Teachers' Competencies in Doing Direct Proof in Geometry

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ABSTRACT
Mathematical proof is necessary in mathematics education for meaningful learning. It can prevent rote learning. Proof should be seen as an important part of mathematics teaching process. It shouldn't be seen as only a special topic in the curriculum. Teaching mathematical proof is valuable for teaching process not only for reaching the correct mathematical statement, but also for knowing and doing mathematics correctly, building the basis of mathematical thinking, and understanding, using and improving mathematical knowledge. This descriptive research examined teachers' proving competency in geometry. The respondents were 123 mathematics teachers consisting of 70 junior high school teachers, 37 senior high school teachers, and 16 vocational school teachers. They were given test consisting problem in geometry that has to be proved. Analysis of the teachers' answers can be classified into three categories. First category showed that some teachers cannot do the proof. They did not know what have to be done to start. Second category showed that majority of the teachers cannot prove with valid argument. They only perform mechanistic steps. Third category showed that none of the teachers can do the proof with complete steps and valid argument. As the result of this research, mathematics teachers have to improve their skills in mathematical communication to support proving competencies.

Keywords: *Proof, Direct proof, Geometry, Valid argument.*

1. INTRODUCTION

One of the aims of mathematics taught at school is students can communicate ideas, develop reasoning, and construct mathematical proofs [1]. Mathematics teachers should master professional competencies include mastering the mathematics material, structure, concepts, and scientific mindset that supports the subjects covered [2].

In teaching process, mathematical proof is necessary not only for teaching the correct mathematical statement, but also for meaningful learning, prevents rote learning, knowing and doing mathematics correctly, building the basis of mathematical thinking, and understanding, using and improving the mathematical knowledge [3]. In mathematics classroom there are many roles of proof, that is: to verify that a statement is true, to explain why a statement is true, to communicate mathematical knowledge, to discover or create new mathematics, to convince mathematical knowledge, to explain mathematical knowledge, and to systematize statements into an axiomatic system [4,5]. Proving is one of the main aspects of mathematical behavior and most clearly distinguishes mathematical behavior from scientific behavior in other disciplines [6]. Proofs should play a larger role throughout school mathematics [7]. The curriculum that require students to explain and justify their ideas could encourage them to refine their thinking, gradually leading them to understand the shortcoming of visual and empirical justifications so that they discover and begin to use some of critical components of formal proof [8].

To be a good proof, there are five essential parts, that is: state the theorem or conjecture to be proven, list the given information, draw a diagram to illustrate the given information if possible, state what is to be proved, and develop a system of deductive reasoning [9]. When trying to present the proof, there are common mistakes students usually made: misunderstanding of definitions, not enough words, lack of understanding, and incorrect steps [12].

Geometry is an example of a mathematical system. It is necessary to have proof in order to believe certain geometric principles. In geometry, a proof is a valid argument that establishes the truth of a statement. It is based on a series of statements that are assumed to be true. An important part of writing a proof is giving justifications to show that every logical step is valid. We can use a definition, postulate, property, or a piece of information that is given. To present the proof, it is important to justify every logical step with a reason [11-13]. A proof in geometry begins with given statement and prove statement. They restate the hypothesis and conclusion of the conjecture. There are two styles of geometric proof. In a two-column proof, we write statements that we know to be true in the left column and the matching reasons why each statement is true in the right column [13]. In a paragraph proof, we write statements that we know to be true and their matching reasons why each statement is true as sentences in a paragraph [13].

A direct proof is a mathematical proof starts with the given statements then uses the laws of logic to arrive at the statement to be proved [13]. An indirect proof or a proof by contradiction is a mathematical proof starts with the negation of the statement to be proved then uses the laws of logic to show that it is false. An indirect proof works because when the negation of a statement is false, the statement must be true [13].

Previous research stated that majority of mathematics teachers did not have enough ability and competency to construct mathematical proof [14,15].

2. METHODS

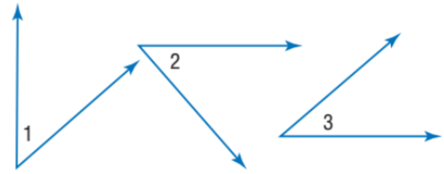
This descriptive research examined mathematics teachers' proving competency in geometry. The respondents were 123 mathematics teachers consist of 70 junior high school mathematics teachers, 37 senior high school mathematics teachers, and 16 vocational school mathematics teachers. They were participants of Higher Order Thinking Skills Teacher Training for junior high school, senior high school, and vocational school mathematics teachers held by PPPPTK Matematika in May and June 2021 (accidental sampling). Data are collected by test. In this test, the respondents were given problem in geometry that has to be proved by direct proof method.

