

Students' Mathematical Concepts Understanding on Fractional Number Operations Using the Discovery Learning Models

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ABSTRACT

This research is a research design with a qualitative descriptive type of research that aims to know mathematical concept understanding skills in fraction operation using Discovery Learning Model. Understanding mathematical concepts is an essential ability possessed by students in learning mathematics. This research was conducted by following the learning system in schools. This research uses instruments in the form of written tests and interview guidelines. The research subjects were students of class VII.6 SMP Negeri 4 Prabumulih, while the focus of the study was three students with varying abilities, were selected based on the results of the tests carried out later. The research was carried out in three stages, namely: (1) the Preparation Stage, (2) the implementation stage, (3) The final stage is data analysis. The data obtained will be analyzed by collecting, reducing, presenting, and drawing conclusions. The results showed that the ability to understand the concept of students on the material of number operations using the Discovery Learning model has emerged. The indicators that appeared the most are stating a concept, clarifying objects according to specific properties according to the idea, and presenting the idea in various forms of mathematical representation. Meanwhile, indicators that rarely appear are using, utilizing, and selecting particular procedures or operations and applying concepts or algorithms to solve problems.

Keywords: *Concepts Understanding Skills, Discovery Learning Models, Fractional Number Operations.*

1. INTRODUCTION

Currently, the applicable curriculum in Indonesia is the 2013 curriculum. One of the objectives of learning mathematics in the 2013 curriculum is to have factual and conceptual abilities in science. In the Permendikbud Number 21 of 2016 concerning the content standards of primary and secondary education, it is stated that one of the core competencies in the field of knowledge that high school students must possess is the ability to understand concepts [1].

One of the learning abilities students need to improve the quality of learning, especially in mathematics, is understanding concepts [2,3]. In the National Council of Teachers of Mathematics, students' knowledge and understanding of mathematical concepts can be seen from students' abilities in (1) defining concepts verbally and in writing; (2) identifying and creating examples and non-examples; (3) using models, diagrams, and symbols to represent a concept; (4) changing one form of representation to another; (5) recognize various meanings and interpretations of concepts; (6) identify the

characteristics of the concept and recognize the conditions that determine a concept; (7) compare and contrast concepts [4]. Understanding concepts is the basis for doing mathematics. That is, every student is required to have a good understanding of concepts to solve mathematical problems. Meanwhile, understanding the concept is very important in solving mathematical problems or problems in everyday life in learning mathematics [5,6]. So that teachers should understand the understanding of concepts to plan the mathematics learning process well so that student's understanding of concepts can increase.

One of the critical mathematics learning materials taught in schools is Fractions. One of the basic materials in learning mathematics is fractional numbers [7]. In Permendikbud No. 37 of 2018, one of the essential materials to learn in fractional operations is adding and subtracting fractional numbers. Fractions are critical topics for students as a basis for studying algebra and others, but many do not understand it [8]. In addition, Siegler et al. revealed that students' knowledge of fractions would affect mathematics learning in the future

and fractions are also commonly used in everyday life [9].

However, students still have difficulty in learning fraction material. The research results show that students have difficulty solving fractions problems due to a lack of understanding of students' concepts in fractional material [10]. In addition, the difficulty in learning fractions material is that students have not been able to add fractions with unequal denominators and write fractional addition and subtraction operations [11].

Based on the research results, students' difficulties in solving fractional arithmetic operations material problems are difficulties in using concepts, principles and difficulties in solving verbal problems [12]. In addition, the way the teacher explains the addition and subtraction of fractions is still using the lecture method, namely the teacher explains how to solve math problems on the blackboard, and then the students imitate how to solve the issues that have been written by the teacher on the blackboard [13]. This causes a lack of understanding of students' mathematical concepts in completing fractional arithmetic operations.

