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Cattle Diet Problem and Methane Emission

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ABSTRACT
One of the first optimization problems is **the diet problem, which** studies during the 1930s. The study is formulated through **linear program** where nutrient requirements are expressed as the minimum and maximum allowable levels for each nutrient. The paper investigates the diet problem relating to raising cattle while paying for feed. Nonetheless, with the rise in idea of stewardship and concerns of global warming. Raising cattle may be considered as a demerit as cattle can release methane waste into the air, contributing the greenhouse gases. Therefore, inspired by “Diet Problem with Methane Emission.

Keywords: *diet problem, methane emission*

1.INTRODUCTION

The diet problem was one of the first optimization problems studied during the 1930s and 1940s. The effort is motivated by the Army's desire to minimize the cost of feeding GIs while providing a healthy diet. Given a group of foods with nutritional information and the cost of each food per serving, the goal of diet is to minimize the cost while meeting the constraints of specified nutritional requirements. The study is formulated through **linear program** where nutrient requirements are expressed as the minimum and maximum allowable levels for each nutrient.

The following essay mainly investigates the diet problem relating to raising cattle while paying for feed. Nonetheless, with the rise in idea of stewardship and concerns of global warming. Raising cattle may be considered as a demerit as cattle can release methane waste into the air, contributing the greenhouse gases. Therefore, inspired by “Diet Problem with Methane Emission,” the following essay also inserts another linear constraint to the cattle diet problem—methane release—into the the optimisation model. Data from “NASEM” have been used with simplex method to search for the optimal diet value [2-4].

This essay is organized as follows: Section 2 provides the background information regarding the model and the set-up for the liner optimization solution. Section 3 discusses the proposed methodology in detail. Experimental results through matlab and analysis is provided in section 4. Finally, conclusion is drawn in section 5.

2.BACKGROUND AND PROBLEM SET-UP

The formulation of cattle diet problem aims to reach three goals: to lower the cattle’s diet cost,to raise the selling price of the cattle, and to decrease methane emissions. Variables are boldfaced, and objectives are underlined in the subsequent detailed descriptions for each goal.

Firstly, for the daily diet cost, a cattle starts with its **initial weight** and gains more **weight** as it is fed on a daily base with N kinds of food, and the market gives a set **daily cost** for each kind of food. <u>The daily diet cost</u> can be calculated by multiplying the diet with a vector of daily cost. Moreover, under a realistic setting, the cattle will have **a dry matter intake**, where it only consumes a set amount of food per day based on its weight.

Secondly, for the daily profit, to determine how much nutrition is absorbed by the cattle from the diet, a set of **nutrient concentrations** is established using laboratory and empirical estimations for each type of food. Every day, a total of M **nutritional requirements** is evaluated to ensure that the cattle are growing in a healthy manner.

The **daily weight gained** by the cattle is based on the amount of food they consume, and the paper presumes that the weight gain is based on the overall energy intake of the cattle. The sale of finishing cattle to meat manufacturers is ultimately the cattle owner's principal source of income. The sale price is based on the weight of each unit. As a result, the daily profit is calculated by multiplying the daily weight gained by the market sale price.

Finally, for the daily emission cost, the IPCC calculated cattle **daily methane emissions** as a linear function of total calorie intake [1]. Then suppose that the government enacts environmental policies aimed at reducing livestock methane emissions and the cattle are subjected to such a policy, with **an emission fine** applied for each unit of excessive **methane output** (over an emission threshold). The daily emission cost is thus the emission fine multiplied by excessive methane emission.

Next, convert the preceding situation into an optimization problem. To keep in mind, the goals are to lower diet costs, to raise selling profits, and to lower methane emission costs. These are transformed into the following equation:

$$maximize.\ p \cdot \Delta W - c^T \cdot x - f \cdot \widehat{M}$$

ΔW - daily increase of weight

c^T - nutrient cost matrix

$f \cdot \widehat{M}$ - fine applied on methane emission

p is the sell price ($/kg), it appears as a constant.

x is a vector, indicating the weight of each food type. Varies x vector can achieve maximum value of objectives.