3. RESULT AND DISCUSSION

Each statement in geometric proof is supported by the reason why we can make that statement (claim). The first claim in the proof is the given statement [11]. The sequence of steps must conclude with a final statement representing the claim to be proved. It is called the prove statement [11].

In this research, respondents were given geometry problem below that has to be proved.

> *"Look at the picture. If ∠1 and ∠3 are complementary and ∠2 and ∠3 are also complementary, prove that ∠1 is congruent with ∠2 (∠1 ≅ ∠2)".*



Analysis of the teachers' answers can be classified into three categories. First category showed that some teachers cannot do the proof (43 mathematics teachers consist of 31 junior high school teachers, 7 senior high school teachers, and 5 vocational school teachers). They did not know what have to be done to start the proof. Second category showed that majority of the teachers cannot prove the problem with valid argument (80 mathematics teachers consist of 39 junior high school teachers, 30 senior high school teachers, and 11 vocational school teachers). They only perform mechanistic steps as they do in ordinary calculations. Third category showed that none of the teachers can do the proof with complete steps and valid argument. Some of respondents' answers are discussed below.

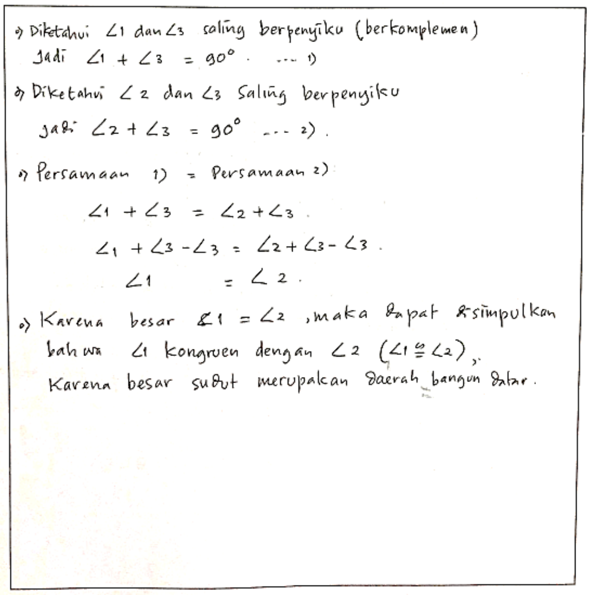
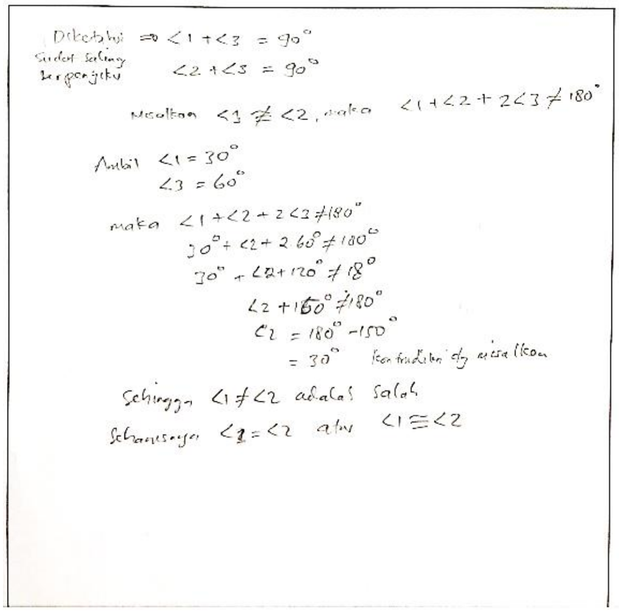

∘) Diketahui ∠1 dan ∠3 saling berpenyiku (berkomplemen)
Jadi ∠1 + ∠3 = 90° . ... 1)
∘) Diketahui ∠2 dan ∠3 saling berpenyiku
Jadi ∠2 + ∠3 = 90° ... 2).
∘) Persamaan 1) = Persamaan 2)
∠1 + ∠3 = ∠2 + ∠3.
∠1 + ∠3 − ∠3 = ∠2 + ∠3 − ∠3.
∠1 = ∠2.
∘) Karena besar ∠1 = ∠2, maka dapat disimpulkan bahwa ∠1 kongruen dengan ∠2 (∠1 ≅ ∠2),
Karena besar sudut merupakan daerah bangun datar.

