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An Empirical Portfolio Study Based on Markowitz Theory

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ABSTRACT
With the further improvement of the financial system and the arrival of the era of big data, the application of new technologies in the financial field is deepening. Based on Markowitz portfolio theory, this paper using "Mean – Variance Model" select three funds and T-bills as risk-free rate in the market for portfolio investment analysis. Through empirical analysis, we obtain the optimal portfolio with the largest Sharpe ratio and the optimal portfolio with the smallest variance and conduct comparative analysis on their expected return rate, standard deviation and Sharpe ratio. The obtained results further explain that Markowitz theory plays an important role in selecting optimal portfolio in financial risk management. At the same time, this paper lays a foundation for the practice and development of R software in the field of financial analysis.

Keywords: *Markowitz mode, Rstudio, Sharpe ratio, Optimal portfolio*

1. INTRODUCTION

Under the background of economic globalization, investors are increasingly using their idle funds to invest in stocks, funds and other financial products to obtain higher returns. With increased investing in frequency and expertise, investors recognize that in the securities market, they can't just focus on the return without considering the risk. Rational investors should try to minimize risk while maximizing projected returns. Portfolio investment is a typical investment management method. Investors pick various types of funds from the market and then diversify their portfolios of risky assets. These actions reduce the non-systemic risk associated with having highly linked funds, so that minimize the risk in investment management.

Modern Portfolio Theory, which was established on Markowitz's pioneering work in 1952, is a fruitful study area supported by both academics and practitioners. Markowitz's (1952) basic paradigm focuses on the tradeoff between mean anticipated return and variance of the return [1]. He used probability theory and quadratic programming to solve the problem of portfolio selection. In his paper, under the assumption that prohibiting short selling and no risk-free lending, this theory proposes quantifying the expected return and investment risk of a portfolio respectively using the mean and the variance. The goal of the investment decision is to find the portfolio with the lowest investment risk or the highest investment return at the same risk level. At the same time, the portfolio should be within the efficient frontier. For the first time, starting from the relationship between risk asset return rate and risk, he discussed and determined the optimal portfolio selection in the economic system with the center of risk diversification. Therefore, there are a considerable amount of researches and practical applications on investment portfolio optimization in various fields based on portfolio theory. By randomly selecting five stocks in the stock market, Xiaomin Wang explained how to apply Markowitz theory from practice [2]. Zola's paper discussed the problem that how investors choose investment strategies in the process of investment decision making based on the mean-variance model [3]. Juha Joenväärä, Mikko Kauppila and Hannu Kahra studied the portfolio selection methods of hedge funds from the perspective of their own and investors' overall portfolio [4]. In Junlan Chen's article, she assumed that the average rate of return and the covariance matrix of each portfolio were known (the average rate of return is usually estimated from historical data), and then constructed the efficient optimal portfolio [5]. Yang Li and Lixia Liu selected 40 securities in Chinese financial market as empirical samples to reveal the investment diversification effect by constructing the optimal portfolio [6]. Yinmiao Zeng, Jun Zhang and Qin Zhang calculated the effective portfolio set under the condition that the mean return rate and covariance matrix of all securities of the portfolio were known [7]. In Guangming Luo and Yong Liu's paper, combining with the actual situation of the Shenzhen and Shanghai stock markets, the Markowitz model was applied to conduct an empirical study on all aspects of the actual market, which proved that the model has obvious advantages in avoiding risks and draws many conclusions that can be reflected with the actual situation of the stock market [8]. Nathapan S's study explored the effect of estimation risk on an admissible efficient set and an optimal portfolio based on a Bayesian framework assuming diffuse prior and informative conjugate prior distribution functions [9]. There were some modern applications of Markowitz approaches which featured in Guerard J's article. This article was written by some of the most prominent scholars and practitioners in the field today. Portfolio selection, data mining testing, and multi-factor risk modeling were topics covered [10].

