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# Prospective Mathematics Teachers' Critical Thinking Processes in Dealing Truth-Seeking Problem with Contradictory Information

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**ABSTRACT**
This study aims to describe the critical thinking process of prospective mathematics teacher students in solving problems with contradictory information. This study is qualitative descriptive research. The subjects consisted of 52 sixth-semester students of Tadris Matematika Programme IAIN Kediri. Collected data through tests and interviews. The test consists of one problem with contradictory information. Analysing data is based on four stages of critical thinking. The results showed that 17% of students chose "yes", 73% chose "no", and the rest answered unclearly. Then four subjects were selected to be interviewed based on the answers "yes" and "no". At the clarification stage, they can describe the primary goal of the problem precisely, even though they did not write it down. At the assessment stage, they can provide appropriate reasons or criteria for concluding. Subjects who answered "Yes" can review other possibilities of conditions of the problem, but this did not happen to subjects who answered "No". At the inference stage, the subjects who answered "Yes" had a hypothesis in advance after doing inductive reasoning, then it was concluded deductively through proof. While the subjects who answered "No", were not able to recognize contradictory information and carelessly conclude. At the strategy stage, the subject who answered "yes" took action to generate other possibilities that met the objectives of the problem. While the subjects who answered "no" recalculated the information on the problem.

***Keywords:*** *Critical Thinking; Truth-Seeking Problem; Contradictory Information*

## 1. INTRODUCTION

Mathematics is not only a subject of study but also a way of thinking. Mathematics can be seen as problem-solving, reasoning, and communication so that students can explore, conjecture, reason with the surrounding environment with confidence [1,2]. Learning mathematics requires values that reflect mathematics in real life and the work environment. In this regard, the mastery of knowledge and information is not enough to compete in entering the workplace in the 21st century. This competition emphasizes the need to prepare a generation that is collaborative, creative, innovative, communicative, and analytical critically thinking to be able to solve complex problems, both in the personal life and workplace, by making effective decisions [3–7]. Critical thinking related to effective decision-making is one of the skills that need to be developed at various levels of education, including as an outcome in higher education [6,8].

Activities in encouraging critical thinking skills are certainly supported by the important role of teachers as the spearhead of educational progress. However, trying to change the mind-set and behaviour of teachers as critical thinkers is not an easy matter, they are mature and difficult to change. Preparing prospective teachers who can think critically becomes more strategic than a long journey to train existing teachers [9]. Prospective teachers will play an important role in developing students' critical thinking skills [10,11]. If this ability is not well-mastered by prospective mathematics teachers well, it will be difficult to carry out the task of developing students' critical thinking.

Students' critical thinking skills can be refined with a stimulus in the form of complex and non-routine problems, that need to be integrated into the educational process [12,13]. Non-routine problems that become the stimulus for learning consist of problems to find and problems to prove [14,15]. Problems in the form of truth-seeking have been developed as predictors of critical thinking [16]. One of them is the problem with contradictory information [17]. Problem-solving process with contradictory information has been investigated at the higher education level [18]. At the secondary school level, it is assessed based on the disposition of critical thinking, namely a person's tendency to be critical [19]. However, critical thinking skills in the process of solving problems with contradictory information need further research.

The problem with contradictory information can be related to mathematical patterns, one of which is mathematical induction. Mathematical induction is one of the tools used to perform mathematics and the development of mathematical maturity [20,21]. So far some research examines how the process or steps of proof construction by mathematical induction. [22–24]. Nevertheless, how the problem of mathematical induction with contradictory information becomes a tool to capture critical thinking skills has not been widely studied. This study aims to reveal the process of students thinking critically in solving a problem with contradictory information.

## 2. RESEARCH METHOD

This research is qualitative descriptive research. The instrument consisted of the researchers, a test about the problem of contradictory information, and semi-structured interview guidelines. The participants were 87 students of a 5th-semester prospective mathematics teacher at one of the state universities in East Java, Indonesia, namely IAIN Kediri. The research subject was participating Real Analysis course, which the principle of mathematical induction as a preliminary.

Problems with contradictory information containing data that appear contrary, it is necessary to make a habit of analysing discrepancies in a question and checking the truth of the information provided before believing it [25]. The test contains the problem with formulated contradictory information in Figure 1.

Consider the following sequence.

$1! - 3 = -2$
$2! - 3^2 = -7$
$3! - 3^3 = -21$
$4! - 3^4 = -57$
and so on.

Investigate whether the sequence can reach a positive value and increase?

If yes, prove your answer. If not, state your reasons.

**Figure 1** Instrument test: truth-seeking problem with contradictory information

Given a sequence with four terms that have negative values and their values are getting decrease. Then it is contrasted with the question of possibilities for a condition where the value is positive and increasing.