Figure 1 Proof step of respondent A

According to Figure 1, respondent A tried to prove it by writing complete sentences. However, he did not write down the arguments for each step of the proof. The answers written in the end were just the usual mechanistic steps.



Diketahui => $\angle 1 + \angle 3 = 90^\circ$
Sudut saling berpenyiku $\angle 2 + \angle 3 = 90^\circ$

Misalkan $\angle 1 \neq \angle 2$, maka $\angle 1 + \angle 2 + 2\angle 3 \neq 180^\circ$

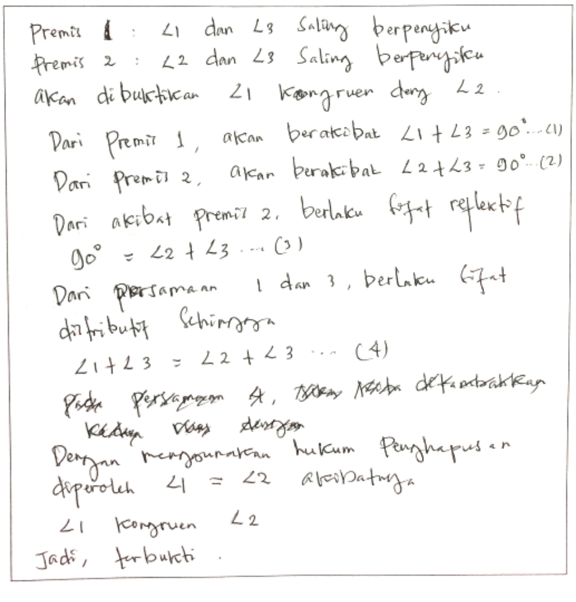
Ambil $\angle 1 = 30^\circ$
$\angle 3 = 60^\circ$

maka $\angle 1 + \angle 2 + 2\angle 3 \neq 180^\circ$
$30^\circ + \angle 2 + 2 \cdot 60^\circ \neq 180^\circ$
$30^\circ + \angle 2 + 120^\circ \neq 180^\circ$
$\angle 2 + 150^\circ \neq 180^\circ$
$\angle 2 = 180^\circ - 150^\circ$
$= 30^\circ$ kontradiksi dg misalkan

Sehingga $\angle 1 \neq \angle 2$ adalah salah
Seharusnya $\angle 1 = \angle 2$ atau $\angle 1 \cong \angle 2$

Figure 2 Proof step of respondent B

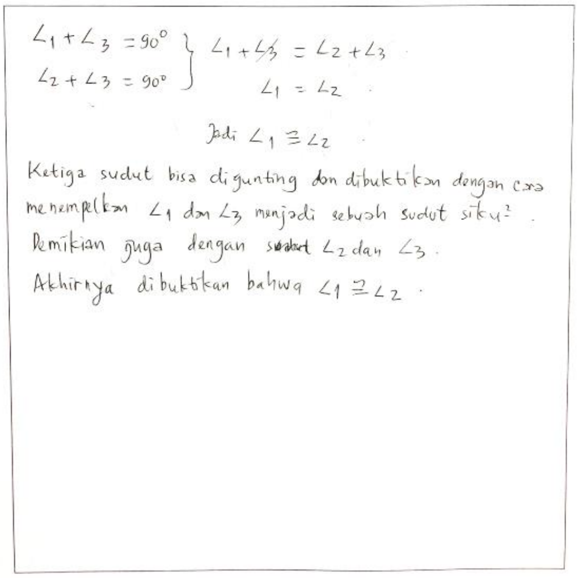
In Figure 2, Respondent B tried to prove it by taking an example in the form of a number for the size of the angle. This is clearly not deductive reasoning and is not a valid proof step.