One of the efforts to improve students' understanding of mathematical concepts in fractional material is to select a suitable learning model. According to the 2013 Curriculum in Permendikbud No. 22 of 2016, choosing and using the right learning model in teaching and learning is necessary [14]. One of the learning models that can be used as an alternative to these problems is the Discovery Learning model. The research results show that the Discovery Learning model is an active learning model that encourages students to absorb knowledge by formulating, discovering, and concluding individually to create new knowledge using relationships from related previous understanding [15]. In this learning model, students will be more guided in understanding the concepts in the material taught by the teacher and how students learn to actively find concepts independently where these concepts can be easier to remember and difficult to forget.

Discovery learning is a learning process in which a concept is not presented in final form, but students are required to organize their way of learning in finding concepts [16]. In addition, one of the advantages of the discovery learning model, according to the Ministry of Education and Culture, is that students will understand basic concepts and ideas better. Discovery Learning is a model that can make students learn actively by finding concepts [17].

Based on the description above, it is necessary to conduct learning that helps students improve their mathematical concept understanding skills, considering the importance of students' mathematical concept understanding skills in fractional number operations material. Therefore, researchers want to understand students' mathematical concepts on fractional number

operations using the Discovery Learning model. The research title is "Ability to Understand Students' Mathematical Concepts on Fractional Number Operations Using the Discovery Learning Model."

2. METHOD

2.1. Research Design

This research is qualitative descriptive research that aims to determine the ability of students to understand the concept of number operations using the Discovery Learning model. The research subjects are a class of VII.6 in SMP Negeri 4 Prabumulih students. This research focuses on understanding students' mathematical concepts after using the Discovery Learning model on fractional number operations. In this study, the ability to understand mathematical concepts of students was measured through several indicators of the ability to understand mathematical concepts, namely (1) restating an idea, (2) clarifying objects according to specific properties according to the concept, (3) presenting concepts in various forms of mathematical representation, (4) using, utilizing, and selecting specific procedures or operations, (5) applying concepts or algorithms to solve problems.

This research was conducted through 3 stages, namely: (1) the preparation stage (2) the implementation stage, (3) the final stage [18].

2.2. Procedure

This preparatory stage is the initial stage of research. The preparation stage of this research is to prepare a proposal, administer a research permit, contact the school that will be used as a research location, prepare instruments in the form of written test questions and teaching materials in the form of lesson plans and worksheets, along with discovery learning models and fractional number operation materials, as well as validation of research instruments and materials. Teach with education experts.

Furthermore, after the researchers carried out the preparation stage, the researchers carried out learning activities following the RPP that had been prepared. Learning is carried out for two meetings. At the third meeting, the researchers gave a written test in 5 questions that matched the indicators of understanding mathematical concepts to students in class.

After the research activities have been carried out, the researcher proceeds to the final stage, where the researcher collects data from the results of tests that students have carried out at the last meeting. Furthermore, the data that has been obtained is analyzed based on which indicators appear on the understanding of students' mathematical concepts. After being diagnosed, the researcher draws conclusions, where the findings generated can answer questions from the formulation of the problem.

3. RESULT AND DISCUSSION

Students' understanding of mathematical concepts was obtained from tests conducted by participants at the third meeting. The test is given to see the students' ability to understand mathematical concepts presented using the discovery learning model. The questions consisted of 5 questions, with each question containing one indicator of the ability to understand concepts. The test is given in class and is only followed by 24 students. From the number of students who have taken the test, three subjects will be selected for analysis. The three subjects

Table 1. Table code indicators

Question Number	Code of Indicators	Indicators	Descriptor
1	A1	Restate a concept	Rewrite existing concepts using their sentences.
2	A2	Clarifying objects according to specific properties according to the concept	Clarifying objects according to specific properties according to the concept.
3	A3	Presenting concepts in various forms of mathematical representation	Presenting a concept in various forms of representation such as pictures or mathematical sentences.
4	A4	Using, utilizing, and selecting specific procedures or operations	Using the proper steps in solving problems.
5	A5	Applying concepts or algorithms to solve problems	Using the right concepts and steps to solve everyday life problems.

were taken based on the test results; namely, all indicators are met, only three indicators are met, and one indicator is met.

