

Industry Diversification and Optimal Industry Combination of Portfolios

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ABSTRACT

The importance of asset diversification has been tested and proven many times in the past few decades. Investors can allocate their money into different asset classifications, such as stocks, golds, and funds. Undoubtedly, returns and risks vary across different types of assets. A huge number of risk lovers are keen to invest in stocks and equity funds due to their fascinating revenues. However, the equity market covers a wide variety of industries. How could one reasonably allocate assets to obtain a fair gain with low risks? We aimed at figure out the optimal combination of 20 industries that construct minimum variance, maximum Sharpe ratio, and maximum drawdown portfolios separately among 64 industries in China. Basically, we gathered industries index data in 2014 and 2015 and utilized Excel functions to do our experiment. Eventually, 7 industries showed up in both years' minimum variance portfolios. If certain industries appeared in minimum variance portfolios over and over again, these industries usually had relatively low risks and could offer suggestions for investors. Some implications could also be found from maximum Sharpe ratio and maximum drawdown experiments.

Keywords: industry diversification; minimum variance portfolio; industry combination, maximum Sharpe ratio portfolio; maximum drawdown portfolio.

1. INTRODUCTION

1.1. Background:

As we all know, eggs cannot be put in the same basket, and the essence of this behavior is diversification of risk. Investment, without doubt, is a risky behavior, and the optimal investing strategy is to make a portfolio to maximize return and minimize or even avoid risk for investors. Portfolio optimization is the process of selecting the best portfolio, out of the set of all portfolios being considered, according to some important factors. Therefore, if we can find the optimal portfolio, the investment risk will be reduced dramatically, meaning that the investment efficiency can be improved. In this paper, we mainly focus on finding out the optimal portfolio. Specifically, we select twenty industries from sixty-four industries in China to form the industry with the smallest fluctuation, the largest Sharpe

ratio, and the maximum drawdown, forming the optimal portfolios. All these three factors are important standards to measure whether the specific industry is worth investing in. The purpose is to offer non-professional investor some useful investment reference, and it can also minimize the risk of investment. To be more specific, there are thousands of industries in China, so it is difficult and complicated for investors to determine which industry they can invest. Especially during economic crisis, investment in industry in Chinese market was almost depressed, so the profitable investment became more difficult for investors with increased risk. By comparing the index, including fluctuation, Sharpe ratio, and maximum drawdown, the investors can choose the industry with low fluctuation and minimize the risk. We also examined how the economic crisis can affect certain industries. Under this circumstance, investors have a basic understanding of the Chinese market, so they can choose the appropriate industries to invest according to fluctuation, Sharpe

ratio, and maximum drawdown. It can also provide the investors with the reference about industry functions. For example, if the investors want to invest in stock, manufacturing, and textile, they can have a better understanding of allocating funds in each industry.

1.2. Motivation:

The three most important indicators in our paper are variance, Sharpe ratio, and maximum drawdown. Maximum drawdown is a crucial indicator to measure strategic risk, which implies the largest loss the investor might face. Sharpe ratio is the measure of excess return beyond the risk-free rate of return for each unit of risk. Without doubt, the main purpose of investing is to obtain the maximum return, but profits and risk coexist. Under this circumstance, choosing the optimal portfolio to maximize return and to minimize risk is something that many investors should consider. As we all know, Chinese investment market is complicated, and different investors have different demand for investment. For instance, different individuals have different risk tolerance, so risk lovers can choose a portfolio with higher risk and higher returns.

However, risk aversion can choose a relatively conservative portfolio according to their demand. There are many factors that can influence the market, such as sources, or public policy. For instance, the prospect of education industry depressed after the Double Reduction Policy, and the real estate industry also faced many problems. However, by calculating these three important factors, the investors can have a better understanding of the stability of the industry directly. After that, investors can obtain more information based on these three indices, deciding whether to invest or not. It also provides investors with the opportunity to realize the challenges and opportunities in this industry. Meanwhile, fluctuation reflects whether the industry is stable or not. Education industry, for example, has high Sharpe ratio and fluctuation rate under the double reduction policy, so the investors can avoid choosing this industry to minimize the risk of investment. Thus, based on the three important indices, we can analyze some industries with higher risk and some industries with lower risk. The investors can choose the portfolio with highest returns at the maximum risk they can afford. Portfolio optimization, at least, offers investors with the opportunity to minimize the risk of investment, and this is also the motivation for us to conduct this research.

