

Application and Comparison of Markowitz Model and Index Model in Hong Kong Stock Market

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ABSTRACT

This paper aims to study the differences between the index model and the Markowitz model in the Hong Kong stock market and American stock market and explore the advantages and disadvantages of these two models, to provide an investment suggestion for investors who want to enter the Hong Kong stock market. This paper uses the price data of 15 stocks in the recent two years to calculate their return, standard deviation, beta, and other parameters. Using the index model and the Markowitz model to build the portfolio through these parameters. The research results are similar to some research results of applying the index model and the Markowitz model to the American stock market. The portfolios constructed by the two models are almost the same, but the result of the index model will be slightly better than that of the Markowitz model, and the amount of calculation required to construct the index model is much less than that of the Markowitz model. Therefore, this paper recommends investors who want to enter the Hong Kong stock market use the index model when building their optimal portfolio.

Keywords: Index model, Markowitz model, Hong Kong stock market, Optimal portfolio

1. INTRODUCTION

The Hong Kong stock market generally refers to the Hong Kong stock exchange. Compared with the American stock market, the volume of the Hong Kong stock market is much smaller than that of the American stock market. Compared with the Chinese mainland stock market, the Hong Kong stock market is more internationalized. It is much easier for international investors to enter the Hong Kong stock market than to enter the mainland stock market. The turnover of overseas investors accounts for about 40% of the total turnover. The Hang Seng Index (HSI index) is one of the most representative indexes in the Hong Kong stock market. It takes the representative listed stocks as the component stock, and it is a weighted average stock price index weighted by its circulation. It is the most representative stock price index reflecting the price fluctuation trend of the Hong Kong stock market, reflecting the rise and fall of the Hong Kong stock market. In recent years, with the opening of the Shenzhen-Hong Kong stock, connect and Shanghai-Hong Kong stock connects, as well as the impact of the trade war and epidemic situation, the stock market in Hong Kong has been fluctuating. Many investors believe that now is a good time to enter the Hong Kong stock

market. The Markowitz model is proposed by Markowitz, which constructs the portfolio through mean and covariance, while the index model decomposes the risk into company-specific risk and macro risk to construct the portfolio.

Mandal found insight into Sharpe's Single Index Model and its limitations. Mandal used Sharpe's Single Index Model to construct an optimal portfolio and determined the risk and return of the optimal portfolio which is constructed by the single index model. Mandal also pointed out that single index model is an easier way to construct an optimal portfolio than Markowitz's Model and the single index model can show the risk of security which is in a well-diversified portfolio [1]. Ivanova and Dospatliev made an empirical study on the Markowitz model on the Bulgarian stock market from 2013 to 2016 and used the Markowitz theory to determine the efficient frontiers with the optimal portfolio of 50 stocks from the Bulgarian stock market. Ivanova and Dospatliev figured out that the efficient portfolios which are determined by the Markowitz theory had a better performance compared with other Bulgarian individual security [2]. Putra and Dana determined the optimal portfolios by the single index model and the Markowitz model. Wilcoxon-Mann-Whitney test is used for the data analysis. Putra and Dana compared the performance and

average return of the single index model with the Markowitz model. Putra and Dana pointed out that risk-averse prefer using a single index model and risk-taker prefer using the Markowitz model [3].

Utami et al. used the index model to determine the optimal portfolio of the L45 index members during one year of the covid-19 pandemic. Utami et al. figured out each weight of L45 member at the minimum risk and each weight of L45 member at the maximum return with a certain risk [4]. Nalini selected 15 companies from the S&P BSE Sensex index and determined the "cut off point" to select the stocks for the optimal portfolio using the single index model. Nalini selected four stocks to determine the optimal portfolio and pointed out both macro and micro factors are important to investing [5]. Mary and Rathika applied single index model to construct an optimal portfolio of 10 companies from NSE and CNX PHARMA price index which came from September 2010 to September 2014. Mary and Rathika figured out the "cut off point" and only one stock's Cut off value is higher than the "cut off point" which means that only one stock in the optimal portfolio [6]. Lam et al. compared 11 methods of forecasting the future correlation matrix of stock returns in the Hong Kong stock market. Lam et al. pointed out that when the homogeneous grouping of stocks is used to construct a multi-index model, the multi-index model based on family grouping is better than the single-index model and the single index model is better than the multi-index model based on industry grouping. Lam et al. also pointed out that none one is better than the overall mean model [7].