3.1. Analysis Test

From the table above, there are five questions. Each question contains one indicator. The first question contains one indicator, namely the A1 indicator. The second question contains one indicator, namely the A2 indicator. The third question contains one indicator, namely the A3 indicator. The fourth question contains one indicator, namely the A4 indicator. And the fifth question contains one indicator, namely the A5 indicator. The following is a complete analysis of the data obtained.

1. Answer the questions below using your sentences!

- How do you add and subtract fractions with the same denominator?
- How do you add and subtract fractions with different denominators?

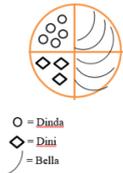
Figure 1 Question number one

2. Group the following fractions based on their denominators!

- $\frac{1}{2} - \frac{3}{4}$
- $\frac{2}{5} + 2\frac{1}{5}$
- $\frac{3}{4} - \frac{1}{4}$
- $2\frac{1}{10} + \frac{3}{8}$
- $\frac{123}{200} + \frac{233}{300}$

Figure 2 Question number two

3. Today is Dini's birthday. Dini brings pizza and wants to share the pizza with Bella and Dinda. The following is a picture of a pizza distribution in the shape of a circle.



○ = Dinda
 ◇ = Dini
 — = Bella

- Make a mathematical sentence from the picture of the division of the pizza!
- How many pieces of pizza do Bella and Dinda have?
- How much share does Dini still have after the pizza is given to her two friends?

Figure 3 Question number three

4. Uncle has a plot of land behind his house. $\frac{1}{3}$ of the land is planted with banana trees, while the other $\frac{4}{9}$ is planted with corn trees, and the rest is made into fish ponds.

- How much land is planted with banana and corn trees?
- How much land does the fish pond make?

Figure 4 Question number four

5. The Gita has a $5\frac{1}{2}$ m long ribbon for wrapping gifts. But Gita's tape was lacking, so Gita bought another $\frac{1}{3}$ m long ribbon. What is the total length of ribbon that Gita needs to wrap the gift? And explain how do you find the answer?

Figure 5 Question number five

3.1.1. Analysis of Understanding Mathematical Concepts of N Subjects

N subjects are categorized as competent subjects, because the subject meets all indicators of the ability to understand mathematical concepts. The following is an analysis of understanding mathematical concepts of N subjects:

In question number one, the subject of N has met the A1 indicator where the subject has been able to add and subtract fraction operations in his sentence repeatedly. This can be seen from the results of the answers from the N subject.

- a. tinggal ditambahkan/dikurangkan pembilangnya
- b. Sama kan dahulu penyebutnya dengan cara kpk

Figure 6 Answer from subject N for question number one

In question number two, the N subject met the A2 indicator, where the subject was able to classify those with the same denominator and those with different denominators. This can be seen from the results of the test answers from N subjects.

yang penyebutnya sama adalah b dan c
yang penyebutnya berbeda a, d dan e

Figure 7 Answer from subject N for question number two

In question number three, the N subject has met the A3 indicator. It can be seen that subject N, in part a, has made a mathematical model from the image presented. In parts b and c, subject N has also answered questions based on the pictures presented in question 3.

a. Dari 1 pizza yang dibawa dimi dibagi menjadi 4 / yang Dinda ditandai dengan bucat, yang Bela ditandai dengan persegi, dan yang Bela ditandai dengan garis yang melengkung. Dinda mendapatkan $\frac{1}{4}$ bagian pizza. Dini mendapatkan $\frac{1}{4}$ pizza dan Bela mendapatkan $\frac{2}{4}$ bagian pizza

b. Bela memiliki $\frac{2}{4}$ pizza dan yang Dinda $\frac{1}{4}$ jika ditambahkan maka hasilnya adalah $\frac{3}{4}$ bagian pizza
$$\frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$$
 bagian pizza

c. Pizza Dini adalah lingkaran yang dibagi menjadi 4. Lalu ia bagi kepada kedua temannya $\frac{2}{4}$, $\frac{2}{4}$ milik Bela. Sedangkan Dinda $\frac{1}{4}$ jadi sisa dari pizza yang dibawa Dini adalah $\frac{1}{4}$ bagian pizza yang dimiliki Dini adalah $\frac{1}{4}$ bagian
$$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$$

