Constructing an Abridged Life Table based on Estimator of the Probability of Death Using Maximum Likelihood Estimation

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ABSTRACT
This research aims to construct an abridged life table by estimating the probability of death using maximum likelihood estimation. This research is a descriptive research with a qualitative approach. The model used is Polya Model. Subjects in this research were five students of the Islamic Economics study program at UIN Sulthan Thaha Saifuddin Jambi. The Indonesian mortality data used is the Taspen Mortality Table 2012. Based on the constructing result of abridged life table using data of the Taspen Mortality Table 2012, the life expectancy for the population aged 0 is 75.25 years.

Keywords: *Abridged Life Table, Maximum Likelihood Estimation, Polya Model.*

1. INTRODUCTION

The high and low level of mortality in a country not only affects population growth, but can also be used as a barometer of the high and low level of health in that country [1]. Lower mortality rate will indicate that a society has a good survival rate and higher mortality rate will indicate otherwise [2]. Mortality data in a country is usually presented in a life table. Basically, life table is a hypothetical table that combines various mortality rates in different ages into a single statistical model [3]. Life table is not only designed to measure mortality rates, but can also be used in the health and insurance sectors [4].

There are four life table models that were developed based on the analysis of calculation of death rate in the real population. The four models include United Nations Model Life Tables, Coale and Demeny Regional Model Life Tables, Ledermann's System of Model Life Tables, and Brass Logit Life Table System [5]. Indonesia currently is still using the Coale-Demeny Model approach, especially Western Model of Coale-Demeny Life Table [1].

Life table can be constructed into two tabular forms i.e. complete life table and abridged life table. Complete life table is a table that containing data of death in a population which presented in one-year age intervals, while abridged life table is a table that containing data of death in a population which grouped in five or ten age intervals [6]. Cohorts can be assumed started from a radix, such as 1,000, 10,000, or 100,000 [7].

The model used in this research is Polya Model. The stages in Polya Model are (1) understanding the problem, (2) planning a solution, (3) solving the problem, and (4) checking again [8]. In this research, life table that will be constructed is an abridged life table. By using Polya Model, students are expected to be able to construct an abridged life table based on mortality data in Indonesia.

2. METHOD

This research is a descriptive research with a qualitative approach. The strategy used is exploration of processes, activities, and events [9]. Subjects in this research were five students of the Islamic Economics study program at UIN Sulthan Thaha Saifuddin Jambi. In the implementation, by using Polya Model, students are given instructions i.e. steps that must be taken for constructing an abridged life table. The Indonesian mortality data used in this research is the values of probability of death in the Taspen Mortality Table 2012 which is presented in Table 1 [10].

Table 1. The Taspen Mortality Table 2012

Age (x)	TMT 2012	Live life Expectancy
0	0.00426377	75.73
1	0.00049113	75.05
2	0.00038199	74.09
3	0.00030559	73.12
4	0.00025830	72.14
5	0.00023647	71.16
6	0.00023283	70.17
7	0.00022556	69.19
8	0.00021464	68.20
9	0.00020373	67.22
10	0.00018918	66.23
11	0.00018554	65.24
12	0.00020218	64.26
13	0.00022031	63.27
14	0.00024007	62.28
15	0.00026160	61.30
16	0.00028507	60.31
17	0.00031063	59.33
18	0.00033849	58.35
19	0.00036884	57.37
20	0.00040192	56.39
21	0.00043797	55.41
22	0.00047724	54.44
23	0.00052004	53.46
24	0.00056667	52.49
25	0.00061748	51.52
26	0.00067285	50.55
27	0.00073318	49.58
28	0.00079892	48.62
29	0.00087055	47.66
30	0.00094860	46.70
31	0.00103364	45.74
32	0.00112630	44.79
33	0.00122727	43.84
34	0.00133728	42.89
35	0.00145714	41.94
36	0.00158774	41.00
37	0.00173003	40.07
38	0.00188506	39.13
39	0.00205398	38.21
40	0.00223801	37.28
41	0.00243850	36.36
42	0.00265694	35.45
43	0.00289492	34.54