Varghese and Joseph compared the Markowitz model and the index model. Varghese and Joseph pointed out that the limitation of the index model is that it does not include the time effect. Varghese and Joseph also pointed out the difference and similar characteristics between the index model and the Markowitz model [8]. Du Plessis and Ward determined the optimal portfolio of the top 40 JSE listed companies by the Markowitz model and re-determined periodically. Du Plessis and Ward compared the return of the optimal portfolio and the FTSE/JSE ALSI40 index. Du Plessis and Ward pointed out that the Markowitz model provided the basis of investing strategy [9]. Chen and Brown used the single index to show the simple decision rules for optimal portfolio selection derived by Elton et al. [10] are not the same under the Bayesian and the traditional methods of analysis. Chen and Brown pointed out that when short sales are not allowed, the number of securities in the optimal portfolio under the Bayesian method may be smaller than the traditional method [11].

This paper aims to do an empirical study which is studying the difference of applications by the index model and the Markowitz model in the Hong Kong stock market and American stock market and explore the

advantages and disadvantages of these two models under the possible real-world investment conditions. It is hoped that this study will provide a relevant investment direction for investors who want to construct their optimal portfolio in Hong Kong stock market.

2. METHOD

In this section, this paper introduces the Markowitz model and single index model which help investors to determine the optimal portfolio. This paper also introduces three constraints that may happen in the real world in this section.

2.1 The Markowitz model

The Markowitz model determines the weights of every asset in the portfolio when there is a max sharp rate by average daily return and covariance between every two assets and average standard deviation.

To construct a Markowitz model, investors need to get the return from objective assets. the return is equal to the value which is the price in t minus the price in t-1, divided by the price in t-1.

$$R_{i,t} = \frac{(p_{i,t} - p_{i,t-1})}{p_{i,t-1}} \quad (1)$$

Where the $R_{i,t}$ is the daily return of asset i in day t, $p_{i,t}$ is the price of asset i in day t, $p_{i,t-1}$ is the price of asset i in day t-1.

The average daily return of each asset is equal to the mean of everyday return.

$$R_{i,mean} = \sum_{t=1}^n R_{i,t} \quad (2)$$

Where the $R_{i,mean}$ is the daily average return, n is the days counted.

The expected return of the portfolio is equal to the sum of the average daily return multiplied by the proportion of its corresponding assets.

$$E(r_p) = \sum_{i=1}^n w_i E(r_{i,mean}) \quad (3)$$

Where the $E(r_p)$ is the expect return of the portfolio, $E(r_{i,mean})$ is the $R_{i,mean}$.

The variance of the portfolio is computed using the equation:

$$\sigma_p^2 = \sum_{i=1,t=1}^n \sum_{j=1,t=1}^n w_i w_j Cov(r_{i,t}, r_{j,t}) \quad (4)$$

Where σ_p^2 is the variance of portfolio, w_i and w_j are the weight of asset i and j, $Cov(r_{i,t}, r_{j,t})$ is the covariance between the two assets.

The Sharp rate of the portfolio is computed using the equation:

$$S_p = \frac{E(r_p) - r_f}{\sigma_p} \quad (5)$$

Where S_p is the Sharp rate of the portfolio and r_f is the risk-free rate. σ_p is the standard deviation of the portfolio.

The Markowitz model can determine the optimal proportion of assets when the average rate of return of each asset, the covariance between two assets, and the average standard deviation are given, according to the equations.

2.2 The index model

The single index model takes the excess return of each asset as the dependent variable and the market excess return as the independent variable for linear regression. According to the regression, investors get the asset's expected return if the market is neutral and systematic risk of an asset and unsystematic risk.

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t} \quad (6)$$

Where α_i is the asset's expected return if the market is neutral, β_i is the asset's responsiveness to market movements or systematic risk of asset i. $\varepsilon_{i,t}$ is the unexpected component or return due to unexpected events that are only to this asset, or unsystematic risk.

The portfolio's return due to unexpected events that are only to this portfolio and responsiveness to market movements is computed using the equation:

$$\alpha_p = \sum_{i=1}^{n+1} w_i \alpha_i \quad (7)$$

$$\beta_p = \sum_{i=1}^{n+1} w_i \beta_i \quad (8)$$

Where the α_p is the portfolio's expected return if the market is neutral and β_p is the responsiveness to market movements of the portfolio.

The residual variance of the portfolio can be computed by the equation:

$$\sigma_{\varepsilon_p}^2 = \sum_{i=1}^{n+1} w_i^2 \sigma_{\varepsilon_i}^2 \quad (9)$$

$\sigma_{\varepsilon_p}^2$ is the residual variance of the portfolio and $\sigma_{\varepsilon_i}^2$ is the residual variance of each asset.

The exceeded return of the portfolio can be computed by the equation:

$$E(r_p) = \sum_{i=1}^n w_i E(r_{i,mean}) \quad (10)$$

Where $E(r_p)$ is the exceeded return of the portfolio.