Four constraints to the food vector x are proposed for the scenario to be realistic. First, all elements in vector x should be greater than or equal to zero since the mass of food cannot be negative:

$$x \geq 0$$

Second, the daily dry matter intake (D) of a cattle is presumed to be equal to the mass of food it is fed with, or the sum of all elements in vector x :

$$e^T \cdot x = D$$

where e is vector of ones

Moreover, the cattle's daily nutrient requirements should also be considered as cattle require satisfiable energy of protein, calcium, and phosphorus etc. for daily activities. Matrix A is introduced to display the number of nutrients in each food kind, and matrix b is introduced to represent the minimum nutrients required. Nutrient consumption should be greater than or equal to the bare minimum:

$$A \cdot x \geq b$$

Finally, the estimations of parameters (NASEM 2000) and the methane emission M (IPCC 2006) are given as below:

$$\Delta W = 13.91 \bullet (Et - Em) \bullet W^{-0.6837}$$

W - the weight of cattle, which increases daily

$Et,s = C^T \bullet x$ - energy intake

C - energy matrix

$Em,s = s \bullet W^{0.75}$ - energy used

$D = 1.8545 + 0.01937 * W0\backslash$

As a result of combining these, the following diet optimization problem is arise as:

$$max.\ p \cdot \Delta W - c^T \cdot x - f \cdot \widehat{M}$$

$$s.t.\ x \geq 0,\ A \cdot x \geq b,\ e^T \cdot x = D,\ \widehat{M} \geq 0,\ \widehat{M} \geq (m{\cdot}k / mec) \cdot C^T \cdot x - M0,\ \Delta W \geq 0$$

The table below shows all variables and their corresponding symbols:

Table 1: Symbolization

symbol	*unit*	*description*
x	kg	N-dimensional vector specifying how much kg of each food types a cattle eats per day.
M^	kg	Estimated kg of methane emitted by a cattle per day
c^T	$/kg	N-dimensional vector specifying how much a kg of each food type costs
A	unit	M × N matrix specifying how much nutrient each food type contains
b	kg	M-dimensional vector specifying daily requirement of a specific nutrient
C	J/kg	N-dimensional vector specifying energy concentration in each food type
e^T	unit	N-dimensional vector of ones
p	$/kg	Selling price of cattle meat
W	kg	Daily weight of a cattle
f	$/kg	Amount of capital punishment per kg of excessive methane emission

D	kg	Daily dry matter intake of a cattle
Em	MCal	Daily energy of maintenance for a cattle
s	$J/kg^{0.75}$	Correlation factor between Em and W , which depends on breed, nutritional state, and sex etc.
k	MJ/MCal	Unit conversion factor
Et	MCal	Total energy intake of a cattle per day
m	unit	Methane conversion factor
mec	MJ/kg	Methane energy constant
M0	kg	Methane emission threshold

3.METHODOLOGY

3.1Data

Most data are selected from NASEM and IPCC

Table 2 Data Acquisition

Parameter	Value/Unit	Source
c^T (Cost Vector) See below		https://fyi.extension.wisc.edu/forage/files/2021/05/05- 24-2021.pdf https://www.ams.usda.gov/mnreports/ams_2707.pdf https://practicalfarmers.org/2017/02/doing-the-math- and-picking-a-market-for-straw-profitability/ https://www.ams.usda.gov/mnreports/ms_gr852.txt https://www.sbreb.org/wp- content/uploads/2018/09/Production6.pdf http://agebb.missouri.edu/dairy/byprod/allcompanies.asp
A (Nutritional Concentration)	See below	NASEM 2000 (pp. 134)
b (Nutritional Requirement)	See below	NASEM 2000 (pp. 106)
C(Energy Concentration)	See below	NASEM 2000 (pp. 134)
p (Price)	4.124($/kg)	https://tradingeconomics.com/commodity/beef
W0 (Initial Weight)	400(kg)	NASEM 2000 (pp. 106)
f (Fine)	0.8248($/kg)	20% of selling price
s(Correction Factor)	0.077	NASEM 2000 (pp. 6)
k(Unit Convertion Factor)	4.184	
N (Number of Cattle)	57.4	https://www.iabeef.org/raising-beef/cattle-industry-facts
m (Methane Convertion Factor)	0.03	IPCC 2006 vol. 4 pp. 10.30
mec (Methane Energy Constant)	55.65	IPCC 2006 vol.4 pp.10.31
M0 (Methane Emission Threshold)	0.125	

