonstrate the method of partial differentiating NPV concerning rate and understand how sensitive NPV is to the discount rate changes under different situations. We study the impact potential discount rate changes could have on the projects' NPV value under two different scenarios, one is the constant discount rate, one is the expected discount rate decrease caused by the decrease in interest rate. The study shows that when a discount rate decrease is predicted, NPV is less sensitive to the decline of current discount rate. The analysis of this paper provides investors a more rigorous and holistic method of understanding the impact of the discount rate change.

The rest of the paper will cover the following part: section two further introduces the methods we are using in the study, and section three presents the case of Mahjong Inc.; section four demonstrates the result of the study, followed by section five discussing the results, limitation and directions for future studies; lastly, section six concludes the whole paper.

2. METHOD

2.1. What is Sensitivity Analysis

Sensitivity analysis (SA) is a method wildly used across different fields of studies, including finance, to determine how different sources of uncertainty from model inputs influence the model output's uncertainty [6]. In investment decision-making, SA is used to study how changes in one independent variable, under several assumptions, would influence the dependent variable.

When making investment decisions, the decision-makers normally build financial models with multiple input parameters against one output variable and use the current data for valuation. However, as the market situation changes rapidly and constantly, it is not enough for decision-makers to merely consider the current data at hand. The decision-makers also need to consider the potential impact changes of these variables, i.e. market risks, have on the output; they would use the sensitivity analysis method to assist them in doing so, thus deriving plans to mitigate the negative impacts. According to Borgonovo et al. [7], there are three indications of the SA method to decision-makers when evaluating investment opportunities: i) to evaluate the robustness of the financial model per se, ii) to determine the changes of the model's output under variations of input parameters, and iii) to understand the impact each of the model's assumptions has on the investment valuation.

There are multiple methods of SA currently used by researchers and industrial experts, including Partial Differentiation, one standard deviation or 20% increase or decrease in the input variables, sensitivity index, and etc. [8]. This paper will mainly focus on the partial differentiation method, which is also known as the direct method of sensitivity analysis, as it helps us explicitly understand the impact of the change in one input variable when holding others constant. By comparing the partial derivatives of different variables, we could realize which variable has the most and which does the most negligible impact on the output.

2.2. NPV in Investment Decision Making

The Net Present Value (NPV) rule is one of the most studied investment decision-making rules, which uses the future cash flows as well as the current discount rate to generate, as its name suggests, the present value of the project. The common formula of NPV is:

$$NPV = \mathrm{f}(C_1, C_2, \ldots C_t, \mathrm{r}, \mathrm{t}) = C0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots + \frac{C_t}{(1+r)^t} = \sum_{i=0}^{t} \frac{C_i}{(1+r)^i} \quad (1)$$

Where Ci is the value of the ith cash flow, r is the discount rate, and t is the overall duration of the investment project.

Investors adopt the NPV rule when making investment decisions: when the projects are independent, a project should be accepted with a positive NPV, and vice versa; when the projects are mutually exclusive, the project with the highest positive NPV should be accepted.

However, it is often risky to rely on the NPV rule when evaluating investment projects because the cash flows, determined by several other exogenous factors, and discount rate may vary in the future, resulting in the fluctuation of the NPV value and thus potential changes of the ultimate decision. Investors usually consider other criteria, such as the sensitivity analysis of the NPV against different variables, to make the decision more solid.

2.3. Sensitivity Analysis on NPV

Following the previous discussions, we could further elaborate the NPV formula into the below equation:

$$NPV = f_1(C_1, C_2, \ldots C_t, \mathrm{r}, \mathrm{t}) = f_2(\alpha_1, \alpha_2, \ldots \alpha_n, \mathrm{r}, \mathrm{t}) \quad (2)$$

Where α_i are exogenous factors that independently influence the cashflows, some examples of these factors are: total revenue, variable cost ratio, terminal market value, etc. [4].

Therefore, the impact changes of discount rate have on NPV could be represented by:

$$\frac{\partial NPV}{\partial r} \quad (3)$$

Similarly, that of α_i could be represented by:

$$\frac{\partial NPV}{\partial \alpha_i} \quad (4)$$

In reality, investors normally adopt a dynamic approach using a data spreadsheet [4], instead of the exact partial derivatives, to study the threshold value where the decision should be altered.

3. CASE STUDY

We use a case study of the Mahjong, Inc., an imaginary corporation, to demonstrate the sensitivity analysis of NPV [9-10].

Mahjong, Inc., has identified the following two investment projects.

Table 1. The cash flow of two projects of Mahjong Inc.

Project	CF0	CF1	CF2	CF3	CF4
A	-430	230	179	124	94
B	-430	70	138	240	260

The discussion of this case will be separated into two scenarios:

1) The analysts expect that the discount rate will remain 10% constantly
2) The analyst expects that the government will adopt an expansionary policy, reduce the interest rate during the issuance of CF2 and CF3, and result in a 10% decrease in the company's discount rate. Therefore, the discount rate of this project will become 10%, 10%, 9%, 8%, 8%, respectively the CF0-4.