Moreover, some have further developed Markowitz's theory of finding an optimal portfolio. For instance, in Mahboubeh Shadabfar and Longsheng Cheng's paper, the unpredictability of hazardous assets was handled through a probabilistic optimization problem in a probabilistic variant of the portfolio selection problem [11]. For a multi-periods portfolio optimization issue, Xiaoyue Li, A. Sinem Uysal and John M. Mulvey used model predictive control. They designed a portfolio whose allocation is determined by modeling predictive control with a risk-parity objective, and presented a sequential convex program technique that yields 30 times quicker and more robust solutions in the trials with the exception of the mean-variance target [12]. In Junwen Xiang's paper, the basic framework of portfolio based on covariance proposed by Markowitz aimed to diversify investment risk through unrelated investment products [13]. Shuli Li studied the Markowitz portfolio model with mean and variance changes at the same time and analyzed the effect of mean and variance changes on the effective frontier curve [14]. In Shanming Li and Pei Xu's paper, he used Markowitz model, single index model and EGP model to calculate the effective portfolio of a specific stock sample at a specific time, measured the risk variation law of stock portfolio, and calculated the return and variance of random equalized stock portfolio [15]. Elton E J , Gruber M J and Padberg M W showed two assumptions that a person was willing to accept the existence of a riskless asset. And then, they tried to find the best answer to the portfolio problem and utilize a simple choice criterion [16]. In Yin G and Zhou X Y's paper, the market characteristics were dependent on the market mode (regime) that jumps between a finite number of states in a discrete-time form of Markowitz's mean-variance portfolio selection issue [17]. In Zhou，X Yu，Yin，G.'s paper, for a market with one bank account and many stocks, a continuous-time variant of the Markowitz mean-variance portfolio selection model was developed and examined [18]. On the basis of Markowitz's mean-variance portfolio optimization theory, Leung P L, Ng H Y and Wong W K's paper deduced the explicit formula of optimal portfolio return estimation, which was significantly better than the traditional estimation method in terms of optimal return and its distribution [19].

When analyzing the optimal portfolio, most of articles used Excel software to calculate the variables of funds and result comparison, such as the mean value, the variance, correlation coefficient, Sharpe ratio and so on. EXCEL is a strong office program. There are also some studies that further verify Markowitz model with the help of other data analysis tools. For example, Libo Sun used Python tools to find the portfolio with the minimum risk or the largest Sharpe ratio in the portfolio and the effective boundary based on Markowitz theory [20].

However, it has a low computational performance. Complex computations take a long time to complete, and the computer itself is prone to failure. The lack of portability between systems is also a problem. There are certainly functional differences between various versions of EXCEL files, and the VBA computing programs sometimes fail to work after switching computers owing to incompatibility. In addition, EXCEL is not a data science tool, so it is difficult to meet the requirements for data processing power in the current era of big data.

Our paper was impaired by Libo Sun's article with the help of Python tools to calculate the expected return, standard deviation and Sharpe ratio of the portfolio, so as to find the optimal portfolio. Hence, this paper studies an application of Markowitz portfolio theory in stock market optimal portfolio selection using Rstudio. R is an open-source statistical computing software that includes a full suite of data processing, calculating, and mapping tools. Its statistical analysis and drawing functions are both outstanding, and it may be quite useful in resolving financial issues. The complicated computation of covariance and correlation coefficient between numerous assets may be solved more efficiently and rapidly when using R software to calculate the optimum portfolio. Also, there are some researches reveals that in Rstudio, the optimal portfolio can be generated fast, and the calculation part can be made easier and more intuitive. As Rong Gao discussed in her article, R software should be used for model analysis and the application of it in investment class should be promoted [21]. In Würtz, Diethelm' book, he described many ways to get the optimal portfolio using formulas written in R software [22]. This new technology is extremely useful for applying Markowitz's portfolio theory to the financial market.

2. METHOD

2.1. Related Concepts

The standard normal distribution test is devoted to determining if the data of funds is drawn from the whole normal distribution. If the test fails, logarithmic transformation or data replacement is required. The normal distribution is a probability distribution. It's also known as the Gaussian distribution after Carl Friedrich Gauss, who was the first to discover it. The normal distribution is a continuous probability distribution that is used in a variety of scientific domains. A family of distributions with the same general form is known as a normal distribution. The general form of its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (1)$$

In this paper, the standard normal distribution was used to compare with the returns of these funds. The standard normal distribution also called unit normal distribution. When μ = 0 and σ = 1, this is a specific instance that is defined by the probability density function:

$$\boldsymbol{\varphi}(x) = \frac{\mathrm{e}^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \quad (2)$$

The factor $1/\sqrt{2\pi}$ in this case ensures that the entire area under the curve (x) is one. The exponent 1/2 assures that the distribution has unit variance (i.e., variance equal to one) and, as a result, unit standard deviation. This function contains inflection points at $x = +1$ and $x = -1$ and is symmetric around $x = 0$, where it reaches its maximum value of $1/\sqrt{2\pi}$.