1.3. Result:

In conclusion, we have figured out 3 optimal combinations of 20 industries that construct minimum variance, maximum Sharpe ratio, and maximum drawdown portfolios separately among 64 industries in

China. Basically, we gathered industries index data in 2014 and 2015 and utilized Excel functions to do our experiment. Finally we find optimized portfolio only containing less than 10 industries, which indicates that constructing portfolio by a single criterion tends to concentrate investment on a few industries. As a result, in order to diversify our investment, we should take more factors into consideration.

1.4. Contribution:

A large number of previous studies have examined the relative importance of country factors in determining equity returns and concluded that country factors prevail over industry factors.[1] To examine the correctness of this finding, some experts gathered industry index data from 21 developed countries and presented evidence to show diversification across global industries providing more risk reduction than country factors. They calculated the Sharpe ratios of different countries, such as Japan, Australia and Britain, and Sharpe ratio of different industries, such as life insurance and commercial banking. Finally, they found that the Sharpe ratios of industry factor combinations were higher than those of national factor combinations, reaching a conclusion that portfolio diversified across industries brought a higher return to risk ratio. An alternative way to examine the performance of industry-based portfolio and country-based portfolio is to compare portfolio's volatility. By using the same data, they discovered that the volatility of industry-based portfolio was obviously smaller, reflecting a lower risk.[2] These studies have showed how industry factors mattered for an investor and paved a solid foundation for our research. Therefore, on the basis of these studies, we would explore more about the industry factors in Chinese financial market. How could investors allocate their assets across different industries to acquire a higher return to risk ratio? Which industries could they avoid in a specific time? These are the main objectives we intend to explore.

2. METHOD

2.1. Data

The data needed for this study are: the 64 industry indexes data in the Chinese stock market, including monthly return rate of these industries and monthly risk-free return rate. In this paper, we selected the sampling interval from January 2014 to December 2015. All the industry indexes were downloaded from Straight flush (<http://www.10jqka.com.cn/>). Industry index data includes almost all industry categories in the Chinese financial market, which can improve the representatives of our data.

(1) *Sharpe ratio*

Sharpe ratio inherits modern portfolio theory about mean and variance. The basic idea of this ratio is to use the ratio of the fund's income in a period of time and the standard deviation of income as the index to measure fund performance. The Sharpe ratio in a broad sense can be reflected by formula (1). $SF = RF - \sigma F$ [1]

RF is the return of the fund over a period of time, and σF is the standard deviation of the return of the fund over the period. This measure describes the level of return per unit of risk (the standard deviation of return). In investment practice, people tend to compare investment returns with some benchmarks, such as risk-free interest rate and market index returns, which to some extent represent the opportunity cost of investment. In order to more clearly show the difference among returns of the fund and the benchmark, another widely used form of Sharpe ratio is calculated by the excess return and standard deviation of the fund relative to the benchmark, with RB representing the return of the benchmark over the same period: If σF uses the returns of a market index as a benchmark, the Sharpe ratio represents the basis. The excess return of exponential investment represents the return of the fund on securities selection, which can be expressed as:

$$SF = RF - RB - \sigma F [4]$$

(2) *Maximum drawdown*

Maximum drawdown is a measure of risk control ability. It refers to the peak of the fund net curve to the bottom of the largest. The maximum drawdown of the fund cannot be measured by a single product, so we need to consider many factors. [5]

Maximum drawdown is an important risk index: it reflects the risk control level of the manager, forms the investment experience of investors and determines the returns of investors.

