Mathematical Reasoning Ability of Grades IX Students in Triangle Congruence Proof Learning Using Two-Column Proofs Strategy

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ABSTRACT
This study aims to describe the mathematical reasoning ability of grades IX students on the congruence triangle material after taking proof learning using the two-column proofs strategy. The subjects of this study were 25 students of class IX.2 SMP Negeri 1 Palembang. At the first and second meetings, students were taught how to prove the congruence of triangles using the two-column proofs strategy. The data was obtained from a written test consisting of 3 essay questions that have been adjusted to indicators of mathematical reasoning ability. The results showed that the students' mathematical reasoning ability was in the fair category with an average score of 64.33. Students are categorized as fair because some students have met the four reasoning indicators. Students also understand that in compiling evidence, first identify the existing facts, then develop arguments based on these facts using valid reasons. Then, when giving arguments to show evidence, students experience conceptual and principal problems so that they fail to provide valid arguments.

Keywords: *Congruent triangles; Mathematical reasoning; Proofs; Two-column proofs.*

1. INTRODUCTION

One of the essential competencies students must acquire in the modern world is mathematical reasoning [1,2]. Reasoning encourages students to think logically in making conclusions by following existing provisions and making new valid arguments [3]. With mathematical reasoning, students can learn more meaningfully, not only remember facts, concepts, and procedures, or imitate examples, but also can understand mathematical concepts in an integrated manner [4,5,6].

However, the mathematical reasoning ability of Indonesian students is still very low. This is shown by the results of the last TIMSS survey followed by Indonesia, which was in 2015, showing that there are only 20% of Indonesian students who can answer correctly in the cognitive field of reasoning [7]. Similarly, the results of the 2018 PISA survey, the average score of Indonesian students for math literacy which included reasoning was only 379, whereas the international average score was 487 [8].

One of the topics of mathematics that has long been believed to be a means for students to learn to reason is geometry. Standard geometry includes a strong focus on developing reasoning and rigorous proofs, using definitions and undeniable facts [1]. One of the geometry materials contained in the 2013 Curriculum is the material of congruence and similarity, which is taught in grade IX SMP [9]. Congruence and similarity can train students' mindset to be structured in learning mathematics because it contains components that are interconnected with each other [10]. Studying congruence can also develop advanced mathematical thinking skills [11].

However, the results of several studies indicate that there are still many high school students who have low mathematical reasoning abilities, especially in congruence and similarity materials [12]. This is because students are still accustomed to memorizing, are rarely given questions that emphasize high thinking processes, are not accustomed to working on non-routine questions, and are less exposed to mathematical evidence [13,14].

The main solution to overcome this problem of low mathematical reasoning ability is to familiarize students with constructing mathematical proofs [15]. When students discover or prove a mathematical principle, automatically the students' inductive and deductive thinking patterns are developed [13]. Students need to get used to doing proofs to develop their thinking capacity and disposition in drawing conclusions through the possibilities given and as a tool for solving mathematical problems [16]. In addition, completing mathematical proofs is also useful for students to (1) verify the truth, (2) grow new insights about the reason the statement is true, and (3) communicate their mathematical knowledge [17].

Referring to several studies [18,19,20], one way that has the potential to give positive results to the development of students' reasoning abilities is through the application of the two-column proofs strategy in classroom learning. Two-column proofs are a way to practice writing proofs and justifying mathematical statements, especially in geometry class [21]. The use of the two-column proofs strategy helps build ideas and evidence while at the same time accommodating the learning process so that teachers can point out and pay attention to errors in the proving process made by students [22]. Compared to other strategies, the two-column proofs strategy has qualities that allow it to facilitate greater flexibility in reasoning and proof [23]. So far, there has been no research that describes how the mathematical reasoning abilities of grade IX students on the material for proving congruence triangles after they have been taught using the two-column proof strategy.

Based on the description above, the researcher wants to know how the mathematical reasoning ability of class IX students in learning to prove the congruence of triangles is using the two-column proofs strategy.