The use of the characteristics model of critical thinking from Perkin & Murphy distinguishes these skills clearly. Each indicator measures one aspect of critical thinking, and no indicator measures the same aspect [8]. There are four stages of critical thinking, namely: a) *clarification*, students define the problem being discussed; b) *assessment*, students evaluate the truth of the criteria/situation/topic, make decisions, and propose proof with valid reasons; c) *inference*, students conclude the relationship between ideas, draw conclusions by deduction or induction, and generalize; d) *strategy*, students describe or evaluate possible actions. Critical thinking indicators in solving truth-seeking problems with contradictory information are in Table 1.

**Table 1.** Model of critical thinking process

| Stages | Description | Indicator |
|---|---|---|
| Clarification | State, clarify, describe (not explain) or define the issue/problem. | • State the goal of the problem<br>• Analyze, discuss the purpose/objective of the problem<br>• Identify one or more assumptions/statements related to the information in the problem<br>• Identify the relationship between statements/assumptions from the information in the problem<br>• Define terms relevant to the problem |
| Assessment | Evaluate some of the aspects discussed; decide in a situation, propose proof to corroborate an argument or its relationship to the problem. | • State/ask for reasons that corroborate the proof submitted is valid and relevant<br>• Determine the criteria met by a condition<br>• Make a right or wrong judgment on a criterion, condition, or topic<br>• Submit appropriate proof for defined criteria |
| Inference | Show the relationship between ideas; draw appropriate conclusions by deduction or induction, generalizing, explaining (not describing), and | • Make appropriate deductions<br>• Draw appropriate conclusions<br>• Finding the final conclusion<br>• Make a generalization |

| Stages | Description | Indicator |
|---|---|---|
| | making hypotheses. | • Summarize the relationship between ideas |
| Strategy | Propose, discuss, or evaluate a possible action. | • Take action.<br>• Describe a possible action/method/procedure<br>• Evaluate possible actions/methods/ procedures<br>• Predict the outcome of an action |

After participants solved a problem with contradictory information in the form of a test for about 30 minutes, then they were classified into two categories, "YES" or "NO" answer. Then selected subjects who met three critical thinking indicators and for each of the two subjects with consistent answers (both from test results and during interviews).

## 3. RESULT AND DISCUSSION

Data retrieval was carried out in a synchronous virtual room with Google Forms and Zoom Meetings on October 2-5, 2021. A total of 52 respondents completed the problem instrument sheet with contradictory information. Next, we analyzed the test results data and determined the subjects to be interviewed based on students answered "Yes" or "No". Furthermore, each category is represented by two students.

Based on Figure 2, shows that 73% of students answered "No" and 10% of students were categorized as "Unclear". Furthermore, about 17% of students could give the correct decision, namely "Yes". These data describe that the critical thinking process carried out by students is not good enough to justify the truth of the given problem with contradictory information.

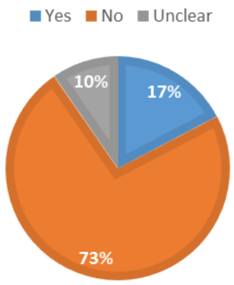



**Figure 2.** Students' answer categories percentage

$$n! - 3^n \quad (1)$$

$$n! > 3^n \quad (2)$$

$$n! - 3^n = 0 \quad (3)$$

Table 2 presents the stages of the critical thinking process of the participants. Students rarely carry out the clarification stage in their sheet. The students' answers obtained that they did not write down the goal of the problem. Although some of them were able to identify general patterns that were relevant to the problem, such as Equation (1). Students who are failed in assessing the truth do not carry out the stages of clarification properly. The average of clarification of students who answered "No" is the lowest.

Other information in Table 2 shows that students who answered "Yes" more often carry out the assessment process, such as providing what criteria so that the resulting pattern can be positive and increase, for example, Equation (2) or Equation (3) must be held. They also state the reasons or criteria that met the specified conditions or why they take a fixed action. Meanwhile, in the "No" and "Unclear" answer categories, it was found that students did less in the assessment stage than students with "Yes" answers. It shows that the assessment process carried out by students is not sufficient to direct students to get the correct decision.

The findings in Table 2 shows that students with "Yes" and "No" answers more often do the inference stage than students with the "Unclear" answer. Because the inference stage becomes the point where students evaluate and make decisions about the problem with contradictory information. This indicator becomes a standard of success whether someone can determine the truth of the statement appropriately. In students with the category "Yes" more often do inference stages than others. It shows that students who answered "Yes" can make sub-conclusions before obtaining the main conclusion as the goal of the problem. Students answered "Yes" indicate other deductions that support the last conclusion. Unlike the students who answered "No", they carried out just the main conclusion at the inference process.