Suppose that we have an underlying asset whose price process at time t is given by St . For example, the price process could be a stock price, index, interest rate or exchange rate. Denote by Mt its running maximum up to time t : (1) $Mt = \max_{u \in [0, t]} Su$

Drawdown Dt is defined as the drop of the asset price from its running maximum: (2) $Dt = Mt - St$.

Maximum drawdown $MDDt$ is defined as the maximal drop of the asset price from its running maximum over a given period of time: (3) $MDDt = \max_{u \in [0, t]} Du$

Assume that you have an investor who enters the market at a certain point and leaves it at some following point within a given time period. Maximum drawdown measures the worst loss of such an investor, meaning that he buys the asset at a local maximum and sells it at the subsequent lowest point, and this drop is the largest

in the given time period. This represents the worst period for holding this asset and could be written mathematically as:

$$(4) MDDt = \sup_{qu \in \{0, 1\}} (-qu) dSu$$

one switch ($0 \rightarrow 1$) and ($1 \rightarrow 0$)

Similarly, we can define the concepts of draw up, and maximum draw up. Draw up Ut is defined as the

increase of the asset price from its running minimum:

$$Ut = St - mt,$$

where

$$(6) mt = \min_{u \in [0, t]} Su$$

Maximum drawup $MDUt$ is given by

$$(7) MDUt = \max_{u \in [0, t]} Uu [3]$$

(3) *Minimum variance*

The risk of a portfolio can be measured by the fluctuation of the stock, the greater the volatility of the portfolio means greater risk. Bali found a negative correlation between volatility and expected returns. [6] For the relationship between the return rate of stock portfolio and its volatility, Haugen and Baker's empirical results show that according to market value plus weighted market portfolios are not efficient, in other words, we can find a portfolio with the same return as the market but less volatility. [7] In this paper we use variance σ to measure the risk coefficient of our portfolio. Since portfolio can disperse the risk of a single stock, the volatility of a single stock cannot constitute the risk of a portfolio through linear portfolio, more studies use covariance matrix as a measure of risk for further analysis. [8] Numerous domestic and foreign empirical evidence shows that, [9] the abnormal returns of stocks are mainly related to market value, value (book-to-market ratio) and momentum, [10] three factors are involved. Considering that there is no monthly momentum effect in China's stock market, therefore, this paper only considers from the perspective of yield. When the expected return of the portfolio is given, the weight of individual assets in the portfolio can be adjusted to get minimum risk. In Chinese stock market the short selling is not allowed, so we do not consider short buying in our portfolio. We set the sum of the weights of all assets is 1, and weight $x \geq 0$ indicates that the weights of all assets are non-negative (i.e., short selling is not allowed), then through optimizer exercise which changes the weight of each industry in portfolio, we can get the minimum variance portfolio.

2.2. Method

These data were utilized to calculate the covariance between each two industries. To find out the optimal combination of industries that construct to a minimum

variance portfolio, we analyzed and took advantage of portfolio variance formula first. Below is the portfolio variance formula 2: $\sigma_p^2 = \sigma^2(R_p) = \sum_{i=0}^n \sum_{j=0}^n W_i W_j Cov(R_i, R_j)$. Assume each industry takes up 5% of the portfolio. For easily calculating, the impact

of weight was ignored. Then we utilized excel data analysis to calculate covariance between industries and obtain a covariance matrix. Below are some results from our experiment.

Table 1 Covariance matrix of 6 industries

	Food processing and manufacturing	Retail	Building materials	Traditional Chinese medicine	Beverage manufacturing	Bus
Food processing and manufacturing	0.001454843					
Retail	0.001477015	0.001826186				
Building materials	0.001425714	0.001384327	0.002032786			
Traditional Chinese medicine	0.001465058	0.001675662	0.001191831	0.001954101		
Beverage manufacturing	0.001265748	0.001428233	0.000753141	0.00174651	0.002373296	
Bus	0.001073907	0.001238685	0.00135734	0.001008799	0.000519082	0.002423857

Assume each industry takes up 5% of the portfolio. For easily calculating, the impact of weight on portfolio variance was ignored. We simply multiply the covariance between industries and industry variance

itself to find the optimal combination. The final 20 industries that construct a minimum variance portfolio is shown in Table 2.