2. METHOD

This research is a descriptive qualitative study that aims to describe student's mathematical reasoning abilities through two-column proofs strategy on the topic of congruence triangle. The subjects are 25 students of class IX.2 of SMP Negeri 1 Palembang in the odd semester of the 2021/2022 academic year. This research implemented on October 2021 until November 2021. Data was collected by a result of test. The research procedure consists of the preparation stage, the implementation stage, and the data analysis stage. In the preparation stage, the researcher prepares an instrument such as lesson plan (RPP), student worksheet (LKPD) uses two-column proofs strategy, a test consisting of three questions, and a scoring guide. Then, the researchers implemented learning with two-column proofs strategy of two meetings. At the third meeting, the activities were working on test questions. The test consists of 3 questions about proving the congruence of two triangles formed by some 2 dimentional shapes with certain information about the sides and angles. After that, the test results are analyzed according to the scoring guidelines that have been made.

The student's mathematical reasoning ability are identified based on the following indicators:

Table 1. Indicators of mathematical reasoning ability

No.	Indicator	Descriptor
1	Making mathematical statements	Students can make mathematical statements Students can make mathematical statements that are of true value
2	Compile evidence; provide reasons or evidence for the correctness of the solution.	Students can provide arguments against each step of the method or strategy that has been determined in the solution by using valid evidence.
3	Draw conclusions from statements.	Students can make new statements that are truly based on several statements whose truth has been proven or assumed previously through mathematical manipulation.
4	Checks the validity of an argument.	Students can re-examine or investigate the truth of the statements made or given.

The data collection technique in this study was a test. The test questions consist of 3 descriptive questions, each of which contains 4 indicators of mathematical reasoning that you want to see. From the test data, it can be seen the indicators that appear on the students' answers. The research data that has been collected will be analyzed qualitatively by describing whether or not indicators of mathematical reasoning ability appear.

3. RESULTS AND DISCUSSION

After the test implemented, then the test data was analyzed to see the mathematical reasoning ability. Student's mathematical reasoning ability after being analyzed and categorized can be seen in Table 2.

Table 2. Distribution of student mathematical reasoning ability

Score Range	Category	Amount of Students	Percentage
86-100	Excellent	9	36%
71-85	Good	3	12%
56-70	Fair	3	12%
41-55	Poor	1	4%
0-40	Very poor	9	36%

Based on Table 2, students mathematical reasoning ability in class IX.2 of SMP Negeri 1 Palembang is categorized into five categories; excellent, good, fair, poor, and very poor. The results of the analysis of data on the mathematical reasoning skills of students in grade IX.2 SMP Negeri 1 Palembang after participating in the triangle partnership proof learning using a two-column proofs strategy showed that the average score obtained by students was 64.33, so it was in the category enough. The results of the data analysis showed that Class IX.2 students were divided into 5 categories, namely: 36% of students categorized as excellent, 12% of students categorized as good, 12% students are categorized fair, 4% of students are categorized as poor, and 36% of students are categorised as very poor.

Table 3. Distribution of student answer type.

Question Numbers	Student Categorized	Type of Answer
1	Excellent	– Proving precisely – Proving with valid statements and reasons, but incomplete in giving conclusions
	Good	– Proving precisely
	Fair	– Prove with the right statements and conclusions, but there is one statement with invalid reasons – Prove incorrectly in giving some statements and conclusions
	Poor	– Prove correctly, but there are statements with invalid and incomplete reasons in concluding
	Very poor	– Prove incorrectly in providing a valid reason for each statement – Prove incorrectly in giving statements and reasons, and not making conclusions
2	Excellent	– Proving precisely – Proving with valid statements and reasons, but not accurate in giving conclusions
	Good	– Prove with valid statements and reasons, but there is one important statement that does not exist – Proving with valid statements and reasons, but not accurate in giving conclusions – Prove correctly, but there are statements for invalid reasons
	Fair	– Proving precisely – Prove incorrectly in giving valid reasons to some statements and giving conclusions
	Poor	– Prove by there is an incorrect statement and do not make conclusions
	Very poor	– Prove correctly, but there are statements for invalid reasons – Proving incorrectly in providing a valid reason for each statement and is not appropriate in making conclusions – Prove incorrectly in giving statements and reasons, and not making conclusions
3	Excellent	– Proving precisely