Age (x)	TMT 2012	Live life Expectancy
44	0.00315417	33.64
45	0.00343661	32.74
46	0.00374428	31.85
47	0.00407945	30.97
48	0.00444455	30.09
49	0.00484225	29.22
50	0.00527544	28.36
51	0.00574727	27.50
52	0.00626117	26.66
53	0.00682086	25.82
54	0.00743039	24.99
55	0.00809417	24.17
56	0.00881699	23.36
57	0.00960404	22.56
58	0.01046098	21.77
59	0.01139393	20.99
60	0.01240957	20.22
61	0.01351512	19.46
62	0.01471843	18.71
63	0.01602800	17.98
64	0.01745305	17.25
65	0.01900358	16.54
66	0.02069040	15.84
67	0.02252522	15.15
68	0.02452070	14.48
69	0.02669054	13.82
70	0.02904951	13.17
71	0.03172822	12.54
72	0.03505881	11.91
73	0.03857564	11.31
74	0.04233899	10.72
75	0.04648031	10.15
76	0.05099961	9.60
77	0.05612148	9.06
78	0.06197740	8.54
79	0.06860023	8.04
80	0.07583658	7.56
81	0.08465607	7.10
82	0.09406716	6.66
83	0.10390006	6.25
84	0.11475734	5.86
85	0.12690193	5.49
86	0.13941902	5.14
87	0.15481203	4.81

Age (x)	TMT 2012	Live life Expectancy
88	0.17113628	4.51
89	0.18841370	4.23
90	0.20541175	3.98
91	0.21846568	3.75
92	0.23527747	3.52
93	0.25573757	3.29
94	0.27936940	3.08
95	0.30669336	2.89
96	0.33209456	2.72
97	0.35875021	2.58
98	0.37353517	2.46
99	0.39535380	2.33
100	0.42297904	2.21
101	0.44870892	2.09
102	0.47665736	1.98
103	0.50701062	1.87
104	0.53995495	1.77
105	0.57534791	1.67
106	0.61216513	1.58
107	0.65047784	1.49
108	0.68992995	1.41
109	0.73138152	1.33
110	0.77448194	1.23
111	1.00000000	1.00

3. RESULT AND DISCUSSION

Suppose there are N independent observations $X_1, X_2, \dots, X_N$ with the probability density function $f(x;\theta), \theta \in \Omega$. Then the likelihood function for θ is defined as

$$L(\theta;x) = \prod_{i=1}^{N} f(x_i;\theta), \theta \in \Omega. \qquad (1)$$

If likelihood function is differentiable in θ, then the value of $\hat{\theta}$ is the value that maximizing $L(\theta)$ function. To make it easier to derive, in most cases, the likelihood function is transformed into a log-likelihood function as follows

$$l(\theta) = \ln L(\theta) = \sum_{i=1}^{N} \ln f(x_i;\theta), \theta \in \Omega. \qquad (2)$$

The value of $\hat{\theta}$ is obtained from solution of equation [11]

$$\frac{\partial}{\partial\theta} l(\theta) = 0. \qquad (3)$$

Suppose there are N individuals whose mortality rate is to be observed in a one year period and assumed following the Binomial distribution. If D represents the number of individuals who death in one year period, with assuming that death of each individual to- i is independent and with same probability i.e. q, then

$$D = \sum_{i=1}^{N} \delta_i, \quad (4)$$

where $\delta_i = 1$ if it's fail (death), $\delta_i = 0$ otherwise (alive). Since it is assumed that $P(\delta_i = 1) = q$, then the probability mass function is [12]

$$P(D = k) = \binom{N}{k} q^k (1 - q)^{N-k}. \quad (5)$$

By using Equation (1), (2), and (3) for Equation (5), so the estimator of q-parameter for each interval of age $(x, x + 1]$ using the maximum likelihood estimator is

$$L(q_x) = \prod_{i=1}^{N_x} \left[\binom{N_x}{\delta_i} q_x^{\delta_i} (1 - q_x)^{N_x - \delta_i} \right]$$

$$\leftrightarrow l(q_x) = \sum_{i=1}^{N_x} [\ln(N_x!) + \delta_i \ln(q_x) + (N_x - \delta_i) \ln(1 - q_x) - \ln(\delta_i!) - \ln((N_x - \delta_i)!)]$$

$$\leftrightarrow l'(q_x) = \frac{\sum_{i=1}^{N_x} \delta_i}{q_x} - \frac{N_x - \sum_{i=1}^{N_x} \delta_i}{1 - q_x} = 0$$

$$\leftrightarrow \hat{q}_x = \frac{D_x}{N_x}. \quad (6)$$

In solving the problems, students are given notations, functions, and formulas of life table. The notations and functions in life table include x which represent the ages of the population, then l_x is number of individuals who survive at the exact age of x [13]. In this research, the radix used was 100,000. The number of death of individuals between the age of x to $x + n$ is denoted by ${}_n d_x$ and defined as

$${}_n d_x = l_x - l_{x+n}. \quad (7)$$

The probability of survival of individuals of aged x will reach the exact aged $x + n$ is denoted by ${}_n p_x$ and defined as

$${}_n p_x = \frac{l_{x+n}}{l_x}. \quad (8)$$

While for special cases, the probability of survival of individuals of aged x will reach the exact aged $x + 1$ is denoted by p_x and defined as [14]