Table 2 20 industries construct minimum variance portfolio in 2014

Food processing and manufacturing	Retail	Building materials	Traditional Chinese medicine	Beverage manufacturing
0.15%	0.18%	0.20%	0.20%	0.24%
Packaging and Printing	Railway transportation	Logistics	Building Decoration	Gas and Water
0.27%	0.27%	0.28%	0.28%	0.29%
Bus	New chemical materials	Real estate development	White goods	Paper industry
0.24%	0.24%	0.25%	0.25%	0.26%
Clothing home textiles	Chemical pharmaceutical	Chemical synthetic materials	Trade	Whole car
0.29%	0.30%	0.30%	0.31%	0.31%

Then we use the same method to get the minimum variance portfolio with 2015 data. The final 20 industries and their variances are shown in Table 3.

Table 3 20 industries construct minimum variance portfolio in 2015.

Bank	Whole car	Gas and Water	Insurance and others	Beverage manufacturing
0.60%	0.80%	1.13%	1.19%	1.31%
Coal mining and processing	Building materials	Retail	Basic chemical	Electricity
1.46%	1.50%	1.60%	1.60%	1.64%
Logistics	Transportation equipment service	Traditional Chinese medicine	Food processing and manufacturing	Railway transportation
1.67%	1.68%	1.73%	1.73%	1.75%
Forestry and Plantation	Extraction service	Non-ferrous smelting processing	Steel industry	Aquaculture
1.76%	1.81%	1.81%	1.89%	1.91%

Finally, we compared the portfolio results of 2014 and 2015 and found that there were 8 industries included in both years' portfolios.

Table 4 Comparison of 2 portfolios

Food processing and manufacturing	Retail	Building materials	Traditional Chinese medicine	Beverage manufacturing
Bus	New chemical materials	Real estate development	White goods	Paper industry
Package and Printing	Railway transportation	Logistics	Building Decoration	Gas and water
Clothing home Textiles	Chemical and pharmaceutical	Chemical synthetic materials	Trade	Whole car
Bank	Whole car	Gas and Water	Insurance and others	Beverage manufacturing
Coal mining and processing	Building materials	Retail	Basic chemical	Electricity
Logistics	Transportation equipment service	Traditional Chinese medicine	Food processing and manufacturing	Railway transportation
Forestry and Plantation	Extraction service	Non-ferrous smelting processing	Steel industry	Aquaculture

Table 5 Industries' monthly close price and maximum drawdown

Industry	Other electronics	Medical equipment service	Electronics manufacturing	Media	New materials	Agricultural service
January	1385.518	3278.041	1598.861	1864.141	1874.889	2071.073
February	1505.31	3616.919	1568.936	1741.163	2036.608	1999.469
March	1404.361	3540.603	1431.128	1580.136	1932.944	1826.217
April	1378.739	3286.051	1434.08	1573.654	1913.057	1758.393
May	1424.974	3432.406	1541.842	1629.931	1995.216	1796.812
June	1536.252	3740.972	1652.063	1813.097	2170.252	1883.326
July	1612.665	3501.067	1651.693	1719.897	2451.806	2023.264
August	1818.911	3770.466	1798.651	1898.063	2494.031	2213.035
September	2067.96	4321.652	2010.459	2053.082	2772.957	2452.912
October	2179.84	4528.151	2030.482	2078.554	2783.24	2475.797
November	2381.028	4359.107	2149.589	2090.388	2916.474	2507.194
December	1955.036	3793.32	1797.891	1880.796	2448.213	2262.449
Max drawdown	17.89%	16.23%	16.36%	15.58%	16.06%	15.10%

Maximum drawdown means the maximum loss from a peak to trough during a specific period. We collected monthly close prices for each industry from 2014 to 2015 to calculate their maximum drawdown rate. For instance, the maximum drawdown for medical equipment service happened in November and

December, which was $(4528.151 - 3793.32) / 4528.151 * 100\% = 16.23\%$. We used the same method to calculate all 64 industries' maximum drawdown and summarized 20 industries which had the highest drawdown in 2014, listing them in table 6.