Question Numbers	Student Categorized	Type of Answer
	Good	– Proving precisely – Prove incorrectly, but there are some statements for valid reasons
	Fair	– Proving precisely – Prove correctly, but there are statements for invalid reasons – Prove incorrectly, but there are some statements for valid reasons
	Poor	– Prove incorrectly, but there are arguments for valid reasons
	Very poor	– Proving incorrectly in giving a valid reason for each argument and not being right in making a point – Prove incorrectly in providing arguments and reasons, and not making conclusions – Not answering

The results of this study indicate that from learning proof using the two-column proofs strategy, there are 48% of students who can prove the congruence of triangles well. This shows better results than the flow proof strategy based on the research results of Sumarni, et al. [24] where there are only 20% of subjects who can prove the congruence of triangles well. This is relevant to the results of research by Adeliza & Ramli [18] which states that the two-column proofs strategy is easier for students to understand than other proof strategies. But in addition, this contradicts the results of research by Linares [25] which states that it is easier for students to compile evidence using the flow proof strategy than the two-column proofs strategy.

Overall, from the 3 questions tested, students in the very good category can prove it correctly and have brought up all indicators of mathematical reasoning ability. First, the indicator makes a mathematical statement by writing a true-valued statement using the right mathematical notation in writing proofs. Then, the appearance of indicators compiling evidence, providing reasons or evidence for a solution's correctness marked by the arguments given are always supported by valid reasons. However, students must be more careful in the indicators section of compiling evidence, providing reasons or evidence for a correct solution because there are 2 very good students who are wrong in using the principle. The first is wrong about the properties of the line and the second is wrong about the naming of the postulates. Then, the indicator of drawing conclusions from statements is shown by the accuracy of students in providing the final answer as a result of concluding statements. Finally, the indicator checks the validity of an argument by providing a final answer in accordance with the proof step. From the students' answers in the very good category, it can also be seen that the students in the very good category have applied the principle of the two-column proofs strategy in doing the proof. Students start proving from known facts or information from the problem, then build arguments from these known facts or information. Each student's argument is also always accompanied by valid reasons to support the truth of the argument.

In addition to students in the very good category, students in the good category have also brought up all indicators of mathematical reasoning. However, students in the good category generally can only prove correctly on 2 of the 3 questions given. Good category students usually have one significant error in 1 of 3 questions. Significant errors that are often experienced are errors caused by the lack of mature understanding of students' principles and concepts about plane shapes, lines, and angles. Students in the good category have also applied the principle of the two-column proofs strategy in doing the proof.

Enough category students can prove exactly 1 of the 3 questions given. For the emergence of indicators, all indicators have appeared, but students in the sufficient category still have many significant errors, so that in certain questions, not all indicators appear. Indicators that rarely appear are indicators of compiling evidence, providing reasons or evidence for the correctness of the solution, because quite a lot of students have difficulty in providing valid reasons. Another indicator that rarely appears is the indicator of checking the validity of the argument, because often students in the sufficient category have a proof step that is not in accordance with the final answer. Errors experienced by students in the moderate category can occur due to a lack of student understanding of the principles and concepts needed to prove.

Poor category students can only come up with 2 indicators, namely indicators of making mathematical statements and indicators of compiling evidence, providing reasons or evidence for the correctness of the solution. The appearance of indicators for making mathematical statements can be seen from the statements given which are partially correct and have used the right mathematical notation. Indicators of

compiling evidence, providing reasons or evidence for the truth of the argument have also appeared, but are less than optimal because students do not provide sufficient evidence to state the congruence of triangles in answer number 1 number 2. For indicators to draw conclusions from a statement and indicators check the validity of an argument not yet appeared well, because in the answers of the US students the completion steps and the final answers were appropriate, but wrong.

