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Providing a Proof and Providing Non-Proof Argument

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ABSTRACT
This research aimed to study about providing a proof and providing non-proof argument. The study began by explaining some ways of verification in math accompanied by troubles faced by students in proof process. Then, it continued by explaining the differences between providing a proof and providing non-proof argument by giving example cases related to verification. Providing a proof using math induction way and providing non-proof argument using rational thinking. This research is a literature study on providing a proof and providing non-proof arguments that are reviewed based on theories and research relevant to this research. Result of the study show that, providing a proof is needed for proving or confirming a form of math statements which are appropriate for university students hence their cognitive development had been in abstract stage. Whereas, providing non-proof argument inclined explains history in a form of particular statement. Thus, it considered as important in learning math at school about providing a proof and providing non-proof argument need to be customized based on the level of students' cognitive development.

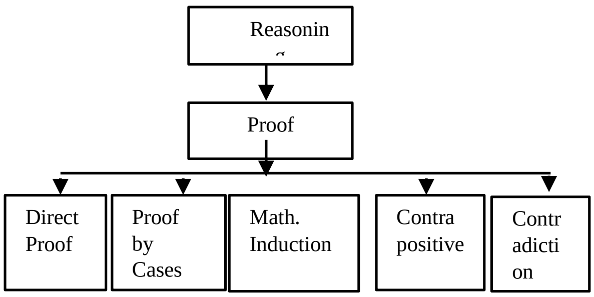
Keywords: *Proof, Non-Proof, Argument*

1. INTRODUCTION

Natures of philosophy are radical thinking and deeper thinking source. If someone is demanded for thinking, the person will always ask "Why". Question "Why" is the first step of reasoning activity in math and needs an answer in the form of an explanation [1]

This explanation used for proving math knowledge in order to correct particular theory [2]. Furthermore, he pointed out that proof is sequence of logic proof that a thing can describe others in which giving explanation why a statement is true. Theorem that has been set before can be used for a new summary; one can refer to axiom which become starting points or rules which are accepted by people is unconditional, meaning that if there is a theorem proved, then the theorem is verified forever [3]. In philosophy, proof is an absolutism. Someone who has this concept will tend to think that proof in math is absolutely true and it can't be revised.

Proof in math becomes a part that cannot be separated from reasoning [4, 5]. Eventually, mathematically proof is a formal way to express kinds of reasoning and certain justification. Figure 1 shows that reasoning and proof position with some kinds of proof [6] explained specifically as following.



1.1. Direct Proof

Direct proof is the easiest proof method for determining theorem, hence does not need knowledge about special technique. Direct proof usually used for proving theorem in implication form $p \to q$. Meaning that, statement $p \to q$ correct where p is known correct.

Example. For instance m and n are integer. Then, if m and n are even, thus $m + n$ is even.

Proof. If m and are even, thus there is integer k and j therefore $m = 2k$ and $n = 2j$. Then, $m + n = 2k + 2j = s(k + j)$. Hence $k, j \in Z, (k + j) \in Z. \therefore m + n$ is integer.

Student's Obstacles in Direct Proof

Students' hindrances in direct proof is they are difficult to determine initial step which should be done and sometimes they are inconsistent to give evidence because they are only memorizing [7, 8]

1.2. Proof by Case

In proving case, one thing should be done is finding out a special case that verify no correctness of a statement.

Example. Statement: "For every n of original number, then $2^{2^n}+1$ is the prime number.

Proof: This statement is true for every original number of n. If we substitute n value using number 3, it results on 257, If n = 4, the result is 65537. Both numbers are prime numbers that prove correctness. However, if n=5, the result is 4294967297 as non prime numbers. This example is a case that proves incorrectness of statement.

Student's Obstacles in Proof by Case

Students may not find a disclaim case since using prime numbers and cause complicated calculation and they are may not be able to specify general cases [2]

1.3. Mathematical Induction

Mathematical induction is often used as a proof theorem. The principle is: For every positive integer n, for example $P(n)$ is a dependent statement on n. If $P(1)$ correct and every positive integer number k if $P(k)$ correct thus $P(k+1)$ correct as statement $P(n)$ is worth true for all positive integers number n. In this mathematical induction, there are three important points that become basic principle of induction as following. Induction Base, check if $P(1)$ correct, statement effective for $n=1$. Induction Hypotheses, assuming $P(k)$ correct, statement effective for $n=k$. Induction Step, shows that if $P(k)$ correct, thus $P(k+1)$ also does.

Example. Show that $\mathbf{2^{3n+1}+5}$ is always a multiple of !

Proof. The statement $\boldsymbol{P(n)}$: $\mathbf{2^{3n+1}+5}$ is always a multiple of $\mathbf{7}$.

Induction Base $(\boldsymbol{n=1})$

$$\mathbf{2^{3(1)+1}+5=2^4+5=16+5=21=7\times 3}$$

$\therefore \boldsymbol{P(1)}$ holds.