Figure 8 Answer from subject N for question number three

In question number four, the N subject has also met the A4 indicator. It can be seen that subject N has been able to work on the addition and subtraction operation of the fraction by using the right steps in solving it. Subject N works on part a by first equating the denominators because the denominators are different, namely the LCM of 3 and 9, which is 9. For part b, subject N immediately subtracts the denominators because the denominators are already the same.

a. $\frac{1}{3} + \frac{4}{9} = \frac{1 \times 3}{3 \times 3} + \frac{4 \times 1}{9 \times 1} = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

b. $1 - \frac{7}{9} = \frac{9}{9} - \frac{7}{9} = \frac{2}{9}$

Figure 9 Answer from subject N for question number four

In question number five, the N subject has also met the A5 indicator. It can be seen that subject N has been able to work on the operation problems of adding fractions related to everyday life by using the right concepts and steps in solving them. Subject N has converted mixed fractions into common fractions to make it easier to solve. Then subject N was able to equalize the fraction's denominator first because the fraction's denominator was different, namely the LCM of 2 and 3 was 6. Then subject N added them up.

$$5\frac{1}{2} + \frac{1}{3} = \frac{11}{2} + \frac{1}{3} = \frac{11 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2} = \frac{33}{6} + \frac{2}{6} = \frac{35}{6}$$

dengan cara dengan cara merubah pecahan campuran menjadi pecahan biasa dan menyamakan dahulu penyebutnya.

Figure 10 Answer from subject N for question number five

3.1.2. Analysis of Understanding Mathematical Concepts of EDP Subjects

EDP subjects are categorized as moderately capable subjects. Because the subject only meets three indicators of understanding mathematical concepts from five existing indicators. The following is an analysis of understanding mathematical concepts of EDP subjects:

In question number one, the subject of EDP has met the A1 indicator where the subject has been able to add and subtract fraction operations in his sentence repeatedly. This can be seen from the results of the answers from the EDP subject.

<input checked="" type="checkbox"/>	Jawablah soal dibawah ini, menggunakan kalimat mu sendiri!
<input type="checkbox"/>	a. bagaimanakah cara menjumlahkan dan mengurangkan pecahan yg berpenyebut sama?
<input type="checkbox"/>	Jawab: dengan menambahkan pembilang keduanya.
<input type="checkbox"/>	b. bagaimanakah cara menjumlahkan dan mengurangkan pecahan yg berpenyebut berbeda?
<input type="checkbox"/>	Jawab = dengan mencari KPK dari penyebutnya

Figure 11 Answer from subject EDP for question number one

In question number two, the EDP subject met the A2 indicator, where the subject was able to classify those with the same denominator and those with different denominators. This can be seen from the results of the test answers from EDP subjects.

Jawab = Pecahan yg penyebutnya sama adalah huruf <u>b</u> dan <u>c</u>
= Pecahan yg penyebutnya berbeda adalah huruf <u>1</u> , <u>d</u> dan <u>e</u>

Figure 12 Answer from subject EDP for question number two

In question number three, the EDP subject meets the A3 indicator. It can be seen that the EDP subject in part a has been able to create a mathematical model from the images presented. In parts b and c, EDP subjects could also answer questions based on the pictures presented in the questions.

a. buatlah kalimat matematika dari gambar pembagian Pizza tersebut.
Jawab = Dinda = $\frac{1}{4}$ Bella = $\frac{2}{4}$
Dini = $\frac{1}{4}$

b. berapa bagiankah jumlah Pizza yg dimiliki oleh bella dan dinda
Jawab = $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
c. berapa bagiankah yg masih dimiliki oleh Dini setelah Pizza tersebut diberikan kepada kedua temanya?
Jawab = $\frac{1}{4}$