In general, students in the very poor category have not brought up any indicators of mathematical reasoning, except for AR students. AR students can only prove question number 2, and the indicators that appear in their answers are indicators of making mathematical statements and indicators of drawing conclusions from statements. Meanwhile, the other very poor category students did not seem to understand the evidence too well. From the answers of students in the very poor category who provided statements and reasons, and did not provide conclusions, students in the very poor category did not have an accurate conception of what constituted mathematical proof.

The most common error experienced by students when proving the congruence of triangles is in the indicators of compiling evidence, providing reasons or evidence for the correctness of the solution which is indicated by the number of errors in the use of concepts and principles. This is due to the lack of mastery of the prerequisite material which is the basic concept and principle to be used in the proving process, so that when proving students are confused about which concepts and principles are appropriate. As explained by Nurkhaeriyyah, et al [26] that the weak understanding of concepts causes students to be unable to complete their work properly. Another difficulty found in the indicators of constructing evidence, providing reasons or evidence for the correctness of the solution is the difficulty of providing sufficient evidence to show the congruence of the triangles. This difficulty is due to errors in reading important information contained in the problem, which provides clues for students to prove the congruence of the triangles. As explained by Nurkhaeriyyah, et al [26] that errors in reading important information contained in questions cause students not to use the available important information to solve problems.

In some students, the errors experienced in the indicators of compiling evidence, providing reasons or evidence for the correctness of a solution also resulted in errors in the indicators drawing conclusions from statements. Some students experience errors in reading important information contained in the questions so that which makes them give inappropriate arguments that result in errors in drawing conclusions. In the indicator of drawing conclusions from statements, the errors that are quite often found are errors in the use of principles, namely mistakes in determining postulates or theorems that are in accordance with the conditions that are met by both triangles. This principle error causes students to misinterpret the problem which then makes it difficult for students to draw the right conclusions.

Then, the indicator checks the validity of the argument, namely the ability to re-examine or investigate the truth of the statement made or given, indicated by the suitability of the statement in the student's final answer with the evidence provided in the completion step. The errors experienced by students on this indicator are the result of carelessness and lack of thoroughness in analyzing the available information so that there is a discrepancy between the completion steps and the final conclusion.

Then the indicator makes a mathematical statement, namely the ability to make a correct value mathematical statement by using the correct mathematical notation based on the information contained in the problem. In general, students are able to make mathematical statements that are true. Of the 25 students, only 1 student did not use mathematical notation in their statements. Most of them have used the right notation, some are still not disciplined in using notation, and some of them have used it, but do not understand the meaning of each notation. This is in line with Solfitri & Roza [27] that the errors experienced by students in solving geometry problems are notation errors and students' failure to understand the problems to be converted into correct mathematical sentences.

In the indicators of compiling evidence, providing reasons or evidence for the correctness of a solution, students are able to provide arguments against each step of the method or strategy by using valid evidence due to learning using the two-column proofs strategy. As Herbst [28] states that two-column proofs help build ideas and evidence as well as accommodate the learning process. Although students are still wrong in principles and concepts, students already know the basics in compiling evidence. The error experienced by students is because students are not used to doing proofs, the mathematics learning they usually do is only to calculate to determine certain values.

Based on the learning activities that have been carried out using the two-column proofs strategy, it can be said that the two-column proofs strategy is able to develop students' mathematical reasoning abilities. as stated by Verzosa, et al [23] that the two-column proofs strategy has qualities that allow it to facilitate higher flexibility in reasoning and proof than other strategies.

4. CONCLUSION

Based on the results of the study, obtained the mathematical reasoning ability of students of class IX.2 SMP Negeri 1 Palembang is a fair category, with an

average score of 64.33. Students are categorized as fair because some students already meet all four indicators of reasoning. Students also understand that in compiling evidence, first identify the facts, then develop arguments based on those facts using valid reasons. Then, when giving arguments to show evidence, students experience conceptual and principal constraints that fail to provide valid arguments.

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