Table 6 20 Maximum drawdown industries in 2014

other electronics	electronics manufacturing	medical equipment service	new materials	media	extraction service	Optical Electrons	Agricultural services	Computer Application	Environmental engineering
17.89%	16.36%	16.23%	16.06%	15.58%	15.32%	15.28%	15.10%	14.63%	13.71%
Transportation equipment service	Computer equipment	auto parts	Communication service	Communication equipment	Household Light Industry	Semiconductor and original	General Equipment	Aquaculture	financial securities
13.68%	13.53%	12.96%	12.48%	12.29%	11.95%	11.92%	11.58%	11.38%	10.76%

We also analyzed the period that maximum drawdown happened for these 20 industries. The same method was used with 2015 data. And 20 industries

which had the highest drawdown were listed in table 7 and table 8.

Table 7 Maximum drawdown time

other electronics	electronics manufacturing	medical equipment service	new materials	media	Extraction service	Optical Detector ranks	Agricultural services	Computer Application	environmental engineering
dec	dec	nov dec	dec	feb-apr	feb-apr	sept-dec	feb-apr	dec	march april
Transport	Compute	auto	communi	Com-	Househol	semicond	General	Aquacult	financial

ation	r	parts	cation	communi	d d Light	uctor and	Equipmen	ure	securities
equipme	equipme		service	cation	Industry	original	t t		
nt service	nt			Equipme					
				nt					
dec	march	dec	feb-apr	dec	dec	dec	dec	dec	feb
	april								march

Table 8 20 Maximum drawdown industries in 2015

bond	communi	oil	optical	Optical	steel	computer		Nonferro	
	cation	mining	electronic	Electronic	industry	applicatio	trade	us	new
	services		s industry	s		n) n		smelting	material s
						industry		and	
54.07%	53.40%	52.94%	46.73%	46.73%	46.68%	46.28%	45.97%	processin	g
Commun	audio-	Semicon	computer	mining	instrume	New	Chemical	Chemical	Airport
ication	visual	ductor	equipme	and	ntation	chemical	products	synthetic	shipping
apparatu	equipme	and	nt	processin		materials		materials	
s	nt	original		g					
44.06%	43.09%	42.94%	42.92%	42.41%	40.36%	40.25%	38.67%	37.88%	36.53%

Table 9 20 industries with highest Sharpe ratio

Special	Securities	Pharmaceutical	building materials	building
		business		decoration
3.30%	9.60%	4.01%	3.08%	3.56%
Bank	Instrument and	Logistics	Trade	Auto Parts
	meter			
4.18%	3.67%	3.51%	4.02%	4.49%
Gas water	Insurance	and	Packaging	and
	Other		and	Electric power
2.74%	4.63%	2.90%		Electrical equipment
Real	estate	Non-automobile	The bus	Port of shipping
development	delivery			Rail transportation
3.39%	4.13%	2.70%	4.33%	4.57%

In the maximum Sharpe ratio portfolio we plan to focus on the Sharpe ratio, for all those industries, we calculated the Sharpe ratio for each industry. Then we include 20 industries in our portfolio, regardless of scale, we rank these 64 industries by Sharpe ratio and choose top 20 industries with the maximum Sharpe ratio. This analysis is displayed in figure 9..

In order to simplify the model and focus on industry characteristics research, We calculated the average interest rate on China's 10-year government bonds in 2014 (a constant) as the risk-free rate, in figure 1.



Figure 1 China 10-year bond average interest rates

Initially all industries have the same weights, further to maximize the Sharpe ratio of our portfolio, we change the weight of each industry. Then by analyzing the weights of industries in the optimized portfolio, we can see which industries are worth investing in 2014. We also did the same research with the data in 2015, to find if these “best industries” are still have a good performance.