Table 3 Nutritional/Energy Concentration and Cost (A,C and c) table

	C	A			c
Food Type	Net Energy Concentration (Mcal/kg)	Crude Protein (%)	Calcium (%)	Phosphorus (%)	Cost ($/ton)
ALFALFA Fresh	1.38	18.9	1.29	0.26	225
Hay	1.31	18.6	1.4	0.28	177
Straw	0.6	4.4	0.3	0.07	53
Gluten Meal	2.2	66.3	0.07	0.61	562.5
Seed	2.24	24.4	0.17	0.62	257.5
Barley Grain	2.06	13.2	0.05	0.35	118
Sugar Beets	1.76	9.8	0.68	0.1	38.77
Cotton Hulls	0.68	4.2	0.15	0.09	275
Wheat Midds	1.6	18.7	0.17	1.01	167.5
Cottonseed Meal	1.79	46.1	0.02	0.02	400
Distiller's Grains Dried	2.18	30.4	0.26	0.83	210
Oat Hulls	0.41	4.1	0.16	0.15	238.13

Table 4 Nutritional Requirement table

Nutritional Requirement	Unit	B
Energy	MCal (NE)	6.38
Protein	kg(MP), where CP=MP/0.64	0.274
Calcium	kg	0.009
Phosphorus	kg	0.007

3.2Methodology

To find the optimal solution, Simplex method is being applied. Simplex method is a linearly programming strategy in which we find the optimal solution or feasible solution by iteration with the help of MATLAB. Simplex method contains following steps:

1. To find a feasible solution, if there is no basic feasible solution, then the problem has no solution. If there is a feasible solution, there will be 2 cases:

Case 1: It is an optimal solution, then there would be a basic optimal solution Case 2: There is no basic optimal solution, and thus there is no optimal solution.

1. If the problem went to case 2, then, the procedure will be terminated to find the output. If it is case 1, the basic optimal solution can be found and by iteration, the feasible solution can be approximate with the help of MATLAB.

The loop of the approximation is:

(1)Find a possible optimal solution

(2)Check whether the solution is the final optimal solution: if yes, there is an output in solution, if no, then continues to

(3)Find a better possible optimal solution and return to step (2)

Whether the method can be applied is based on whether there is degeneracy appears. Degeneracy is defined as one variable that takes 0 value when iterating, and this situation will make iteration converge slower. On the other hand, if no degeneracy occurs, the simplex method will converge to the expected error bound after iteration. If this situation occurs, there will be 2 cases:

Case 1: There is a unique optimal solution

Case 2: There is an infinitely feasible solution satisfying the requirement as the solution set is not bounded.

To apply the simplex method in diet problem, all the collected data are input into MATLAB as matrix. After that, we transform the original ob. and st.

$s.\ t.\ x \geq 0, A \cdot x \geq \hat{b}\ \hat{M} \geq 0,$

$\hat{M} >= m \cdot k/(\ mec)\ \hat{C}\ T\, x - M0$

To general LP form: *Min* $(\hat{c})\ \hat{}\ T^*(\hat{x})$ *St.*

$(\hat{x}) \geq 0,$

$\hat{A}\ \text{-}(\hat{x}) \geq 0$

Where: slack variables

$v\ def = A \cdot x - (\hat{b}) \geq 0\ (1)$

$\mu\ def = M0 -\qquad m \cdot k/(mec)\ (\hat{C}\ T)\ x - \hat{M}$

(2) $\hat{A}$ = |A 0 -I 0|

|e^t 0 0^t 0|

|m*k/mec*Ct -1 0^t 1|

$\hat{x} = x$

$(\hat{M})\ V$

μ

$\hat{c} = \_x0012\_c - 13.91\ p\ \hat{W}\ (-0.6837\ C)$

$f\ 0$

0

$b = \_x0012\_\ b\text{-}$

$D\ M0$

To make sure whether Simplex method can be applied, the degeneracy existence is first being tested. If there is no degeneracy, the outcome is expected in form of:

1. Unique optimal solution
2. Infinite solution

If the program runs successfully, a unique vector (X^) or a solution set are expected. And vector (x^) is rewritten into the original nutrition facts.