4. RESULT

4.1. Fixed Discount Rate

We shall first examine the case when the discount rate is fixed at 10%. By the data above, we could calculate the NPV of the two projects, and it is clear that Project B yields a higher NPV than does Project A under the current discount rate of 10%.

$$\text{NPV}_A = \text{CF0} + \frac{CF1}{1+r} + \frac{CF2}{(1+r)^2} + \frac{CF3}{(1+r)^3} + \frac{CF4}{(1+r)^4}$$

$$= -\$430 + \frac{\$230}{1.1} + \frac{\$179}{1.1^2} + \frac{\$124}{1.1^3} + \frac{\$94}{1.1^4} = \$76.72 \quad (5)$$

$$\text{NPV}_B = CF0 + \frac{CF1}{1+r} + \frac{CF2}{(1+r)^2} + \frac{CF3}{(1+r)^3} + \frac{CF4}{(1+r)^4}$$

$$= -\$430 + \frac{\$70}{1.1} + \frac{\$138}{1.1^2} + \frac{\$240}{1.1^3} + \frac{\$260}{1.1^4} = \$95.99 \quad (6)$$

To take a step further and study the sensitivity of the two projects' NPV to the discount rate changes, we consider the projects' NPV under different discount rates and plot it against the discount rate.

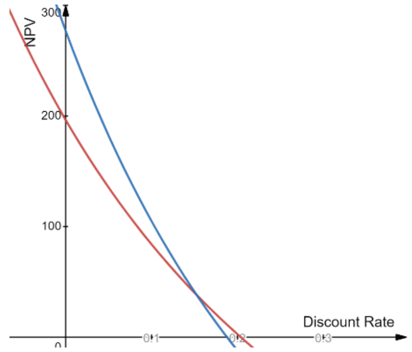

Figure 1 The graph of discount rate against NPV of the two projects

According to the figure, the increase of discount rate negatively impacts the projects' NPVs. Project A and Project B's NPVs turn negative when the discount rate is more significant than 20.44% and 18.84 %, respectively, and the two intersections with the x-axis are the internal rate of return (IRR) for the two projects. Therefore, if the investors predict a future increase in the interest rate, they should consider the possibility of the projects having a negative NPV when the interest rate is too high, and take corresponding preventive and mitigation actions. Moreover, we could see that Project B's NPV is more sensitive to the discount rate change as its graph has a steeper slope, within the interval of the discount rate larger than 0 and smaller than 30%, and thus more subject to interest rate risk.

To examine the sensitivity of the two projects' NPV to the changes of discount rate more rigorously, we take the derivative of the two NPVs for r, holding the cash flows constantly:

$$\frac{dNPV}{d\,r} = -\frac{CF1}{(1+r)^2} - \frac{2CF2}{(1+r)^3} - \frac{3CF3}{(1+r)^4} - \frac{4CF4}{(1+r)^5} \quad (7)$$

Thus,

$$\frac{dNPV_A}{d\,r}\Big|_{r=0.1} = -\frac{238}{(1+r)^2} - \frac{358}{(1+r)^3} - \frac{372}{(1+r)^4} - \frac{376}{(1+r)^5} = -953.2 \quad (8)$$

$$\frac{dNPV_B}{d\,r}\Big|_{r=0.1} = -\frac{70}{(1+r)^2} - \frac{276}{(1+r)^3} - \frac{720}{(1+r)^4} - \frac{1040}{(1+r)^5} = -1402.74 \quad (9)$$

Both derivatives are harmful, indicating a negative correlation between the NPV and the discount rate. Moreover, when the discount rate is less than 82.8%, the result of Project B has an enormous absolute value, showing that Project B is more sensitive to interest rate risk within this range.

4.2. Various Discount Rate

After examining the case where the discount rate is constant throughout the investment, let's look at the situation where discount rates vary across the year. Project discount rate is company-specific and is influenced by other exogenous factors, such as the interest rate. When government lowers the interest rate, companies could borrow at a lower rate, thus would have a lower discount rate, and vice versa. When calculating NPV of projects and making investment decisions, the analyst should also consider the possibilities that future cash flows have different discount rates than the current rate. Otherwise, any unexpected change in the discount rate after adopting the project could result in a loss in the investment.

Assume that the company expects the government to take an expansionary monetary policy. They expect the interest rate to be reduced at the issuance of CF2 and CF3, and they estimate the discount rate to be r, r, 0.9r, 0.8r, 0.8r, respectively for CF 0-4. Thus, the NPV of the two projects will become:

$$\mathrm{NPV}_A = \mathrm{CF0} + \frac{CF1}{1+r} + \frac{CF2}{(1+0.9r)^2} + \frac{CF3}{(1+0.8r)^3} + \frac{CF4}{(1+0.8r)^4} = -\$430 + \frac{\$230}{1.1} + \frac{\$179}{1.09^2} + \frac{\$124}{1.08^3} + \frac{\$94}{1.08^4} = \$97.28 \quad (10)$$

$$\mathrm{NPV}_B = CF0 + \frac{CF1}{1+r} + \frac{CF2}{(1+0.9r)^2} + \frac{CF3}{(1+0.8r)^3} + \frac{CF4}{(1+0.8r)^4} = -\$430 + \frac{\$70}{1.1} + \frac{\$138}{1.09^2} + \frac{\$240}{1.08^3} + \frac{\$260}{1.08^4} = \$131.42 \quad (11)$$

The calculation shows that under an expected expansionary monetary policy, projects will have a higher NPV.