3. RESULT AND DISCUSSION

Table 10 Final output of minimum variance portfolio

Industry	Building materials	Beverage manufact uring	Real estate developm ent	Railway transportat ion	paper industry	Packaging and Printing	bus
weight	28.09%	8.53%	13.23%	16.16%	0.62%	10.36%	23.01%

We could clearly see that there were only 7 industries included in the final output. Based on our discussion, we believed that this was due to two reasons. First, there were only two restrictions, and we didn't take other things, like return, into consideration. Also, since there were 20 industries in this portfolio, some would be excluded for the optimal result.

For the equal weighted portfolio, the monthly expected return is 4.01%, and the Sharpe ratio in this case is 0.810. Then we use solver function to maximize the expected Sharpe ratio by changing the weight on each industry. The maximum Sharpe ratio we get is 1.054, and the monthly expected return in this case is 4.47%.

Table 11 Output with 2014 data

Equally weight result	
Sum of weights	1
Portfolio expected return	4.01%
Portfolio sd	4.53%
Expected Sharpe ratio	0.810
Max Sharpe ratio result	
Sum of weights	1
Portfolio expected return	4.47%
Portfolio sd	3.91%
Expected Sharpe ratio	1.054

We find there are five industries in the optimized portfolio, and they are securities, bank, auto parts, bus and rail transportation. We find that these industries are all related to finance and transport. The weight of each industry was displayed in table 12.

3.1. Result

Then we want to do further study to find the real minimum variance of the portfolio by changing weights of each industry. All calculations are based on formula 3: $\sigma_p^2 = \sigma^2(R_p) = \sum_{i=0}^{20} \sum_{j=0}^{20} W_i W_j Cov(R_i, R_j)$. We utilized Excel solver to test this by changing weight cells. There are 2 constraints: first, the sum of weights equals to 1; second, all weight cells are greater than 0. The solver output for this test is listed in tale 10.

Table 12 Weight of each industry

Securities	0.015808264
Bank	0.225873045
Auto parts	0.489500483
The bus	0.027909023
Rail transportation	0.240909186

The results of the same exercise with 2015 data can be seen in table 5. The maximum Sharpe ratio we get is 0.907, and the monthly expected return is 10.72%.

Table 13 Output with 2015 data

Equally weight result		
Sum of weights		1
Portfolio expected return		10.14%
Portfolio sd		14.90%
Expected Sharpe ratio		0.657
Max Sharpe ratio result		
Sum of weights		1
Portfolio expected return		10.72%
Portfolio sd		11.44%
Expected Sharpe ratio		0.907

Only building materials, building decoration, and Real estate development were included in the optimizer portfolio. Their weights were listed respectively.

Table 14 Weights of 3 industries

Building materials	0.482709413
Building decoration	0.415506133
Real estate development	0.101784455

3.2. Discussion

From the results we can learn that in 2014 we can get relatively high returns for the risks they take by investing in finance and transport. While these industries did not have a good performance in the next year. The industries with the highest Sharpe ratio is building and real estate. Another difference in 2015 is that the average return is much higher than that of the previous year, but the Sharpe ratio did not improve. We guess this is because Chinese stock market experienced a financial crisis in 2015, so many investors choose to invest in real estate instead of stocks.

4. CONCLUSION

In conclusion, we have figured out 3 optimal combinations each contains 20 industries. At first we calculate the The average historical monthly returns, standard deviation, and Sharpe ratio for each month. Then we construct minimum variance, maximum Sharpe ratio, and maximum drawdown portfolios separately among 64 industries in China. Basically, we gathered industries index data in 2014 and 2015 and utilized Excel functions to do our experiment. We find that the minimum variance portfolio mainly includes the consumer beverage industries, that electronics industry has the largest portion in the maximum drawdown portfolio and that maximum Sharpe ratio portfolio mainly includes finance and transport industries. Finally we find optimized portfolio only contains less than 10 industries, which indicates that constructing portfolio by a single criterion tends to concentrate investment on a few industries. As a result in order to diversify our investment, we should take more factors into consideration. Investors who want to choose a portfolio based on a certain metric, such as volatility, Sharpe ratio, etc., rather than focusing on a particular industry may be concerned by our finding. The results of our research help to study the characteristics of different industries in China's financial market.

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