A collection of random variables is considered independent and identically distributed in probability theory and statistics if each random variable has the same probability distribution as the others and is mutually independent. i.i.d., iid, or IID are common abbreviations for this attribute. Because it is the most common, the term i.i.d. is used here.

Assume that the random variables X and Y are defined to have values in the range of I ∈ R. Let $F_X(x) = P(X \le x)$ and $F_Y(y) = P(Y \le y)$ be the cumulative distribution function.

In general, if we say that the two random variable X and Y are i.i.d., which means that each one is independent of all the others (i), and they all have the same probability distribution (i.d.).

In order to find the efficiency frontier of portfolio by the mean and the variance of the return of each fund, the "Mean – Variance Model" will be used in our paper. Suppose that the investor invests in a portfolio of N risky assets in a single investment period, and then the expected return rate of the portfolio is:

$$\mathrm{E}(r_p) = \sum_{i=1}^{n} x_i E(r_i) \quad (3)$$

In this case, r_i represents the expected return rate of the ITH asset, x_i means the investment proportion of the ITH asset.

As an indicator that can concurrently consider the benefits and risks, the higher the Sharpe ratio, the higher the return for the unit risk of the fund.

$$\text{Sharpe ratio} = \frac{r_p - r_f}{\sigma_p} \quad (4)$$

In this formula, r_p = Expected annualized rate of return on investment portfolio, r_f = Annualized interest rate on risk-free assets and σ_p = Standard deviation of annualized return on investment portfolio.

2.2. Methodology

This paper selected nine different funds which had the market risk and the Treasury bonds as risk-free rate, firstly, two assumptions (the normal distribution and the I.I.D sample) of these 10 funds were considered by using every month's historical data in the past 15 years. Secondly, selected three funds of nine funds and randomly constructed 8 portfolios as one observation group with the sum of weights in each portfolio equal 1 (not included T-bills). Also, regarded another 8 portfolios of these three funds and T-bills with random weight as another observation group. Thirdly, two optimization methods: the Sharpe ratio maximization and the variance minimization were used to find the optimal portfolio weight parameters, and calculate the two kinds of optimal portfolio expected return and Sharpe ratio.

2.3. Data collection and Pre-Processing

Rstudio is an "Interactive Development Environment" (or IDE for short) that supports R programming. Started by creating a new Rstudio file and then imported the original data and pre-processed it. Below are the codes.

Collected nine representative funds with the market risk involved drefus, fided, keystone, putnminc, scudic, windsor, eqmrkt, valmrkt, mkt and the Treasury bonds as risk-free rate and obtained the daily return of data of 180 months from January 1, 1968 to December 1, 1982. Using apply function to figure out the mean returns on annual basis. Table.2 shows the name of these 10 funds and the annual return.

Table 1 The mean returns of ten funds on annual basis (%)

Fund names	drefus	fidel	keystne	putnminc	scudinc
Annual mean	8.120	5.636	7.851	6.620	5.319
Fund names	windsor	eqmrkt	valmrkt	mkt	T-bills
Annual mean	12.026	12.990	8.176	8.423	7.174

*The decimal point retains three significant digits

Three kinds of funds from the above nine funds with higher annual mean returns were screened and regarded as the elements of the second analysis, which were drefus, keystne and windsor.

2.4. Data i.i.d assumption analysis

In order to check if the i.i.d assumption is reasonable in the data, compared the distribution of the returns of nine funds with the standard normal distribution. As shown in Fig1-Fig9, we found that all the funds obeyed the i.i.d assumption, which means each fund is independent of all the others (i), and they all have the same probability distribution (i.d.).

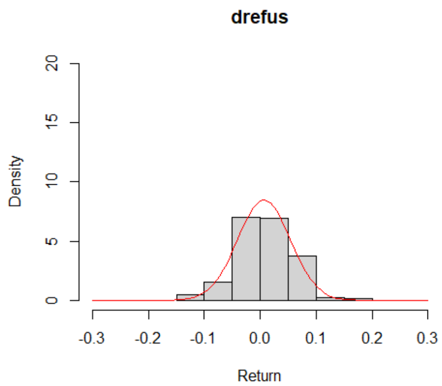

Figure 1. Comparison of the distribution of return of Drefus and Fidel with the standard normal distribution.