**Table 2.** Descriptive Data of Students' Coded Critical Thinking Processes

| Solution Category | Freq. | Average in One Category | | | |
|---|---|---|---|---|---|
| | | Clarification | Assessment | Inference | Strategy |
| Yes | 9 | 0.89 | 2.11 | 2.56 | 5.11 |
| No | 38 | 0.78 | 1.37 | 1.63 | 1.68 |
| Unclear | 5 | 1.2 | 1.40 | 0.80 | 2.60 |
| Total | 52 | | | | |

Furthermore, it shows that the strategy stage of students who answered "No" becomes the most minimal in taking action to support the three stages of the critical thinking process. The drawing conclusion is not logically made using insufficient strategies. For example, recalculating equations but seems not to continue the process for the larger natural number. Additionally, students with "unclear" perform strategic actions, they do not assess the results and/or draw conclusions. Therefore, the critical thinking process carried out cannot lead to a conclusion. This is contrary to the students who answered "Yes", they more often take actions in the form of strategic steps to provide proof or draw conclusions, for example, doing calculations, defining an element related to the problem, and determining a method or a proof.

After selecting the students as the subject of the interview, qualitative data related to the critical thinking process when solving problems with contradictory information was completed. Subjects were selected in the "Yes" and "No" answer categories with the consideration that by being able to provide a definitive decision, the critical thinking process achieves the outcome. While in the answer "Unclear" did not conduct an in-depth interview. The selection was based on coding obtained from the written solutions of students who did all stages of the critical thinking process according to Perkins & Murphy.

### *3.1. Subjects Answered "Yes"*

Two subjects in this category are S1 and S2. The results of their answers can be considered in Figures 3 and Figure 4, respectively. Exposure to interview results was carried out at each stage of critical thinking conducted by each subject.

#### *3.1.1. Clarification's Subject Answered "Yes"*

Figure 3 and Figure 4 indicate that the subject could identify one or more statements related to the information on the problem, which is a pattern consisting of four equations that form a pattern such as Equation (1). These two subjects stated the relationship between statements of the problem information as in Table 1.

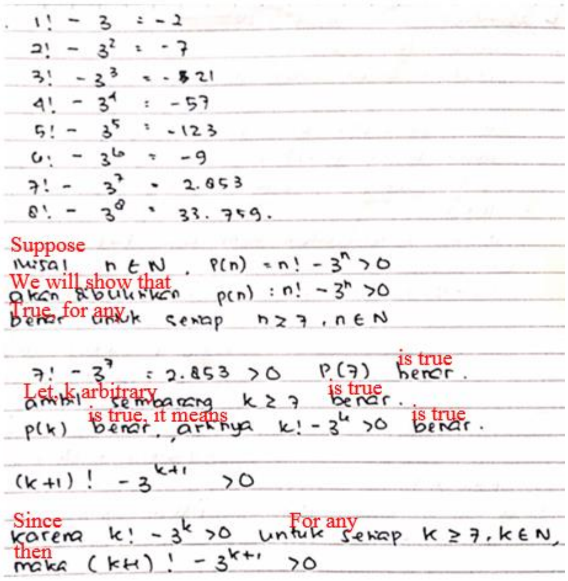



**Figure 3.** S1 Written Solution to Problem with Contradictory Information

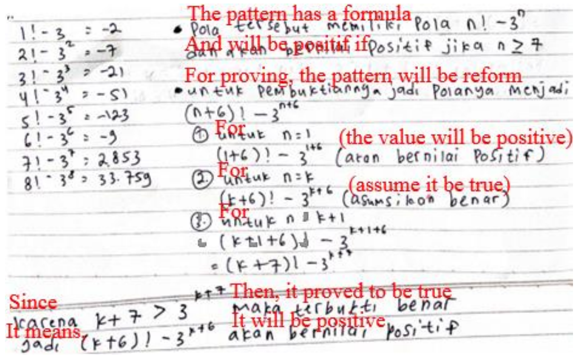



**Figure 4.** S2 written solution to problem with contradictory information

Although they did not write down, but describe the primary goal of the problem precisely. After that, discuss what the problem means. The subjects were able to state that the problem asked them to investigate the possible conditions of the given pattern equation whether it gave a positive value. The transcript of interviews with both subjects is as follows.

R : *What you think when resolving this problem related to "What modal can I find? So what does this problem mean? What goal do I want to achieve?"*

S1: *First, I relate the factorial concept of subtraction with the concept of exponential. The pattern of the results has the symbol of factorial and exponential, then whether it forms a pattern or not --- I asked to investigate whether the pattern may be positive or not --- I will prove the statement, investigate whether there may be a condition where the above pattern is always a positive value. It means this problem asks me to prove a statement, the subtraction between factorials from one to the next, subtracted by three* *power n, and so on (whether it can be of always positive value). To prove it already exists (information) from (n=)1 to 4.*

R : *What did you think at the beginning when you faced this problem?*

S2: *In this problem there is a pattern, from 1!-3 equal -2, 2!- 3 power 2 is -7, from this equation, I thought a pattern has been obtained, namely Equation (1) --- This problem asks when (pattern) is positive. Then "what is the value of n that satisfies these conditions?"*

### 3.1.2. Assessment's Subject Answered "Yes"

Based on Figure 3 and Figure 4, S1 and S2 determine the criteria on the results of the calculations they obtained, which must be positive, such that the conditions at the clarification stage are met, i.e. the pattern can be positive. They also checked the results of calculations for the formula obtained, then rated positive or negative. They pursued to provide proof that matched the criteria he had set, i.e. when *n* was more than or equal to 7 through a series of mathematical inductions. They also provided a reason for the completeness of the truth of statement *P(k+1)* in the induction step to strengthen the draw conclusion, i.e. the resulting pattern is always positive.

