Advances in Social Science, Education and Humanities Research, volume 656
2nd National Conference on Mathematics Education 2021 (NaCoME 2021)

The Factors Influencing Students' Proof Comprehension on Mathematical Induction

Latifah Mustofa Lestyanto[1,*], Desi Rahmadani[1], Asmianto Asmianto[1], Ety Tejo Dwi Cahyowati[1]

[1]*Universitas Negeri Malang, Indonesia*
[*]*Corresponding author. Email: latifah.mustofa.fmipa@um.ac.id*

ABSTRACT
Mathematical Induction (MI) is a subject that mathematics students have known since senior high school. However, many students lack understanding about MI and still incorrectly prove propositions using this proof method, even though they are 4-year students. Hence, investigating factors that influence students' comprehension of MI is needed. This study is a qualitative study that aims to investigate and analyze factors that could be influenced how students comprehend MI. The data were obtained within 2 phases. Participants were asked to write down the proof of proving-question in phase 1, and they were interviewed to deepen the information in phase 2. The evidence suggests that conceptual aspects have to be focused more using many different ways of teaching strategies than the procedural one. Some factors which influence students' comprehension of MI include content knowledge, the experience of proving a variety of problems, the strategy of teaching used by lecturers, the language of the textbook, and the ability to do some algebraic manipulation.

Keywords: *Mathematical induction, Proof comprehension.*

1. INTRODUCTION

Mathematical Induction (MI) as one of methods in mathematical proof has been taught to Indonesian students in senior high school. In MI, they learn how to establish the validity of statements that are given in terms of the natural numbers [1]. Baker [2] argued that this technique is particularly valuable in developing the theoretical foundations of computer science in which very popular recent years. For mathematics' undergraduate student, this method is used when they learn discrete mathematics. Hence, comprehending this mathematical proof method is very important for them.

Even though MI has been taught since senior high school, many mathematics students at Universitas Negeri Malang still find difficulties proving statements using MI. Consequently, its produce errors in their attempt of proving. Moreover, the errors even occur in proof written by 4-year students. In previous study, Lestyanto, et al [3] find that there are just few students who can achieve chaining element level when they read proof by mathematical induction. Chaining element level is the third level of reading comprehension model introduced by Yang and Lin [4]. Students in this level can identify critical procedures, premises or conclusions and
identify critical ideas in proof. The finding of previous study by Lestyanto, et al [3] indicates that most students comprehend MI procedurally and even still find difficulties in it. In general, many students do not know how to apply the premises in other situations.

Studies regarding students' difficulties in mathematical induction have been conducted by other researchers [2], [5–7]. Most of the findings in those researches agree that students focused on the procedural aspects of mathematical induction far more often than on conceptual aspects. The students linked their proof to the algebraic method and do not have a deep comprehension of the concept of mathematical induction. Even Gonzalez [5] found that students experienced difficulties at every step of mathematical induction, particularly in stating $P(k+1)$. These results are in accordance with our observation in mathematics' students at Universitas Negeri Malang. Many students still write an unclear and messy proof. In addition, they

cannot develop their proof when dealing with other cases in which different from given examples. Hence, knowing the factors that influence students comprehending concept of mathematical induction is important.

In other hand, Walida and Hasana [7] have been conducted a research in studying the factors that influence the students' algebra misconceptions in mathematical induction. Yet, the research only focused on algebraic way and has not been investigate the external factors outside the students. In order to find out the factors that influencing students in comprehending mathematical proof internally and externally, through this research, investigation and analysis of the factors are conducted. The study in this article is a preliminary study on development of an electronic module in introduction of mathematical proof.

2. METHODS

This study was conducted on July to September 2021 at mathematics department of Universitas Negeri Malang (UM). Participants were recruited from a summer class of 2020/2021 academic year in this department. They already took fundamental mathematics course which include MI at their first semester of study at UM and retake the same course at the time to enhance their GPA. This study employed a qualitative approach using a case study design.

To collect data, participants completed two phases of data collection. In phase 1, a written-test was given to 15 students. The test consisted of 3 questions with different types of question. Question 1 is a scaffolding-problem, while in question 2, students were given a proof of theorem and they were asked some questions related to the proof. Question 3 is in the form of "prove that". In phase 2, two participants were interviewed to deepen the information. One participant is the participant who answer 3 questions correctly, and the other one is who answer almost incorrectly for all questions.