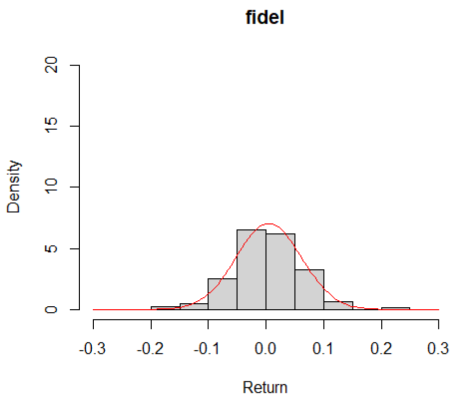

Figure 2. Comparison of the distribution of return of Fidel with the standard normal distribution.

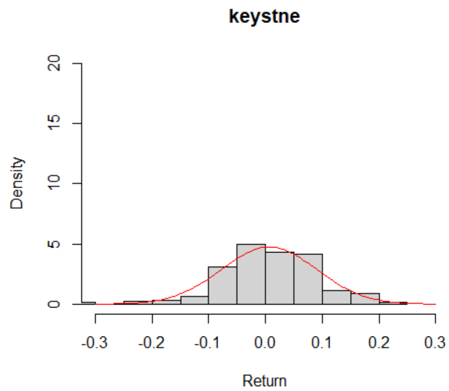

Figure 3. Comparison of the distribution of return of Keystne with the standard normal distribution.

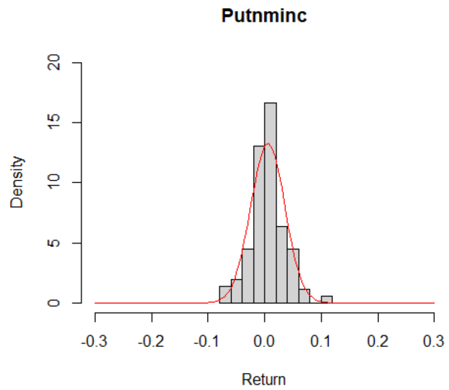

Figure 4. Comparison of the distribution of return of Putnminc with the standard normal distribution.

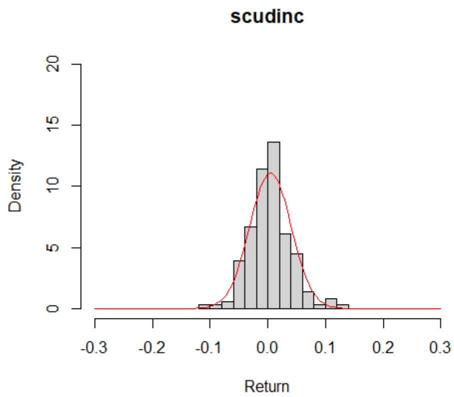

Figure 5. Comparison of the distribution of return of Scudinc with the standard normal distribution.

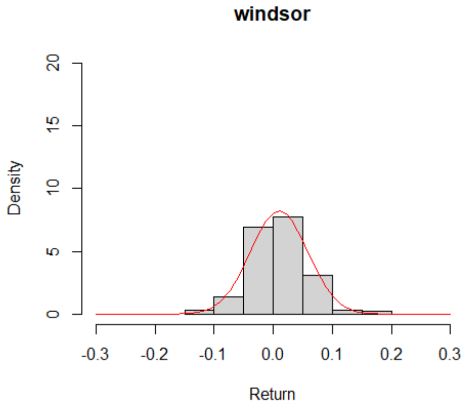

Figure 6. Comparison of the distribution of return of Windsor with the standard normal distribution.

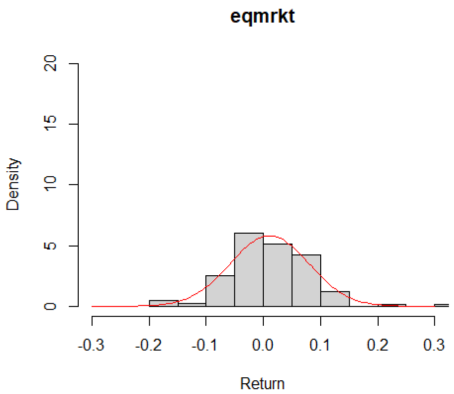

Figure 7. Comparison of the distribution of return of Eqmrkt with the standard normal distribution.