Subjects used established criteria to devise a strategic move and confirm conclusions as Table 1. But the criteria have not been guaranteed the truth when making it as a reason to conclude. Subjects experienced blocks in the process of presenting proof, induction steps failed to be executed. The findings are confirmed from the transcripts of interviews in S1 and S2 as follows.

$$p(n): n! - 3^n > 0 \tag{4}$$

$$(n+6)! - 3^{n+6} \tag{5}$$

$$(n+6)! < 3^{n+6} \tag{6}$$

S1: *--- from 1!-3, then 2!-3 power 2, so on n=8, so I have a conjecture there is a pattern --- the first factorial to the sixth factorial that turns out to be negative--- when I calculated in factorial of the seventh subtracted by 3 power 7, up to n equals 8, it turned out to be positive. Are the remaining 7 all positive results? Finally in Equation (4) is (true) for every n≥7.*

R : *In the third step, from which you can be sure that (k+1)!-3 power k+1 will always be positive. Why do you conclude that?*

S1: *Because I took n≥7. Well, if k<7 could be the result is not positive, or negative.*

R : *Is there another process when you conclude that?*

S1: *There was a process of counting, but it stopped and I couldn't continue.*

S2: *---I got a positive value at 7!-3 power 7--- we have to prove the pattern that is Equation (5) in order to get n≥7 --- obtained (1 +6)! -3 power (1+6) well this will obviously be a positive value. For the second proof I can obtain it (k+7)! It will be more than 3 power (k+7) so that (the pattern) is positive value --- After being substituted (to the formula) will be positive if n≥7. So that the pattern we change to Equation (6) --- (but) I do not use the assumption for the natural number k is correct to infer that (k +7)! is more than 3 power (k+7).*

R : *You say that Equation (1) is more than zero, how do you think so process from the pattern you have and finally appear the pattern in Equation (6)?*

S2: *That's from one-on-one substitute for n, 1!-3 is -2, up to 7!-3 power 7, the result is positive. Well, that's why I think that the pattern will be positive, definitely if n≥7.*

R : *What made you decide to look for pattern values up to n =8?*

S2: *Well here the calculation of n=4, then (n=) 5, (n=)6, it is the factorial value is higher ---Moreover, the value (rank) is also getting bigger, but I think, surely the value of this factorial is greater than the results of the exponent, then I continue to try until I find this positive value.*

### 3.1.3. Inference's Subject Answered "Yes"

Figure 3 gives information that S1 made an appropriate deduction at the base step he performs on the proof by mathematical induction for the natural number *n* equal to 7, the statement Equation (1) is greater than zero, and it is true. In the induction step, S2 stated that if Equation (6) is truly positive for the natural number *k*, then Equation (7) is also true. It means they made a precise conclusion, although in the assessment stage they had difficulty presenting valid proof for the process of drawing such conclusions. In the end, S1 and S2 generalized that true patterns will be positive starting at n more than equal to 7. In contrast to S1, before proving with the principle of mathematical induction, S2 made a conclusion advance with induction reasoning from the trial-error experimenting strategy that he did.

The findings of the test results are in line with interview footage on S1 and S2. As Table 1, they made generalizations by using the trial-error and draw the main conclusion after doing the mathematical induction process. S2 made a precise deduction not only at the end of the conclusion, but also he made a new generalization of the pattern that he concluded it is true for all natural numbers. Here are the transcripts of the interview.

S1: *For n = 1,2,3 (the pattern) turns negative, I continue until the pattern forms a positive, it turns out, the value is positive when in (natural numbers) 7 and 8. So that there is a possibility that 7, 8, and so on follow (the pattern) result is positive (A). --- Equation (6), it turns out to be true (positive). What is the condition? Every n≥7. So for n=7 onwards, it's positive. As for the n<7 he is worth the negative. Is there a condition (that meets)? Oh, maybe positive when it's n≥7*

S2: *(For the natural number) n = 4, then (n = ) 5, (n = ) 6, the higher the factorial value the value of the number will automatically be greater than the power*

*3, although the value (power) is also getting bigger, but I think, surely the value of this factorial is greater than the previous--- (from the results of this calculation) It began to be positive at 7! Minus 3 to 7, so automatically n (starts from) 7. ---Since it starts n =1 for equation patterns (5), 1+6(=7). Means (for Equation (1) value positive starts when n =7.--- Therefore, I can get it (k+7)!. It will be more than 3 power (k +7), so that (the pattern) is positive. That statement is proved for (k+7)!>3 power (k + 7), so it can be concluded Equation (5) will always be a positive value. So it will be positive when n≥7.*