$$p_x = \frac{l_{x+1}}{l_x}. \quad (9)$$

The probability of survival of individuals aged 0 will reach certain aged x is denoted by ${}_x p_0$ and defined as

$${}_x p_0 = \frac{l_x}{l_0}. \quad (10)$$

So by using Equation (9), Equation (10) can also be expressed as

$${}_x p_0 = \frac{l_1}{l_0} \cdot \frac{l_2}{l_1} \cdot \ldots \cdot \frac{l_x}{l_{x-1}} = p_0 \cdot p_1 \cdot \ldots \cdot p_{x-1}. \quad (11)$$

x	${}_n q_x$	l_x	${}_n d_x$	${}_n L_x$	T_x	$\mathring{e}_x$	${}_n m_x$
0	0.00426	100000	426	99787	7525060	75.25	0.00427
1	0.00144	99574	143	398010	7425273	74.57	0.00036
5	0.00112	99431	111	496878	7027263	70.69	0.00022
10	0.00104	99320	103	496343	6530385	65.75	0.00021
15	0.00157	99217	156	495695	6034043	60.82	0.00031
20	0.00240	99061	238	494710	5538348	55.91	0.00048
25	0.00368	98823	364	493205	5043638	51.04	0.00074
30	0.00567	98459	558	490900	4550433	46.22	0.00114
35	0.00869	97901	851	487378	4059533	41.47	0.00175
40	0.01331	97050	1292	482020	3572155	36.81	0.00268
45	0.02036	95758	1950	473915	3090135	32.27	0.00411
50	0.03115	93808	2922	461735	2616220	27.89	0.00633
55	0.04743	90886	4311	443653	2154485	23.71	0.00972
60	0.07197	86575	6231	417298	1710833	19.76	0.01493
65	0.10842	80344	8711	379943	1293535	16.10	0.02293
70	0.16474	71633	11801	328663	913593	12.75	0.03591
75	0.25379	59832	15185	261198	584930	9.78	0.05814
80	0.39208	44647	17505	179473	323733	7.25	0.09754
85	0.57280	27142	15547	96843	144260	5.32	0.16054
90	0.74532	11595	8642	36370	47418	4.09	0.23761
95	0.75178	2953	2220	9215	11048	3.74	0.24091
100+	1.00000	733	733	1833	1833	2.50	0.40000

Figure 1 Abridged life table using data of the Taspen Mortality Table 2012 with estimator of the probability of death using maximum likelihood estimation

Furthermore, the probability of death of individuals aged x will die before reaching the exact aged $x + n$ is denoted by ${}_n q_x$ and defined as

$${}_n q_x = 1 - {}_n p_x = \frac{{}_n d_x}{l_x}. \quad (12)$$

The length of time spent by number of individuals (l_x) in the interval of aged x to $x + n$ is denoted by ${}_n L_x$ and defined as

$${}_n L_x = \int_0^n l_{x+s} \, ds. \quad (13)$$

The remaining total of life time that will be passed by number of individuals of the exact aged x is denoted by T_x and defined as

$$T_x = \sum_{a=0}^{\omega} {}_nL_{x+an}. \quad (14)$$

Where ω is the maximum age of the population. Then the life expectancy of the population aged x is denoted by $\dot{e}_x$ and defined as

$$\dot{e}_x = \frac{T_x}{l_x}. \quad (15)$$

Next, the death rate of the population between the aged x to $x+n$ is denoted by ${}_nm_x$ and defined as [15]

$${}_nm_x = \frac{{}_nd_x}{{}_nL_x}. \quad (16)$$

The following is an abridged life table made by students based on the Polya Model with estimator of the probability of death using the maximum likelihood estimation using data of the Taspen Mortality Table 2012.

4. CONCLUSION

Based on Polya Model, students understand how to construct an abridged life table based on Indonesian mortality data. Based on estimator of the probability of death using maximum likelihood estimation, with assuming follows the Binomial distribution, we get the estimator of the probability of death is the number of deaths compared all individuals observed in that period. Based on the constructing result of abridged life table using data of the Taspen Mortality Table 2012, the life expectancy for the population aged 0 is 75.25 years. That result is not significantly different from the life expectancy of the population aged 0 in the Taspen Mortality Table 2012 (in Table 1) i.e. 75.73 years.

ACKNOWLEDGMENTS

The author would like to thank to my beloved wife dr. Resty Mauliana and my beloved son Muhammad Syamil Al Fatih who always support and pray for me. The author also thanks to students of Ekonomi Syariah UIN STS Jambi who were involved in this research. And also thanks to all committees of the National Conference on Mathematics Education Universitas Sriwijaya for comments and corrections for greatly improved the manuscript.

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