Induction Hypothesis. Assume that $\boldsymbol{P(k)}$ is terue, so $\mathbf{2^{3k+1}+5}$ is always a multiple of $\mathbf{7}$, $\boldsymbol{k\in N}$

Induction Step. We want to show that $\boldsymbol{P(k)\rightarrow P(k+1)}$, where $\boldsymbol{P(k+1)}$: $\mathbf{2^{3(k+1)+1}+5=2^{3k+4}+5}$ is a multiple of $\mathbf{7}$. We know from induction hypothesis that $\mathbf{2^{3k+1}+5}$ is always a multiple of $\mathbf{7}$, so we can write $\boldsymbol{2^{3k+1}+5=7\times x}$ for some $\boldsymbol{x\in Z}$

$$\boldsymbol{(2^{3k+1}+5)\times 2^3=7\times x\times 2^3}$$

$$\boldsymbol{(2^{3k+4}+5)+40=7\times x\times 8}$$

$$\boldsymbol{2^{3k+4}+5=56x-35}$$

$$\boldsymbol{2^{3k+4}+5=7(8x-5)}$$

So $\mathbf{2^{3k+4}+5}$ is a multiple by $\mathbf{7}$ holds $\boldsymbol{P(k+1)}$ is true, provided that $\boldsymbol{P(k)}$ is true. According to the step of mathematical induction, we have snown that $\boldsymbol{P(1)}$ holds, $\boldsymbol{P(k+1)}$ holds then $\boldsymbol{P(k)}$ also true. It follows that $\boldsymbol{P(n)}$ holds for all natural $\boldsymbol{n}$.

Student's Obstacles in Mathematical Induction.

Students can do procedural mistakes by passing one step proof of induction and they are confused to reduce numbers in induction step [9,10].

1.4. Proof by Contrapositive

Correctness value is an implication of proof $p\rightarrow q$ equivalent with the counter balance value is$-q\rightarrow -p$. Therefore, procedure of proof by contrapositive apply proof through the counter balance.

Example. Show that, $n^2 even\rightarrow n\ even$!

Proof. For solving this, we use contrapositive.

n^2odd⇒ n odd.

$$n=2k+1 n^2=(2k+1)^2 n^2=4k^2+4k+1 n^2$$
$$=2(2k^2+2k)+1 n^2=2l+1$$

⇒holds

So, n^2 even ⇒ n even is true.

Student's Obstacles in Proof by Contrapositive.

Students write implication statement as should be shown and they don't know contrapositive form thus proves by giving example.

1.5. Proof by Contradiction

Contradiction happened if there is one or more contradiction statements. For example, contradiction statement : $\mathbf{1=2}$, $\boldsymbol{-1<a<0}$ and $\boldsymbol{0<a<1}$, "$\boldsymbol{m}$ and $\boldsymbol{n}$ two integer number relative prime" and ""$\boldsymbol{m}$ and $\boldsymbol{n}$ both integer number".

Example. For instance A compilation defined as half open interval $\boldsymbol{A\coloneqq[0,1)}$. Maximum prove there is no A.

Proof. If it is written in implication form as following.

"if $\boldsymbol{A\coloneqq[0,1)}$ thus there is no A maximum"

If there is maximum A, case $\boldsymbol{p}$. Thus constitute $\boldsymbol{0<p<1}$ that cause $\frac{1}{2}\boldsymbol{p}<\frac{1}{2}$and $\frac{1}{2}\boldsymbol{(p+1)<1}$, so

$$\boldsymbol{p=\frac{1}{2}p+\frac{1}{2}p<\frac{1}{2}p+\frac{1}{2}=\frac{1}{2}(p+1)<1}$$

Covered two statements are: $\boldsymbol{p}$ maximum A, is the biggest element from A compilation. There is $\boldsymbol{q} \in \boldsymbol{A}$ ($\boldsymbol{q} \coloneqq \frac{1}{2}(\boldsymbol{p}+\mathbf{1})$ is bigger than $\boldsymbol{p}$
The two statements are contradict, suppose A has wrong maximum thus there is no maximum.

Student's Obstacles in Proof by Contradiction

Students can't untangle contradiction forms a statement [11]

2. METHOD

Method in this research is a literature study where the data comes from previous studies and theories that support the literature study of this research. The data collection in this research is by examining literature books, technique of collecting data is by reviewing literature books, reports, and studies that have a relationship with proof in learning mathematics. So this research literature study describes the difference between providing a proof and providing non-proof argument by providing examples of questions related to these two things and explain the importance of both proofs in learning mathematics.

3. RESULT AND DISCUSSION

Table 1. Mathematical component of reasoning and proof

	Reasoning and Proving	
Mathematical Component	Providing Support to Mathematical Claims	
	Providing a Proof	Providing Non-Proof Argument
	• Generic Example • Demonstration	• Empirical Argument • Rationale

Learning math in school, [5] to prove reasoning and proof as fundamental aspect in math. Someone which has reason and analytical thinking tend take no pattern, structure, orderliness in real world situation and symbolic object; they will ask is there any that unpredictable pattern happened because any particular reason, and they are carious then trying to prove. Proof in curriculum 13 for math mostly discuses about proof form is math induction.