### 3.1.4. Strategy's Subject Answered "Yes"

Figure 3 and Figure 4 show that these two subjects take an action by writing patterns in sequence for the natural numbers 5, 6, 7, and 8, then by substituting, they calculate the result. S1 constructed a statement P with such a natural number n, and Equation (1) greater than zero. S2 performed similar actions but defined a different but relevant pattern such as Equation (6). S1 proposed proof that statement is true for n more than equal to 7, while S2 followed the changes then he proposed the statement truth to all natural numbers. They both established a method of proof by principle of mathematical induction although, in different versions, S1 used the second version and S2 used the initial version. Next, they executed the base step and bridge step, respectively, by substitute method then performed the assessment and inference stages. However, on the move, the two did not make deductive efforts to guarantee the stated conclusion.

The emergence of critical thinking indicators in the strategy stages of S1 and S2 subjects is confirmed through the results of interviews. S1 predicted an action that is possible, namely by determining the formula of the equation result, but it did not get a pattern, then he determined the result in the form of a positive or negative value. Meanwhile, S2 revealed that strategies using principal of mathematical induction version 2 may be done to solve the problem. The interview transcriptions of S1 and S2 respectively as follows.

*R : You make an effort to show that the statement is true for n≥7. Can you tell me what you think before it?*
*S1: (Initially) I tried one by one (for n=1, 2, 3,..), so I think there is a pattern.---From (results) the first factorial to the sixth factorial turns out to be negative. Only when I calculate in the seventh factorial subtracted by 3 power 7, until n equals 8 --- This is supposed Equation (1) as P(n) is more than 0.*
*R : How was your process of mathematical induction to make sure the statement truth?*
*S1: We assume for n equal 7 is true. Then we assume n=k was correct. Then n=k+1. It could be substituted into the equation for n=k+1--- but it stopped, and I couldn't continue.*
*R : About these two steps, do you think there is another strategy or other method?*
*S1: At first, I thought that suppose the result formed a pattern, such as an arithmetic series or geometry, it could be inserted the formula. But because it (the result) does not form a pattern, finally I just tried count manually until the results are positive.*
*S2: I substituted one by one so that I get a positive value (I got that) 7!<3 power 7. Well, I had to prove that the pattern will be positive when n≥7. In other words, the pattern in Equation (5), to obtain n≥7. First for n=1, I get (1+6)! < 3 power (1 +6) well this will obviously be positive. For the second, I suppose n=k, so (k+6)!-3 powers (k +6). This is assumed to be true. Then for n = k + 1, I substitute. Then I got it, (k +7)!-3 powers (k+7). Well from here I can get it (k+7)!>3 power (k+7). So that (the pattern) is positive.---Equation 5 will be positive if n≥7. So that the pattern we change to Equation (6) --- I think of the pattern, if (true) n≥7, then Equation (5) (will be true for n≥1). It means n must be added 6.*
*R : What do you think about proof in mathematical induction, does proving it has to start from 1?*
*S2: It may depend on the value of n, because there is a bounded, so I don't think n starts from one (maybe), so it depends on the form of the problem.*

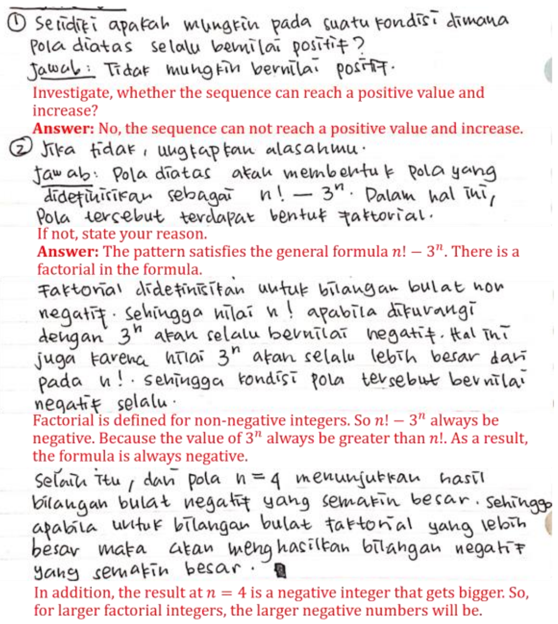
## 3.2. Subjects Answered "No"

S3 and S4 are subjects in this study who answered "No". The results of their answers can be considered in Figures 5 and Figure 6, respectively. An analysis of the critical thinking stages of S3 and S4 will be carried out by triangulating data from their answer sheets and interview results.