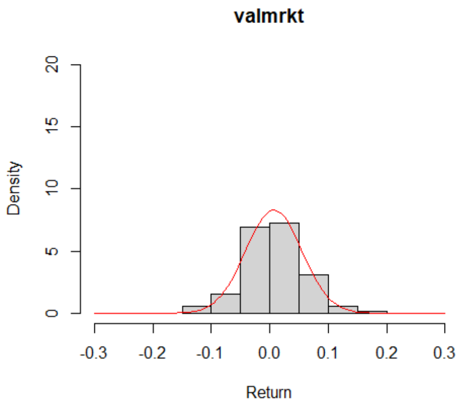

Figure 8. Comparison of the distribution of return of Valmekt with the standard normal distribution.

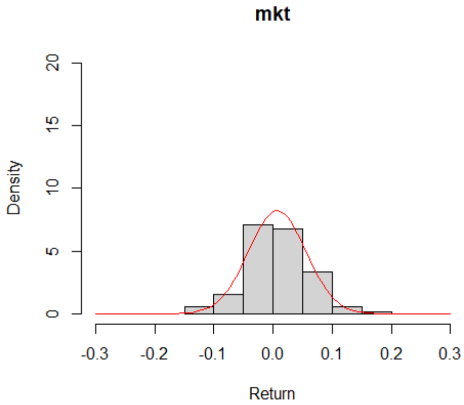

Figure 9. Comparison of the distribution of return of Mkt with the standard normal distribution.

3. RESULT

3.1. Simulate two groups of random portfolios

Aim to find the optimal portfolio that meets the conditions, firstly, the three funds (drefus, keystone, windsor) taken out before were divided into two portfolios including and excluding Treasury bonds. By Rstudio simulation, eight groups of random weight vectors were obtained with the sum of weights in each portfolio equals 1. There were some negative weights in these portfolios, which means people had an unfavorable attitude toward this fund. So these funds would be sold out at first, and then bought back and paid to the creditor. Table 12 and Table 13 describe the distribution of these random weight combinations about two groups, which constituted the feasible set of asset portfolios.

Table 2. The random weight of eight portfolios with risk-free rate

	Portfolio1	Portfoli2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
drefus	0.1	1.41	0.18	0.44	0.01	0.71	0.68	0.42
keystne	0.69	-3.63	1.5	-3.28	-0.02	0.73	0.29	1.17
windsor	-0.13	0.07	-1.19	3.85	0.7	-0.21	0.49	-0.6

T-bills	0.34	3.15	0.51	-0.01	0.31	-0.23	-0.46	0.01

* The sum of weights in each portfolio equals 1

Table 3. The random weight of eight portfolios without risk free rate

	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
drefus	1.72	0.288	0.3	-0.26	0.85	-0.85	1.16	0.74
keystne	-1.83	0.626	0.6	0.47	-0.01	1.44	-0.67	0.37
windsor	1.11	0.086	0.1	0.79	0.16	0.41	0.51	-0.11

* The sum of weights in each portfolio equals 1

Firstly, we select the optimal portfolio with the smallest variance.

Through the functions of the mean, the variance and the Sharpe ratio in Rstudio, found out the returns of mean, standard deviation and Sharpe ratio of each portfolio of combination, which were recorded in Table 14 and Table 15. When calculating the Sharpe ratio, the risk-free rate was set as the return of T-bills.

Table 4. The mean and the standard deviation of returns of eight portfolios with risk free rate

	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
Mean	0.0711	0.0637	0.0258	0.2408	0.1055	0.0733	0.1039	0.0547
SD	0.1982	0.8604	0.3192	0.5506	0.1147	0.2874	0.2652	0.3222

*The decimal point retains four significant digits

Table 5. The mean and the standard deviation of returns of eight portfolios without risk free rate

	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
Mean	0.1296	0.0829	0.0835	0.1107	0.0877	0.0934	0.1029	0.0758
SD	0.2716	0.2371	0.2332	0.2183	0.1212	0.3591	0.1413	0.2051

*The decimal point retains four significant digits

By the comparison, as for the portfolios with risk free rate, the portfolio 5 had the lowest standard deviation (0.1147) among these eight portfolios, and the variance either. By contrast, there were also some portfolios had the higher standard deviation, even above the standard deviation of drefus, keystone and windsor, like portfolio 2, which was 0.8604. According to Mean-Variance Model, this illustrated the portfolio 2 was less valuable than the three individual funds.