Figure 13 Answer from subject EDP for question number three

In question number four, the subject of AF has not met the A4 indicator. It can be seen that the EDP subject has not been able to work on the addition and subtraction operation problems of these fractions by using the right steps in solving them. The EDP subject did not understand the meaning of the question and did not understand how to solve it, so the EDP subject could not answer the question in question number 4.

a. berapa bagian tanah yg ditanami pohon Pisang dan Jajug ?
$\frac{1}{9}$
b. berapa bagian tanah yg dibuat kolam ikan = $\frac{7}{36}$

Figure 14 Answer from subject EDP for question number four

In question number five, the subject of EDP has met the A5 indicator. It can be seen that the EDP subject has not been able to work on the fraction addition operation problems that are related to everyday life by using the right concepts and steps in solving them. EDP subjects have first converted mixed fractions into common fractions to make them easier to solve. However, the EDP subject has not been able to equalize the denominator of the fractions; the EDP subject immediately adds up the fractions regardless of the denominator. So the answer from the EDP subject on question number 5 is still wrong.

Dik = gita mempunyai Rita sepanjang $5\frac{1}{2}$ dan ia membeli lagi $\frac{1}{3}$

Dit = pita diperlukan gita untuk membungkus kado adalah

$$\text{Dijawab} = 5\frac{1}{2} + \frac{1}{3} = \frac{11}{2} + \frac{1}{3} = \frac{12}{3}$$

Figure 15 Answer from subject EDP for question number five

3.1.3. Analysis of Understanding Mathematical Concepts of AP Subjects

AP subjects are categorized as low-ability subjects. Because the subject only fulfills one indicator of understanding mathematical concepts from the five existing indicators. The following is an analysis of understanding the mathematical concepts of AP subjects:

In question number one, the subject of AP has met the A1 indicator where the subject has been able to add and subtract fraction operations in his sentence repeatedly. This can be seen from the results of the answers from the AP subject.

jawab

a. caranya adalah mengarangkan pembilangnya

b. caranya adalah mengubah pecahan tersebut menjadi pecahan yg memiliki penyebut yg sama dan pembilang yg sesuai

Figure 16 Answer from subject AP for question number one

In question number two, the subject of AP does not meet the A2 indicator. It can be seen that the subject of AP has not been able to name and distinguish between fractions with the same denominator and fractions with different denominators. AP subjects misunderstood the question's meaning, wherein question number 2 they were only asked to group fractions with the same denominator and different denominators. At the same time, the subject of AP answered the question by operating it.

2. kelompokkan pecahan berikut berdasarkan penyebutnya!

a. $\frac{1}{2} - \frac{3}{4} = -\frac{2}{4}$ c. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

b. $\frac{2}{5} + 2\frac{1}{5} = \frac{2}{5} + \frac{11}{5} = \frac{13}{5}$ d. $2\frac{1}{10} + \frac{3}{8} = \frac{21}{10} + \frac{3}{8} = \frac{29}{40} = \frac{3}{5}$

e. $\frac{123}{200} + \frac{233}{300} = \frac{356}{600} = \frac{18}{60} = \frac{3}{10}$

Figure 17 Answer from subject AP for question number two

In question number three, subject AP meets indicator A3. It can be seen that the AP subject, in part a, has been able to make mathematical models from the images

presented. In part b, the subject of AP has also been able to answer questions based on the pictures presented in the questions. However, in part c, the subject of AP has not been able to answer the question based on the picture presented.

jawab

a. Dinda = $\frac{1}{4}$ bella = $\frac{2}{4}$

Pini = $\frac{1}{4}$

b. $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

c. (1) bagian

Figure 18 Answer from subject AP for question number three

In question number four, the subject of AP has not met the A4 indicator. It can be seen that the subject of AP has not been able to work on the addition and subtraction operation of the fraction by using the right steps in solving it. The AP subject did not understand how to solve it, so the AP subject could not answer the question in question number 4.