### 3.2.1. Clarification's Subject Answered "No"

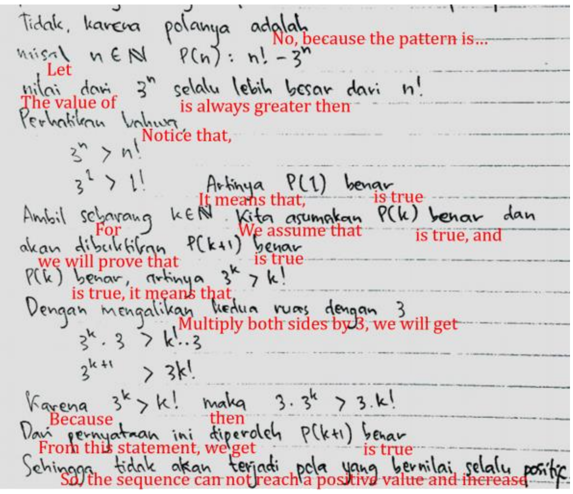
In Figure 5 and Figure 6, S3 and S4 do not mention the purpose of the problem. However, based on the interview results, S3 and S4 were able to explain and discuss the purpose of the questions well. Before they write down the ideas on the answer sheet, they describe the meaning, clues, and objectives of the problem on another sheet. This statement can be observed from the transcript interview as follows.

*R: What do you understand about this problem?*
*S3: There are two problems in this matter. First, we were asked to investigate whether the sequence can reach a positive value and increase. Second, we are asked to give reasons.*
*R: What is the purpose of the problem?*
*S4: To prove whether the statement is true or false.*
*R: Why don't you rewrite the information in the question on your answer sheet?*
*S3: I have written down the information and the purpose of the problem in another sheet.*
*R: What elements can you find in the problem?*
*S3: There are factorial numbers and powers of 3. Factorial numbers are non-negative integers.*
*R: Can you find the pattern in the problem?*
*S3: Yes. I can find the pattern for the components on the right-hand side of the equations, namely Equation (1). But I can't find the formula for the elements on the left-hand side equations. The values vary.*



① Selidiki apakah mungkin pada suatu kondisi dimana pola diatas selalu bernilai positif?
Jawab: Tidak mungkin bernilai positif.
Investigate, whether the sequence can reach a positive value and increase?
**Answer:** No, the sequence can not reach a positive value and increase.
② Jika tidak, ungkapkan alasanmu.
Jawab: Pola diatas akan membentuk pola yang didefinisikan sebagai $n! - 3^n$. Dalam hal ini, pola tersebut terdapat bentuk faktorial.
If not, state your reason.
**Answer:** The pattern satisfies the general formula $n! - 3^n$. There is a factorial in the formula.
Faktorial didefinisikan untuk bilangan bulat non negatif. Sehingga nilai $n!$ apabila dikurangi dengan $3^n$ akan selalu bernilai negatif. Hal ini juga karena nilai $3^n$ akan selalu lebih besar dari pada $n!$. Sehingga kondisi pola tersebut bernilai negatif selalu.
Factorial is defined for non-negative integers. So $n! - 3^n$ always be negative. Because the value of $3^n$ always be greater than $n!$. As a result, the formula is always negative.
Selain itu, dari pola $n = 4$ menunjukkan hasil bilangan bulat negatif yang semakin besar. Sehingga apabila untuk bilangan bulat faktorial yang lebih besar maka akan menghasilkan bilangan negatif yang semakin besar.
In addition, the result at $n = 4$ is a negative integer that gets bigger. So, for larger factorial integers, the larger negative numbers will be.

**Figure 5.** S3 written solution to problem with contradictory information



Tidak, karena polanya adalah
No, because the pattern is...
misal $n \in \mathbb{N}$ $P(n): n! - 3^n$
Let
nilai dari $3^n$ selalu lebih besar dari $n!$
The value of ... is always greater then
Perhatikan bahwa
Notice that,
$3^n > n!$
$3^1 > 1!$ Artinya $P(1)$ benar
It means that, ... is true
Ambil sebarang $k \in \mathbb{N}$. Kita asumsikan $P(k)$ benar dan
For ... We assume that ... is true, and
akan dibuktikan $P(k+1)$ benar
we will prove that ... is true
$P(k)$ benar, artinya $3^k > k!$
is true, it means that,
Dengan mengalikan kedua ruas dengan 3
Multiply both sides by 3, we will get
$3^k \cdot 3 > k! \cdot 3$
$3^{k+1} > 3k!$
Karena $3^k > k!$ maka $3 \cdot 3^k > 3 \cdot k!$
Because ... then
Dari pernyataan ini diperoleh $P(k+1)$ benar
From this statement, we get ... is true
Sehingga tidak akan terjadi pola yang bernilai selalu positif
So, the sequence can not reach a positive value and increase