Proof activity in school mostly play a part on asking student to verify and confirm which statement is specific into more general form. It trains students for testing the truth of any particular statement. Whereas, based on the nature of radically thinking philosophy, students have ever directed to think why a special statement has its general form. It is obvious here, that there are two different things namely proof for proving and proof for explaining.

Proof for proving and proof for explaining are similar to a research which has done by [2] said that, there are two math components in proof which are providing a proof and providing non-proof argument. First, providing a proof is proof of a valid argument based on the truth that can be accepted in order o receive and reject a math problem. Definition of "valid" shows that an explicit statement can be accepted if proven correctly, definition of "accepted truth" used extensively included by axiom, theorem, definition, reasoning ways and representational tool used for proving. Second, providing non-proof argument regards proof as argument for opposing mathematics' claims which is not eligible as a proof. For example, argument is claimed as reason (vs. proof). First category is providing a proof using generic example and demonstration one of math proof. Meanwhile, second category is providing non-proof argument using rational argument or opinion. To differentiate both of this categorize, look at the example bellow.

1. Proof that $1.2+2.3+3.4+4.5+\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

2. Calculate the result of $1.2+2.3+3.4+4.5+\cdots+n(n+1)$

Two statements above are absolutely different. First statement as providing a proof form whereas second statement as providing non-proof argument form. According to [12], providing a proof only shows that a theorem is right. Answer for first statement through math induction as the following explanation.

The statements, $P(n)\colon 1.2+2.3+3.4+4.5+\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

Induction Base $(n=1)$

$$1.2=\frac{1(1+1)(1+2)}{3}\ 2=\frac{6}{3}$$

$\therefore P(1)$ hold.

Induction Hypothesis. Assumes that $P(k)$ correct, so $1.2+2.3+3.4+4.5+\cdots+k(k+1)=\frac{k(k+1)(k+2)}{3}$, $k \in N$

Induction Step. Show that $P(k) \to P(k+1)$, where $P(k+1)\colon 1.2+2.3+3.4+4.5+\cdots+k(k+1)+k+1[(k+1)+1]=\frac{k+1(k+2)(k+3)}{3}$ for proving similarities of the above form we try to simplify the left segment into the right segment form.

The following is left segment;

$$1.2+2.3+3.4+4.5+\cdots+k(k+1)_{\smile}+k+1[(k+1)+1]$$

$$= \frac{k(k+1)(k+2)}{3} + k + 1[(k+1)+1]$$

$$= \frac{k(k+1)(k+2)}{3} + \frac{3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

Evidently, the left segment form = the right segment form thus for $P(k+1)$ is correct proven and effect $P(k)$ also correct. So $P(n)$ Correct for all positive integer numbers n.

Mathematicall induction becomes a strong proof form usually used for determining validity of statement that is given in original number case [13]. But, it is contradictory with [12] who said that, proof and induction are generally unclear. Hence on math induction incomplete to explain why $1.2 + 2.3 + 3.4 + 4.5 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

Even though, math system on math induction is incomplete, based on Godel's opinion math system is still consistent on math induction stage. For overcoming the incompleteness, there must be other proof that explains the second statement. Therefore, number 2 can be answered using as following pattern number.

$$1.2 + 2.3 + 3.4 + 4.5 + \cdots + n(n+1)$$

Table 2. Result

S_n	Result	Pattern
$S_1 = 1.2$	2	$\frac{1(2)(3)}{3}$
$S_2 = 1.2 + 2.3$	8	$\frac{2(3)(4)}{3}$
$S_3 = 1.2 + 2.3 + 3.4$	20	$\frac{3(4)(5)}{3}$
$S_4 = 1.2 + 2.3 + 3.4 + 4.5$	40	$\frac{5(6)(7)}{3}$
$S_5 = 1.2 + 2.3 + 3.4 + 4.5 + 5.6$	70	$\frac{6(7)(8)}{3}$
...	...	...
$S_n = 1.2 + 2.3 \ldots + n(n+1)$	...	$\frac{n(n+1)(n+2)}{3}$

Proving a proof also needs to explain some proof, thus its purpose or clarify a proof is for explaining the beginning of a proof by using the consist knowledge in math [14]. The purpose of proof activity in school is to train students' ability in processing their brain for solving proof problems in order to reach the objective of learning math [15].

4. CONCLUSION

Proof in learning math is needed for separating between providing a proof and providing non-proof argument. Proof that should be taught to students for improving their knowledge about reason related to the theorem pattern, meanwhile the proof explanation needs to be taught so the students can train their way of thinking in getting particular results. Proof study needs to pay full attention on students' cognitive development that proof is a strong abstract thing should be mastered

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