4.RESULTS AND ANALYSIS

We ran the above algorithm in MATLAB using the data retrieved. The algorithm converges and outputs an optimal solution with one non-zero component in the diet: about 9.6 kg of sugar beets.

```
ans =

         0
         0
         0
         0
         0
         0
    9.6025
         0
         0
         0
         0
         0
         0
   10.0133
   10.5204
   93.6764
    6.5207
    0.9533
    0.0869
```

Figure 1 A screenshot of the complete program output

There are several insights that could be shown by the solution. First, there is only one type of cattle food included in the final solution, pointing out that sugar beets has a higher cost performance under the diet problem. A small number of non-zero outputs is significant as it could indicate that the problem is degenerate, as the number of the non-zero outputs is dramatically lower than those of constraints. However, the number of equality constraint, in this case, is one as we only have

$$e\text{^}T^*x=D$$

as our sole equality constraint. Therefore, if the inequality constraints are loose enough, a small variety of diet could become possible, as in this case.

A_hat in the cononical form of the question has a rank of 7, de facto. Which corresponding exactly to the seven components we've got, including the slack variables. Six inequality and one equality constraints could be transformed into a 7-variable linear system, which could be solved to give 7 outputs. Therefore, the result is mathematically consistent.

In term of slack variables, except for M_hat, which is the methane production, all the slack variables are non-zero. However, the requirement for Phosphorus is only slightly exccessed, indicating that if the requirement for Phosphorus increases, the boundry will extend and this solution may not apply very well.

Nevertheless, the requirements for Protein, Energy and Calcium are moderately exccessed, meaning that the solution will still apply effectively if cattles require more Protein, Calcium or Energy.

Next, observe that M_hat is exactly 0. This means that the optimal methane emission is on the emission threshold M_0. As methane emission and growth in weight both depend linearly on energy intake, a small change in x should result in larger changes in the profit generated by growth in weight p · ΔW and the cost resulted from excessive methane emission. Sensitivity analysis on p and f confirms this intuition. It turns out that the objective is − f ·M_hat sensitive to p but quite stable with respect to f, unless f is increased dramatically. As methane is a gas, the unit kg is unrealistic measurement of weight for methane, due to its low density. Hence, it makes sense for us to conclude that the objective is quite sensitive to f provided that f is scaled to be within a reasonably large interval [5].

Finally, the food cost is comparatively lower than the price of cattle meat\(p) and fine taken for methane emission. Therefore, it is obvious that the food price only takes a small part in the objective. D acts as an upper bound for the diet vector x. As D increase in a good amount, it is expected that, under the problem set up, cattle grow in weight indefinitely. Then the objective simply goes unboundly. This analysis of D indicates the importance of placing a realistic constraint on D [6].

5.CONCLUSION

In the planning process of this project, we made a difficult choice in the choice of theme. From the beginning, we wanted to discuss the use of land resources, and at the end, we considered a revised cattle diet, which increased the composition of methane emissions. This is a very common phenomenon, and then we present this phenomenon as an optimization problem, using the simplex method to solve it.

Many people have done similar research on this problem in the past. Inspired by them, we made clear the problem structure and solution, and precise results are obtained. Through the results, we can provide scientific guidance for farmers all over the world. Sensitivity analysis shows that methane emission fee is an effective way to affect cattle industry.

In addition, our project can also adapt to different kinds of animals and different nutritional needs by simply changing several parameters in the model. Farmers' returns are based mainly on two sectors. The first is that the weight of cattle affects the price that farmers can sell, so they need to optimize the cost of growing cattle. Then there is the tax issue of methane emissions. Overall, the whole problem is really split into two parts. In a strict range, we expect our method to still converge to the optimal. In this way, we can help farmers maximize their profits.

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