**Figure 6.** S4 written solution to problem with contradictory information

## 3.2.2. Assessment's Subject Answered "No"

Based on the interview results, S3 and S4 observed four equations in the problem and formulated a pattern for the components on the left-hand side equations, namely Equation (1). S3 and S4 also found that the components on the right-hand side equations have various values, cannot be generalized, and are negative numbers. S3 and S4 used a trial and error strategy by entering the values of n=1,2,3,4,5,6. However, they did not continue the calculation, so they did not realize that a positive value would be obtained when n = 7.

$$3^n > n \quad (7)$$

S4 derived this claim by comparing the value of $3^n$ with n! for n=1,2,3,4,5,6. S3 derives this claim by considering the condition that the factorial number is a non-negative integer. From Figure 6, S4 proves her claim by using the Principle of Mathematical Induction. However, the S4 assessment is not sufficient to prove the truth of the assumption.

$$3^k > k! \quad (8)$$

$$3^{k+1} > 3k! \quad (9)$$

S4 multiplied both sides of Equation (8) by 3. This action aims to obtain the number $3^{(k+1)}$ on the left side. S4 discontinued its calculations and made an error in concluding that p(k+1) is correct, using Equation (9) as the reference.

On the other hand, S3 devised an idea to prove Equation (7) with the Principle of Mathematical Induction, but she did not realize it. This statement can be observed from the interview results as follows.

R: *How do you get Equation (1)?*
S3: *First, I looked at the four equations in the problem. I pay attention to the values on the right-hand side equations. I'm confused because the results vary. It cannot be generalized, and the numbers are always negative. Then I saw the components on the left side. In each of the equations, I see the factorial number subtracted to the power of 3. In the first row, there are 1! -$3^1$. In the second row, there are 2! -$3^2$, and so on until the fourth row. So I conclude, the components on the right-hand side form Equation (1).*
R: *You said that Equation (1) is negative. What's the reason?*
S3: *I concluded it from trial and error. I tried for n=5 and n=6. First, I calculated the value of n!. Second, I calculate the value of 3n. Then I put it in Equation (1), and the result is always negative.*
R: *Why don't you write a proof of that statement?*
S3: *I know that statement must be proven by the Principle of Mathematical Induction, but because of the results of trial and error, I did not find a positive result. I finally gave up.*
R: *You say that Equation (1) is negative, then you prove it by principle of mathematical induction, right?*
S4: *Yes.*
R: *In the proof of p(k+1), why do you multiply both sides by 3?*
S4: *Because I want to show the number $3^{k+1}$*

## 3.2.3. Inference's Subject Answered "No"

Starting from Equation (7), S3 and S4 concluded that the pattern in the problem will never meet the condition where the right-hand side is always positive. This statement can be observed in Figures 5 and Figures 6, as well as the results of interviews with the two subjects as follows.

S3: *Since factorial numbers consist of non-negative integers, I think that 3n is always greater than n!. Consequently, when we subtract n! with $3^n$, the result is always negative.*
S4: *I think of proof is correct. So, $3^n$ will always be greater than n!. So Equation (1) is always negative.*

### 3.2.4. Strategy's Subject Answered "No"

S3 and S4 used several strategies even though the arguments built are not sufficient to generalize the conclusions. The strategies were used by S3 include (a) formulates an idea of proving deductively through subtracting n! with $3^n$, (b) formulates an idea of proving inductively by looking at the results in the 4th pattern, (c) predicting that the pattern results will always be negative by entering the values of n=1,2,3,4,5, (d) describes the general form of the pattern in the problem, namely Equation (1), and (e) formulates the idea of proving Equation (7) but it is not realized. The strategies used by S4 include: (a) predicting the result of the pattern will always be negative by entering the values of n=5 and n=6, (b) develop the idea of proving Equation (7) using Principle of Mathematical Inductions, and (c) evaluate the calculation results using a calculator.

R: *You said that Equation (1) is negative. What's the reason?*
S3: *I concluded it from trial and error. I tried for n=5 and n=6. First, I calculated the value of n!. Second, I calculate the value of 3n. Then I put it in Equation (1), and the result is always negative.*
S4: *I think $3^n$ will always be greater than n!. I tried to enter n=5 and n-6, then I compiled the proof using the Principle of Mathematical Induction.*
R: *Are there any other strategies you use to prove that your statement is true?*
S4: *I evaluate the results of my calculations using a calculator.*

## 3.3. Critical Thinking Processes of Subject in Solving Truth-Seeking Problem with Contradictory Information

The truth-seeking problem with contradictory information given sequence that has negative values and their values are getting decreased. Then it was contradictory with the question of possibilities for a condition where the value is positive and increasing. It contains conflicting data. Truth-seeker is accustomed to analysing the irregularities in the question and checking the truth before trusting [25]. Truth-seeker strongly emphasizes the proof and reasoning even on problem that have been recognized as true [16]. The subjects' processes of critical thinking in this study have similarities and differences in every stage of clarification, assessment, inference, and strategy.