Premis 1 : $\angle 1$ dan $\angle 3$ saling berpenyiku
Premis 2 : $\angle 2$ dan $\angle 3$ saling berpenyiku
akan dibuktikan $\angle 1$ kongruen dengan $\angle 2$.
Dari premis 1, akan berakibat $\angle 1 + \angle 3 = 90^\circ$ ...(1)
Dari premis 2, akan berakibat $\angle 2 + \angle 3 = 90^\circ$ ...(2)
Dari akibat premis 2, berlaku sifat reflektif
$90^\circ = \angle 2 + \angle 3$ ...(3)
Dari persamaan 1 dan 3, berlaku sifat distributif sehingga
$\angle 1 + \angle 3 = \angle 2 + \angle 3$ ...(4)
Dengan menggunakan hukum penghapusan diperoleh $\angle 1 = \angle 2$ akibatnya
$\angle 1$ kongruen $\angle 2$
Jadi, terbukti.

Figure 3 Proof step of respondent C

Respondent C tried to prove it by writing complete sentences with arguments for each step, as in Figure 3. However, the argument that is written is not correct. An error occurred in the use of reflective and distributive properties as arguments.



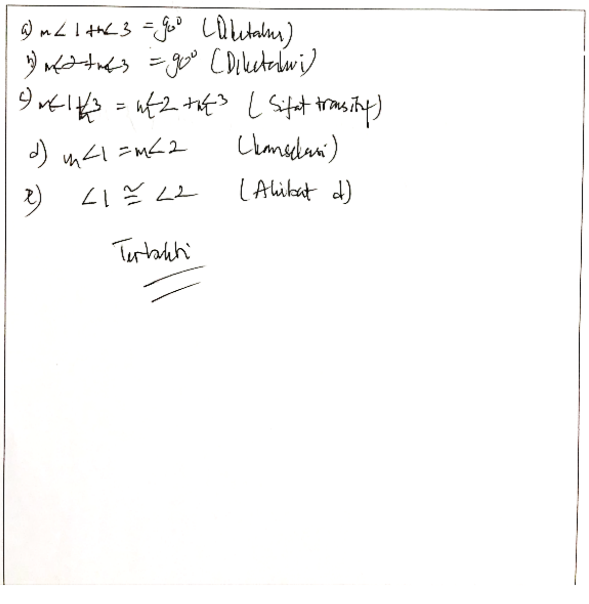
$\angle 1 + \angle 3 = 90^\circ$
$\angle 2 + \angle 3 = 90^\circ$
$\angle 1 + \angle 3 = \angle 2 + \angle 3$
$\angle 1 = \angle 2$

Jadi $\angle 1 \cong \angle 2$

Ketiga sudut bisa digunting dan dibuktikan dengan cara menempelkan $\angle 1$ dan $\angle 3$ menjadi sebuah sudut siku². Demikian juga dengan sudut $\angle 2$ dan $\angle 3$.
Akhirnya dibuktikan bahwa $\angle 1 \cong \angle 2$.

Figure 4 Proof step of respondent D

In the Figure 4 above, Respondent D tried to prove it by demonstrating the cutting of the three corners and then pressing them together. Analogous to the method used by respondent B, this is clearly not deductive reasoning and is not a valid proof step.



a) $m\angle 1 + m\angle 3 = 90^\circ$ (Diketahui)
b) $m\angle 2 + m\angle 3 = 90^\circ$ (Diketahui)
c) $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$ (Sifat transitif)
d) $m\angle 1 = m\angle 2$ (kanselasi)
e) $\angle 1 \cong \angle 2$ (Akibat d)

Terbukti

Figure 5 Proof step of respondent E

According to Figure 5, Respondent E was incomplete in writing the proof structure because it did not contain given and prove Statements. There is an error in using the transitive property. There are no argument in every step.

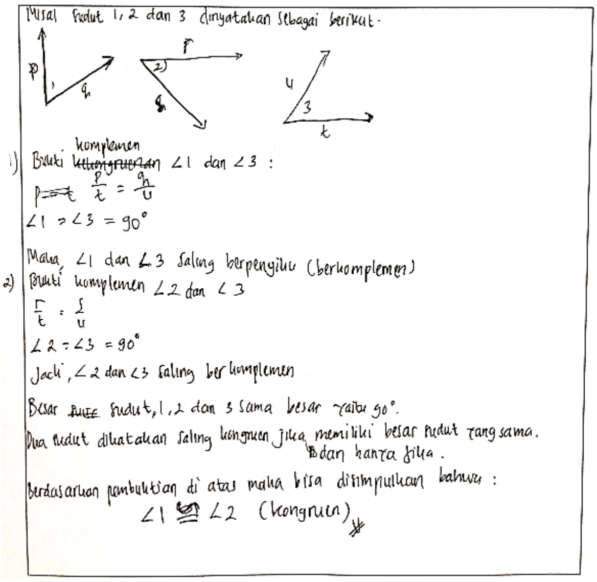


Figure 6 Proof step of respondent F

In the Figure 6, Respondent F made several mistakes in the proof step. The error in writing the ratio of the legs of the irrelevant angle is used in proving this problem and the error in writing the measure of the angle.