The standard deviation of the portfolio can be computed by the equation:

$$\sigma_p = (\beta_p^2 \sigma_m^2 + \sigma_{\varepsilon_p}^2)^{\frac{1}{2}} \quad (11)$$

σ_p is the standard deviation of portfolio and σ_m^2 is the variance of the market.

The Sharp rate of the portfolio is computed using the equation:

$$S_p = \frac{E(r_p) - r_f}{\sigma_p} \quad (12)$$

The single index model can determine the optimal proportion of assets when the daily average return of each asset, the residual variance of each asset, the asset's expected return, and the asset's responsiveness to market movements are given, according to the equations.

2.3 Three constraints

The first constraint is that leverage greater than 50% is not allowed, the second constraint is that the absolute value of each investment proportion is not allowed to exceed 1, and the third constraint is that shorting is not allowed.

3. RESULT

In this section, this paper calculated the annualized average excess return, annualized standard deviation, and other parameters of each stock, and index. The obtained results are shown in Table 1, as follows:

Table 1. The parameters of stocks such as annualized average excess return, annualized standard deviation, beta, etc.

Stock code	Annualized Average Excess Return	Annualized StDev	Beta	Annualized Alpha	Residual StDev
HSI(Index)	-41.96%	79.46%	100.00%	0.00%	0.00%
2007.HK	-61.84%	88.49%	101.63%	-19.20%	36.18%
175.HK	13.04%	101.79%	108.67%	58.64%	53.91%
981.HK	25.66%	106.38%	97.92%	66.74%	72.55%
2318.HK	-65.07%	80.63%	96.34%	-24.65%	25.33%
388.HK	15.12%	85.36%	101.40%	57.67%	28.21%
1299.HK	-29.62%	84.08%	99.99%	12.33%	27.51%
9988.HK	-62.95%	90.67%	101.47%	-20.38%	41.47%
700.HK	-8.16%	90.03%	104.59%	35.72%	34.62%
2319.HK	2.11%	87.99%	101.54%	44.72%	35.10%

2331.HK	78.60%	106.20%	111.86%	125.53%	58.13%
386.HK	-38.70%	81.31%	92.70%	0.20%	34.42%
2380.HK	71.48%	94.83%	97.04%	112.20%	55.19%
2688.HK	17.12%	91.84%	101.52%	59.71%	43.90%
6862.HK	-58.06%	99.48%	99.48%	-16.32%	60.41%
1579.HK	-28.10%	108.41%	108.03%	17.22%	66.21%

Most stocks had negative excess returns and large standard deviations. The standard deviation of the HSI index was the smallest, the standard deviation of stock 1579.HK was the largest, and its fluctuation was also the largest. The stock 2331.HK had the largest excess return, and it was the most sensitive to market fluctuations, and its alpha was also the largest. The stock 981.HK had the largest residual standard deviation.

Based on Table 1, we can determine the optimal proportions of each investment and outcome under different constraints and construct by the index model and the Markowitz model that resolve three optimization problems: maximum sharp rate, minimum standard deviation, and maximum excess return. The obtained results are shown in the six tables, as follows:

Table 2. The three optimal results under constraint 1 are determined by the Markowitz model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	137.91%	-63.58%	147.88%
StDev	101.34%	76.21%	126.92%
Sharpe	136.08%	-83.42%	116.51%

Table 3. The three optimal results under constraint 1 are determined by the Index model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	142.29%	-73.25%	147.88%
StDev	103.05%	75.68%	129.65%
Sharpe	138.08%	-96.79%	114.06%

In the case of constraint 1, the maximum sharp rate calculated by the Markowitz model is 2% smaller than that of the index model, while the minimum standard deviation calculated by the Markowitz model is 0.53%

larger than that of the index model, and the maximum excess return calculated by the Markowitz model is the same as that calculated by the index model.

Table 4. The three optimal results under constraint 2 are determined by the Markowitz model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	468.10%	-73.04%	573.19%
StDev	164.25%	75.97%	219.34%
Sharpe	284.99%	-96.15%	261.32%

Table 5. The three optimal results under constraint 2 are determined by the Index model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	490.20%	-85.97%	573.19%
StDev	163.18%	75.38%	200.41%
Sharpe	300.40%	-114.04%	286.00%

In the case of constraint 2, the maximum sharp rate calculated by the Markowitz model is 15.41% smaller than that of the index model, while the minimum standard deviation calculated by the Markowitz model is 0.59%

larger than that of the Index model, and the maximum excess return calculated by the Markowitz model is the same as that calculated by the index model.