3. RESULTS AND DISCUSSION

The main question for this study is to investigate and analyze factors (internally and externally) that influence students comprehend concept of mathematical induction. In this article, we will present and analyze two students' answer which written in Indonesia's language.

3.1. Student 1

Student 1 is the participant who answer 3 questions correctly, and she is a 3-year student. Figure 1 is her answer to question 1. Question 1 is a scaffolding-problem which asking students to fill blanks in incomplete proof that were provided. Figure 1 shows that Student 1 had no difficulty in filling the blanks. She could complete the proof almost correctly and have a deep understanding on the concept of MI. Based on the interview regarding to this question, she could explain the concept of MI without hesitance and knew that this method of proving has limitation on the natural numbers. However, there are some minor errors in her answer and mostly in the part of algebraic manipulation. Yet, she could realize these errors.

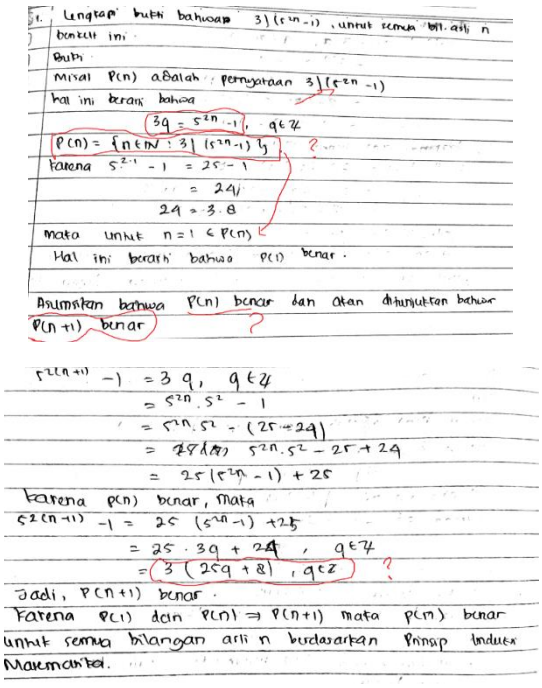


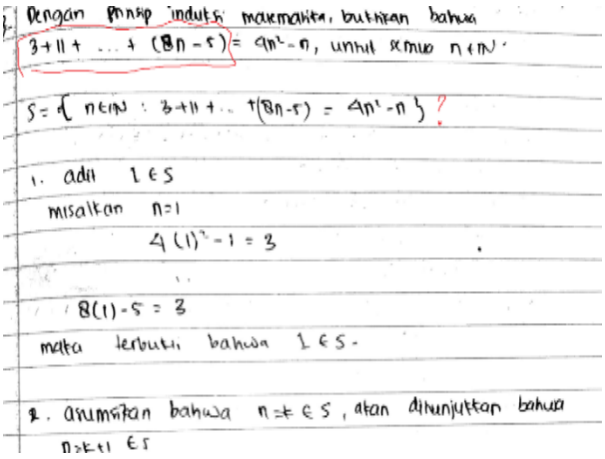
Figure 1 Student 1's answer to question 1

Student 1 also has a good concept on the topic given in question 1 which is related to divisibility. She could describe correctly the meaning of symbol $3|(5^{2n}-1)$. Moreover, she knew that she had to form the $P(n+1)$ part into a final multiplication of 3 with some integer. She knew what is the assumption and what is to be proved. She put it this way, "$P(n)$ is the assumption, which means $5^{2k}-1=3q$, with q is integer. While $P(n+1)$ is have to be proved using the assumption of $P(n)$". Moreover, she could make a reasoning of the inductive step on MI.

Investigation of Student 1's comprehension on MI and factors that influencing it were deepen through interview. Based on the results of interview, she said that she already knew MI since senior high school but mostly as a procedural process of proving. While at university level, her lecturer used an English textbook as a source of learning and rarely made handout or power point slide to deliver the concept of MI in the class. The difference of language used on the textbook with her mother language quite influence her understanding to the concept. On the first time she took the course, she could not adjust and engage the learning. As the result, her grade on this course was not quite good. However, on the second time of taking the same course, she made an improvement. On the topic of MI, she comprehends it by doing many exercises with different variety of problems, and searching some information through internet to deepen her understanding. On the second time of course, because of pandemic's effect, her lecturer made some learning videos and power point slide using Indonesia's language. Discussion of exercises was also held during the class using virtual medium.