In the process of critical thinking clarification as Table 1, the four subjects did not write completely on the results of the answer related to the purpose of the problem. However, the results of the interview support that each subject is able to describe the main objective of the problem precisely and accurately. At the clarification stage, students mention important information from what kind of questions are known then raise that the main problem is checking the truth of a given statement. [10,11]. The clarification stage can also be in the form of reformulating the problem into facts, symbols, and other representations precisely and clearly [1,2]. Students need to declare or write down that the problem has contradictory information becomes one of the determinants in order to find the right answer [16].

Furthermore, at the assessment stage as Table 1, the subject answered "Yes" and was able to provide the proper criteria to draw conclusions. They also reviewed the results of the n factorial pattern for larger natural number values, but this was not the case for the subject who answered "No". They did not manage to find the turning point of a natural number that caused a positive value pattern. However, all four subjects could state the required criteria or reasons appropriately, although on the subject answered "No", do not manage to reach a mathematically valid conclusion. At the assessment stage, students sort out important information and also provide reasons and proof to corroborate their statements [10,26]. Indicators such as students checking the process of the truth of information provided on the problem before solving it are also significant in dealing with a problem with contradictory information [16]. Therefore, this process becomes one of the most important parts so that the student's critical thinking process can be directed to the precise solution and reach a valid conclusion.

The process of critical thinking of inference categories in the four subjects appears with different nuances as Table 1, if in the subject the answer "Yes" before writing the criteria to conclude, they already have a hypothesis through the process of induction reasoning carried out. Then, conclude at the end after the proof with Mathematical Induction is presented. In contrast to the subject answered "No", they do not recognize the contradictory information presented, then use the information given carelessly to conclude. The proof presented by mathematical induction also does not produce precise conclusions. One of the characteristics of a critical thinker is always seeking alternative hypotheses, explanations, conclusions, plans, sources, etc., and being open to them [27]. Students' knowledge of how to think critically in solving mathematics problems seems not owned by some students yet [28]. Students are less precise in making the settlement plan and the steps, do not resolve the problem accurately [29].

The last stage of the critical thinking process is strategy as Table 1. It appears to be very contrasting in that the subject answered "Yes" establishes a trial-error action to find a result that meets the criteria of the problem, whereas this does not appear in the subject answered "No". The proofing method with mathematical induction used by the subject always fails on the bridge step. This shows that students have not been able to be good executors in providing proof. Students need to reconnect the explanation results with other concepts that may be related and try to rework the new information to generate problem solutions in several different ways [11]. They have to use all the correct information and the universal set given in the problem in the problem-solving process [16]. In addition, attitude in critical thinking is also a factor in failure in solving problems, such as student involvement, limited motivation, lack of scepticism in solving mathematical problems [28,30].

## 4. CONCLUSION AND IMPLICATIONS

The test results revealed that 17% of participants used their critical thinking process and managed to get a “Yes” answer, while 73% answered “No” and others answered unclearly. These data indicate that students' critical thinking processes have not been able to direct students reflectively and reasonably conclude when facing truth-seeking problems with contradictory information. The critical thinking process stages that are dominant in students with the answer "Yes" are assessment, inference, and strategy. These three stages determine the participants' success in recognizing contradictory information and determining the valid truth of the truth-seeking problem. The strategy has a bigger role. An action or step taken can complete the continuity of the assessment and inference process until the truth statement determination can be obtained as the primary goal of the truth-seeking problem.

The clarification stage appears with good criteria for subjects who answered "Yes" and students who answered "No". Although they do not write down complete information related to the problem, they can explain the primary purpose of the problem precisely and accurately. At the assessment stage, students who answered "Yes" could provide appropriate criteria for drawing conclusions and reviewing possible results for larger natural number values, but this was not the case for students who answered "No". At the inference stage, students who answered "Yes" already had a hypothesis through the reasoning induction process that was carried out before they provided proof of mathematical induction. In contrast to students who answered "No", they did not recognize the contradictions in the information presented, then they used the information provided carelessly to conclude. The strategy stage appears in contrast. Students who answered "Yes" set a trial and error action to produce other possibilities that met the criteria for the problem. On the other hand, students who answered “No” did the calculations again through the information presented. However, the proof result by mathematical induction shows that the subject has not been able to be good executors in providing proof.

Based on these results, further study of the critical thinking process of students with other criteria, such as those who are unable to provide decisions on a truth-seeking problem. In general, students' critical thinking skills in solving truth-seeking problems still need to be improved, especially on problems with contradictory information. Students need to use all their resources to gain the best understanding and realize how to think critically. In addition, the weakness of students at the strategy stage makes the critical thinking process stop. Therefore, further studies to improve students' critical thinking still need to be explored, especially on strategy indicators.

## AUTHORS’ CONTRIBUTIONS

The first author explored the topic, examined the research problem, and formulated the practice of the method. The first author collaborated with the second author to collect and analyse the data, discuss the result, and draw the research conclusion.

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