jawab

a. $\frac{1}{3} + \frac{4}{9} = \frac{5}{9}$

b. $1 - (\frac{1}{3} + \frac{4}{9}) = 1\frac{5}{9} - \frac{11}{9}$

Figure 19 Answer from subject AP for question number four

In question number five, the subject of AP has not met the A5 indicator. It can be seen that the subject of AP has not been able to work on the fraction addition operation problems that are related to everyday life by using the right concepts and steps in solving them. AP subjects have first converted mixed fractions into common fractions to make them easier to solve. The subject of AP has also been able to find the LCM from the denominator of the fraction, namely the denominator of 2 and 3, namely 6. However, the subject of AP is confused about how to change the fraction's numerator after obtaining the LCM. So that the answer to the subject of AP on question number 5, there are still errors.

$$\frac{5}{2} + \frac{1}{3} = \frac{15}{6} + \frac{2}{6} = \frac{17}{6}$$

Handwritten notes: "jawab", "disederhanakan", "kpk dari 3 dan 2 = 6"

Figure 20 Answer from subject AP for question number five

The results showed that of the five indicators of the ability to understand mathematical concepts used, three indicators often appear on the subject, including the indicator of restating a concept, namely students can restate how to add and subtract fractional operations with their sentences, indicators present concepts in their own words. Various forms of mathematical representation, namely, students can make mathematical models and solve problems from the images presented and indicators of classifying objects according to specific properties according to concepts. Students can group fractions with the same denominator and different denominators. The indicators that have not appeared dominant in all subjects are indicators of using, utilizing, and selecting specific procedures or operations and applying concepts or algorithms to solve problems.

In the first indicator appears in each student. Students can write or restate the concept of how to add and subtract fractions using their sentences and language. It can be concluded that the indicators of restating concepts, high, medium, and low ability students can restate concepts that have been studied quite well. This is in line with the research results conducted by Khairani [19].

Only 2 out of 3 subjects raised this indicator in the second indicator. The subject is given several fractions with the same denominator and different denominator in the given problem. Then the subject is asked to group the fractions based on the denominator. Subjects can classify and distinguish which fractions have the same denominator and different denominators. However, some subjects are still wrong in answering the questions because they are still wrong in understanding them.

In the third indicator, only 2 out of 3 students can also bring up this indicator. In this indicator, almost all subjects can make mathematical models of the problems given in problem number 3. Subjects can also read the images presented and then write them in mathematical form. From the picture presented, it can be seen that the subjects N and EDP have been able to make a fraction of the pizza picture served.

In the fourth indicator, only subject N can use the proper steps in solving the problems given in question number 4. At the same time, the EDP and AP subjects

have not solved the problems in question number 4. Some students are still confused about using the steps to solve addition and subtraction operations with fractions with different denominators. This is in line with research conducted by Kurniawan et al., namely the difficulty in studying fractions material. Students cannot add fractions with unequal denominators and write down fractional addition and subtraction operations [11].

In the fifth indicator, many students have not been able to bring up this indicator. There are still student errors in applying the concept of fractional operations. There are still many students who are confused about changing the form from mixed fractions to ordinary fractions. In addition, students can still change the numerator after the LCM has obtained the denominator. Students also directly add or subtract fractions regardless of different denominators. This is in line with research conducted by Yulianingsih et al., namely the difficulties of other students in doing addition and subtraction operations on fractions, namely, students directly adding or subtracting fractions regardless of different denominators [8].

4. CONCLUSION

Based on the results of the research that has been carried out, it can be concluded that the ability to understand the concepts of students in the material of addition and subtraction of fractional numbers using the Discovery Learning model has emerged, and the student's response to learning using the Discovery Learning model is quite good and interesting.

The indicators that appear the most are restating a concept, classifying objects according to certain properties according to concepts, and presenting concepts in various forms of mathematical representation. Meanwhile, indicators that rarely appear are using, utilizing, and selecting certain procedures or operations, and applying concepts or algorithms to solve problems.

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