Advanced reasoning-problem was given through Question 2 on the written test which related to Fibonacci number. Student 1 also had no difficulty answering the question. She understands the provided-proof proving that f_{3n}on the Fibonacci number is even for every $n \in \mathbb{N}$. She could explain why the proof is started by showing f_3 is even, and she put it this way, " Because if $n = 1$, then $f_{3n} = f_3$. And it should be even because $f_3 = 1 + 1 = 2$". Student 1 can implement the concept of MI in other situations which are rarely given during the class. She could give a reasoning behind the steps of proof given on the problem.

Student 1 could also give a clear proof to the Question 3 which asked to prove that $3 + 11 + \cdots + (8n - 5) = 4n^2 - n$, for all$n \in \mathbb{N}$. Figure 2 shows that she knows what is to be proved and steps that have to be done to prove it. It is also evidence that Student 1 have no difficulty in using MI to prove variety of problems.



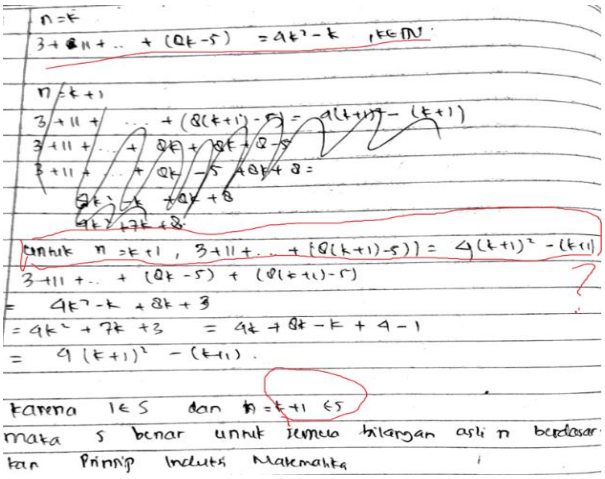


Figure 2 Student 1's answer to question 3

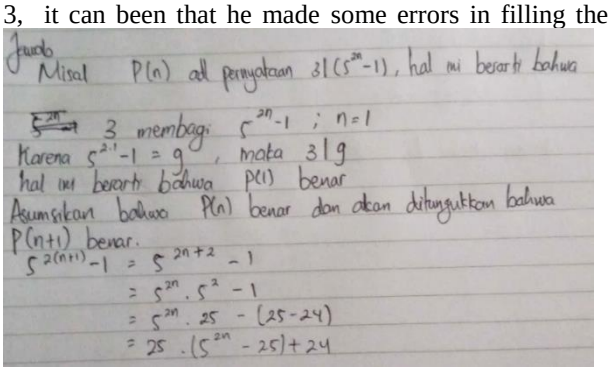
From Figure 2, it can be seen that Student 1 initially defined S as a set of natural number in which $3 + 11 + \cdots + (8n - 5) = 4n^2 - n$. Based on the interview's result, Student 1 knew that the form on the left-hand side is equivalent with $\sum_{n=1}^{k}(8n + 5)$. She then showed that $1 \in S$ by subtituting $n = 1$ to the form of $4n^2 - n$ and $8n + 5$. She also could prove the rest of proof using MI, but made some mistake in the middle of proof. However, in the end she could fix those mistakes and proved that $k + 1 \in S$.

3.2. Student 2

Like Student 1, Student 2 is also a 3-year student. However, Student 2 is a student who does not comprehend MI in both aspects, conceptually and procedurally. He made many errors on his answer in the written test. The following is an excerpt from the interview:

Student 2: When I used mathematical induction, I usually can only prove the basis step (for $n = 1$), and I difficult to prove the next step which involved $n = k + 1$.