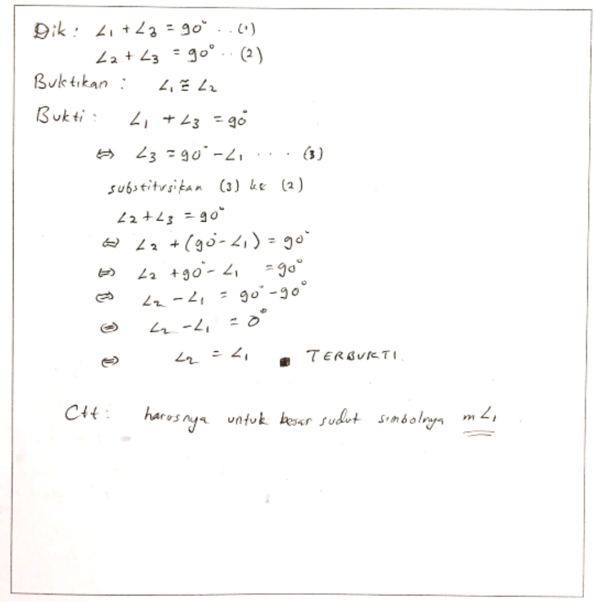


Figure 7 Proof step of respondent G

Respondent G wrote the equivalence notation in an inappropriate situation because it is not an equation, as in Figure 7. This proof is incomplete, there are no argument in every step.

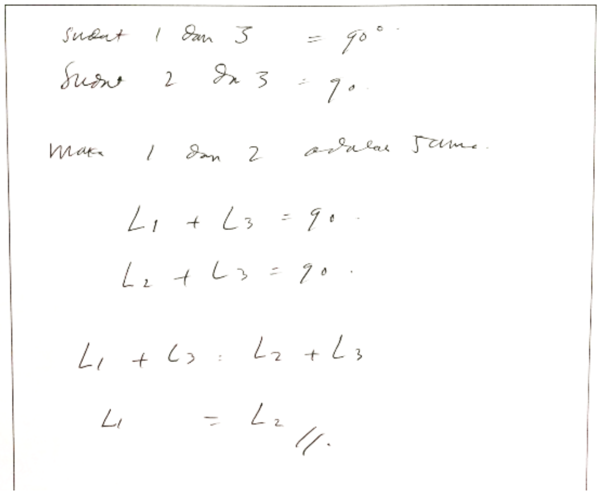


Figure 8 Proof step of respondent H

In the Figure 8 above, Respondent H made a mistake in writing the definition of two complementary angles. This proof is incomplete, there are no argument in every step.

4. CONCLUSION

The result of the analysis of the teachers' answers can be classified into three categories. First category showed that some teachers cannot do the proof. They did not know what have to be done to start the proof. Second category showed that majority of the teachers cannot prove the problem with valid. They only perform mechanistic steps as they do in ordinary calculations. Third category showed that none of the teachers can do the proof with complete steps and valid argument. It can be concluded that there are still many mathematics teachers who have not competent yet in doing proof. Some of mistake made by respondents are: (1) did not write down the arguments for each step of the proof, (2) doing proof by using an example in the form of a number for the size of the angle, (3) using incorrect argument, (4) using demonstration of cutting three corners and then pressing them together, (5) did not contain given and prove statements in the step of the proof, (6) writing incorrect equivalence notation in an inappropriate situation, and (7) writing incorrect definition of two complementary angles.

Some suggestion based on this research are: (1) teachers should refresh their understanding of proof and some methods of proof, (2) teachers should have habit to do proof some basic mathematical concepts usually taught in classroom, (3) teachers' communities should have effort to strengthen the ability to compose mathematical proofs in their professional development, and (4) teachers' training should have more pay attention to the competence of reasoning and proof.

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