In regard of the portfolios without risk free rate, the standard deviation of portfolio 5 and 7 were lower than the standard deviation of any fund (0.1212 and 0.1413). And the variance either. In contrast, some portfolios, such as portfolio 6, had a greater standard deviation than drefus, keystne, and windsor. This indicates that the portfolio we put together was worth less than the three separate funds.

Through comparing the Sharpe ratio of eight portfolios, we can select the optimal portfolio. By building the equally weighted portfolio, the Sharpe ratio increase, which means the investors can get more expected return for every unit of risk. Table 16 and Table 17 combined the Sharpe ratio of two groups of portfolios.

Table 6. The Sharpe ratio of eight portfolios with risk free rate

	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
Sharpe ratio	0.3287	0.0671	0.0621	0.4264	0.8680	0.2342	0.3693	0.1511

*The decimal point retains four significant digits

Table 7. The Sharpe ratio of eight portfolios with risk free rate

	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7	Portfolio8
Sharpe ratio	0.4551	0.3241	0.3326	0.4796	0.5069	0.2435	0.6866	0.3406

*The decimal point retains four significant digits

As for the portfolios with risk free rate, portfolio 5 had the highest Sharpe ratio among these portfolios. The higher sharp ratio means higher expected return, which means we will get more expected return per unit if we invest portfolio 5. When it comes to portfolios without a risk-free rate, portfolio 7 has a greater Sharpe ration than any other fund, implying that if we invest in portfolio 7, we would obtain a larger anticipated return per unit.

3.2. The comparison of two optimal investments

The portfolio with the largest Sharpe ratio and the portfolio with the smallest variance obtained above are sorted into Table 18, 19, so that investors can compare them more intuitively and select the optimal portfolio suitable for themselves.

Table 8. A comparison of the two optimal investments (with risk free rate)

Optimal Portfolio	Weight per equity	Standard deviation	Sharpe ratio
With the smallest variance	0.01,-0.02,0.7,0.31	0.1147	0.8680
With the largest Sharpe ratio	0.01,-0.02,0.7,0.31	0.1147	0.8680

*The decimal point retains four significant digits

Table 9. A comparison of the two optimal investments (without risk free rate)

Optimal Portfolio	Weight per equity	Standard deviation	Sharpe ratio
With the smallest variance	0.85,-0.01,0.16	0.1212	0.5069
With the largest Sharpe ratio	1.16.-0.67,0.51	0.1413	0.6866

*The decimal point retains four significant digits

The experimental results show that Markowitz portfolio theory can be used to find the portfolio with the minimum risk or the maximum Sharpe ratio and the efficient boundary among the portfolio composed of multiple assets. Investors can make rational investment according to their actual ability and risk preference. Rstudio facilitates to a large extent the calculation of expected return rate and portfolio variance in Markowitz's portfolio theory. It can quickly find the optimal portfolio, which is of great value for the application of Markowitz's portfolio theory in financial market.

4. CONCLUSION AND PROSPECTS

In this paper, we discussed the relationship between risk and return in investment based on portfolio investment theory, and conducted an empirical study on Markowitz model. This study contributed to the literature in two ways. Firstly, we extended the basic portfolio investment using the risk-free rate. Secondly, we combined the traditional financial theory with emerging programming language, and used R studio to calculate the expected return, portfolio variance and Sharpe ratio in Markowitz's portfolio theory. In the calculation experiment, we used the historical returns of nine funds which selected from the market and the risk-free interest rate to evaluate the returns of the mean-variance model. In addition, from the empirical analysis results, it could be seen that the Markowitz model can achieve the ideal goal of maximizing investment utility by changing the investment proportion of securities through the optimal asset portfolio. As a new programming language, Rstudio is of great practical significance for obtaining the optimal investment fund portfolio and solving financial problems more quickly and efficiently.

Financial risk management is a very complex system engineering. With the rapid development of technology, theories of Markowitz's concept are being refined and refined, and there are more and more applications based on it, such as stocks, securities and insurance. At the same time, various financial markets are also changing, which obviously puts forward new requirements for the structure of Markowitz's theory. Therefore, we need to constantly optimize the optimal investment theory, apply new technologies, and make the application of these theories more perfect and in-depth. With the regard of selecting the optimal portfolio in risk management, there are many new areas waiting to be explored.

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