From the excerpt, Student 2 had difficulty proving the step of $P(k + 1)$ in mathematical induction procedure. Figure 3 shows his answer to Question 1. From Figure 3, it can been that he made some errors in filling the



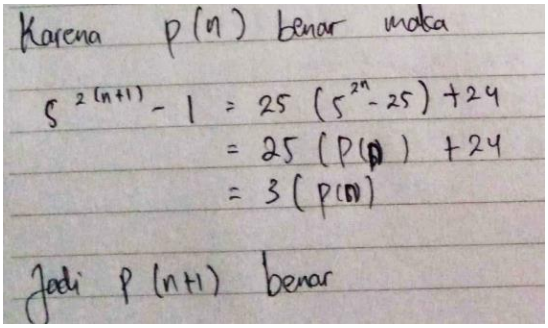


Figure 3 Student 2's answer to question 1

Based on the interview, Student 2 could not correctly define $3|(5^{2n}-1)$. At the beginning, he thought that this topic related to modulo operation. He believed that any real number divisible by 3. The first error on his proof was writing that $5^{2\cdot1}-1=9$. However, during the interview, he realized this error and could answer the right calculation. He hesitated when he was asked about the meaning of "Assume that $P(n)$ is true" on the statement. He thought that to prove $P(n+1)$, he had to prove $P(n)$ first. Even, he assumed that $n$is a variable that can be replaced by any integer or real number. This assumption and his lack of concept on mathematical induction also affect his answer to Question 2 and Question 3.

On the Question 2, related to Fibonacci number, he just could evaluate the value of f_n and could not understand the given proof of the theorem given on the Question 2. As the consequence, he did not answer most of sub questions on Question 2. From the interview, he said, "I could evaluate the value of f_6, but I did not understand the proof. Therefore, I did not answer the other questions.". This indicate that Student 2 cannot implement the concept of MI in other situations, and it is as part of the consequences of his poor concept knowledge on MI.

Through deep investigation using interview to Student 2, it can be identified some reasons why he has a poor concept on MI. First, Student 2 rarely solve some exercises with variety of problems which using MI. Mostly, he read the examples which have solution. When he tried to prove a proposition using MI, he always stuck on the second step and difficult in stating $P(n+1)$. Second, he feels that the English language used on the textbook was difficult to understand. The following excerpt supports this finding.

Student 2: My mathematics textbooks since elementary to high school were in Indonesia's language. Hence, because the course is using English textbook, I usually translate it with google translate and sometimes I find that there are some mistakes on the translation. I think the textbook was difficult to understand.

When he was asked about the learning strategies used by his lecturer, he said that discussion and doing the exercises from the textbook were the main strategies. Yet, the lecturer did not make any handout or power point slide when delivering the materials.

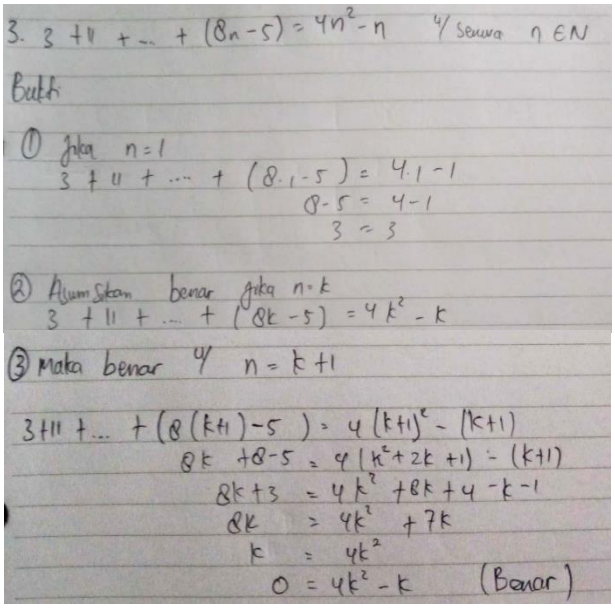


Figure 4 Student 2's answer to question 3

Figure 4 shows Student 2's answer to Question 3. From this figure, Student 2 divided his proof into three parts without define an initial set like Student 1. In part 1, he evaluated left-hand side and right-hand side of the equation simultaneously by substitute $n=1$ to both sides, and it ended by writing "3=3". Indeed, he knew the stating of $P(k)$ in part 2, but did not know how to prove $P(k+1)$ in part 3. He assumed that $P(k+1)$ is hold by the statement, so he directly wrote $4(k+1)^2-(k+1)$ on the right-hand side. Though he stated the $P(k)$, but he did not know how to use it to show that $P(k+1)$ holds the statement. Like in part 1, he operated simultaneously between left-hand side and right-hand side of the equation.