Now take the derivative of the NPV under the new equation:

$$\frac{dNPV}{d\,r} = -\frac{CF1}{(1+r)^2} - \frac{1.8CF2}{(1+0.9r)^3} - \frac{2.4CF3}{(1+0.8r)^4} - \frac{3.2CF4}{(1+0.8r)^5} \quad (12)$$

Thus,

$$\frac{dNPV_A}{d\,r}\Big|_{r=0.1} = -\frac{230}{(1+r)^2} - \frac{322.2}{(1+0.9r)^3} - \frac{297.6}{(1+0.8r)^4} - \frac{300.8}{(1+0.8r)^5} = -862.3 \quad (13)$$

$$\frac{dNPV_B}{d\,r}\Big|_{r=0.1} = -\frac{70}{(1+r)^2} - \frac{248.4}{(1+0.9r)^3} - \frac{576}{(1+0.8r)^4} - \frac{832}{(1+0.8r)^5} = -1239.28 \quad (14)$$

The values of the derivatives show the sensitivity of NPV to the change of discount rate when r is around 10%. By the calculation, NPV is still negatively correlated with the increase in the discount rate, and when r=10%, project B is more sensitive to the change of discount rate than does project A. Moreover, by comparing it to the result of the previous sub-section, we could see that the absolute value of the derivative with an expected decreasing discount rate that with a constant one. This shows that with a desired expansionary policy and reducing interest rate, NPV reacts less to the change of interest rate, and hence is less sensitive to the discount rate change when expansionary monetary policy is expected.

5. DISCUSSION

The above session used the case of an imaginary company, Mahjong Inc. to study the sensitivity analysis of NPV under different predictions of future discount rates, constant discount rate or decreasing discount rate. Under both scenarios, the NPV has a negative correlation with the discount rate increase, given that the derivative of NPV with respect to the discount rate is always negative. Moreover, the NPV of project B is more sensitive to the change of discount rate than does project A, because project B has more significant cash flow in later stages with a higher discount factor, resulting in a larger effect on the change of NPV when the discount rate varies.

Meanwhile, when there is an expected change in the discount rate, the sensitivity of the projects' NPV to the change of discount rate is different. The project is less sensitive to the change of discount rate when the discount rate is lower in later time periods, presumably due to an expected expansionary monetary policy. This is due to the fact that a lower discount rate gives lower weight on the cashflows in terms of the impact they have on the overall changes of NPV; thus, when the current discount rate increases, the NPV decrease less if the future discount rate is lower than the current one.

The result indicates that project NPV might have different sensitivity to the discount rate change and thus different tolerance to risk when the expected discount rates used to calculate the NPV vary. This shows the importance of doing sensitivity analysis to the NPV. By taking into consideration of the potential changes in the discount rate, the firm could take more accurate mitigation methods targeting the potential risks and minimize the negative impact the future change might bring to the investment project.

There are certain limitations of this study. One limitation is that it lacks the support of real-world companies' data. The discount rate data is company-specific and determined by the company considering numerous factors. The information gathered online is normally insufficient and not comprehensive enough to generate an accurate estimation to be used for the NPV calculation, let alone the sensitivity analysis of NPV.

Further study could be done in the future by cooperating with companies and collecting more accurate and comprehensive data. With data covering a longer time period, the impact of historical interest changes had on NPV of investment projects could be better studied. Meanwhile, a more sophisticated model could be built, with more factors incorporated into the system. This could help us to understand how different factors independently influence the discount rate or cashflows and therefore impact the NPV and investment decision.

6. CONCLUSION

This paper has presented how sensitivity analysis could complement the NPV method in making investment decisions under different scenarios of discount rate risk. The research was demonstrated based on the case of an imaginary company, using the sensitivity analysis method of partial differentiation. The study showed the NPV's sensitivity to change of interest rate under different predictions of interest rates and compared the sensitivity of NPV of various projects.

The study showed that the project's NPV is negatively related to the change of discount rate; when the projects' cashflows are more concentrated in the future part of the project, the project is more sensitive to the discount rate changes. Moreover, if there is a predicted decrease in interest rate in the future and the project has a lower discount rate in the future period than in the current period, the project is less sensitive to the change in discount rate than other projects having the same constant current discount rate.

Unlike the typical way of using a data spreadsheet to study NPV's sensitivity, this paper gives a more rigorously yet holistic view. Future studies could be done by cooperating with real-world companies with more accurate and comprehensive data to demonstrate the analysis better.

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