Some difficulties faced by Student 1 and Student 2 at the first-time taken the course which include MI were mostly because of lack of concept knowledge on MI. This finding was confirmatory with previous research [2], [5], [7]. Not only lack of concept, student also find difficulties using MI as a procedural technique like the finding proposed by Baker [2]. Difficulty in doing algebraic manipulation in the proof was also faced by

the students. This difficulty makes the student stuck in the middle of writing proof. Hence, poor production of proof happened. Student who stuck in doing algebraic manipulation is the student who has no strong engagement in learning and less effort in comprehending MI. Doing exercise with variety of problems also influence students' performance in writing proof using MI. Internal factors from the students in willingness to comprehend MI play a significant role on their proof. This finding is supported by previous research proposed by Walida and Hasana [7].

In the other hand, external factors also more or less influence students' comprehension on mathematical induction. From the research's results, an English-textbook used as the main resource in the learning influence their understanding on the concept of MI. It because English is not the mother language of the students. In addition, there are no other media learning developed by the lecturers in Indonesia's language which could help students to understand more about the concept of MI. Having the educational language as a second language are all considered risk factors for language and literacy failure [8]. Besides, teaching strategies employed by lecturer also affect students' comprehension on MI. When discussion is implemented, it should be the discussion which deepen the students' reasoning skill and not only discussion about the answer of problems which using MI.

4. CONCLUSION

Mathematical induction is a mathematical proof method that has been taught since senior high school. Yet, many students still find difficulties in proving propositions using this method. Based on the results, some factors influencing students at mathematics department of Universitas Negeri Malang comprehend the mathematical induction are come from internally and externally of the students. These factors include content knowledge, experience of proving variety of problems, strategy of teaching used by lecturers, language of textbook, and ability to do some algebraic manipulation. The conceptual aspect of mathematical induction should be focused more without neglect the procedural one. One of further researches that can be developed is the development of learning material in Indonesia's language which help students in comprehend mathematical induction.

ACKNOWLEDGMENTS

Universitas Negeri Malang (UM) for providing the grant to do this research through "Inovasi Pembelajaran" scheme in 2021.

REFERENCES

[1] R. G. Bartle, D. R. Sherbert, Introduction to Real Analysis, 4th ed, John Wiley & Sons, Inc., New Jersey, 2011.

[2] J. D. Baker, Students' difficulties with proof by mathematical induction, in Annual Meeting of the American Educational Research Association, 1996, pp. 1–19.

[3] L. M. Lestyanto, Sudirman, Susiswo, E. Hidayanto, The level of students' reading comprehension on proof by mathematical induction, 2215 (2020). DOI:10.1063/5.0000564.

[4] K.-L. Yang, F.L. Lin, A Model Of Reading Comprehension Of Geometry Proof, Educational Studies in Mathematics 67(1) (2008) 59–76. DOI: 10.1007/s10649-007-9080-6.

[5] M. A. Gonzales, Undergraduate Students' Difficulties and Convictions with Mathematical Induction, Undergraduate thesis, The University of Texas at Austin, 2020.

[6] J. R. Doyle, R. Núñez, Characterizing Students' Conceptual Difficulties With Mathematical Induction Using Visual Proofs, International Journal of Research in Undergraduate Mathematics Education 7(1) (2021) 1–20. DOI: 10.1007/s40753-020-00119-4.

[7] S. Walida, S. Hasana, The Identification of Students' Misconceptions in Mathematical Induction, Journal of Education and Learning Mathematics Research (JELMaR) 1(2) (2020). DOI:10.37303/jelmar.v1i2.28.

[8] K. Rogde, Å. M. Hagen, M. Melby-Lervåg, A. Lervåg, The Effect Of Linguistic Comprehension Instruction On Generalized Language And Reading Comprehension Skills: A Systematic Review, 15(4) (2019). DOI: 10.1002/cl2.1059.