Table 6. The three optimal results under constraint 3 are determined by the Markowitz model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	74.68%	-40.81%	78.60%
StDev	91.14%	77.79%	106.20%
Sharpe	81.94%	-52.47%	74.01%

Table 7. The three optimal results under constraint 3 are determined by the index model.

Outcome	Maximum sharp rate	Minimum standard deviation	Maximum excess return
Return	74.67%	-40.56%	78.60%
StDev	91.61%	77.77%	106.20%
Sharpe	81.51%	-52.16%	74.01%

In the case of constraint 3, the maximum sharp rate calculated by the Markowitz model is 0.43% larger than that of the index model, while the minimum standard deviation calculated by the Markowitz model is 0.02% larger than that of the index model, and the maximum excess return calculated by the Markowitz model is the same as that calculated by the index model.

4. DISCUSSION

With the spread of Covid-19, the stock market in Hong Kong had been falling in recent years which is also mentioned in Cao et al.'s research [12]. This paper uses the data of the last two years, so the HSI index and most of the stocks' excess returns used in this paper are negative. Moreover, due to the environmental impact in the past two years, Hong Kong's stocks fluctuated greatly.

Because the number of companies, transaction costs, liquidity, and investment tools in the U.S. stock market are better than those in Hong Kong, the development of the U.S. stock market is better than that in Hong Kong, and its anti-risk degree is stronger. Therefore, the return of HSI index is less than SPX 500 index, and its risk is greater.

By comparing different constraints, constraint 2 had the best result in three conditions. The reason is that constraint 2 is the least stringent constraint which means in this constraint investors can short and have leverage. In addition, constraint 3 is the most stringent.

According to the results of the third section, the Markowitz model and the index model can be well applied in Hong Kong stock market under different constraints. The reason is that although the American stock market has some differences from Hong Kong stock market their essence is the same. The Markowitz model uses stock's covariance to construct a portfolio and

the index model uses stock's beta and alpha to construct a portfolio that both models can apply in the two stock markets.

As shown in Table 2 to table 7, the optimization portfolio calculated by the index model is very similar to that calculated by the Markowitz model, and in most cases, the optimization portfolio calculated by the index model is better than that calculated by the Markowitz model. This result is very similar to Qu et al.'s research which compared the difference between the Markowitz model and the index model [13]. The reason is also mentioned in Bodie et al.'s book [14], although the Markowitz model is more flexible in asset covariance structure than the index model, however, this advantage is unrealistic if the covariance cannot be estimated from any confidence degree. Using total covariance requires estimating thousands of risk values. Even if the Markowitz model is better in theory, too much estimation error accumulation had an impact on the portfolio, which leads to its inferiority to the portfolio determined from the index model. In addition, the index model also can separate macro analysis and securities analysis. In a word, the index model not only can reduce the calculation but also can determine a better solution than the Markowitz model. So, this paper suggests that investors, which are interested in Hong Kong stocks but don't understand Hong Kong stocks market, use the index model to determine their optimal portfolios.

5. CONCLUSION

This paper applied the Markowitz Model and the index model to fifteen stocks in Hong Kong stock market by two years' data of each stocks' price and compares their optimal solutions under three different constraints. The main findings of this paper were similar to some research that applied two models in other stock markets like the American stock market. The Markowitz model

and the index model can be applied to the Hong Kong stock market. In the Hong Kong market, the index model is also better than the Markowitz model, not only because the amount of calculation required by the index model is much less than the Markowitz model, but also the optimal result calculated by the index model is better than the Markowitz model. Therefore, it was recommended that investors who want to enter the Hong Kong stock market use the index model to build their portfolios. This paper made an empirical study for the difference between the index model and the Markowitz model in the Hong Kong stock market and provided a direction for investors who are interested to enter the Hong Kong stock market but know less about the Hong Kong stock market.

REFERENCES

- [1] N. Mandal, Sharpe's single index model and its application to construct optimal portfolio: an empirical study, *Great Lake Herald*, vol.7, no.1, pp.1-19, 2013.
- [2] M. Ivanova, L. Dospatliev, Application of Markowitz portfolio optimization on Bulgarian stock market from 2013 to 2016, *International Journal of Pure and Applied Mathematics*, vol.117, no.2, pp.291-307, 2017. DOI:10.12732/ijpam.v117i2.5
- [3] I. K. Putra, I. M. Dana, Study of Optimal Portfolio Performance Comparison: Single Index Model and Markowitz Model on LQ45 Stocks in Indonesia Stock Exchange, *American Journal of Humanities and Social Sciences Research (AJHSSR)* 2020, vol.3, no. 12, pp. 237-244, 2020.
- [4] E. M. Utami, L. Amaliawati, S. Komariah, D. M. Puspitasari, O. Sinaga, The Analysis of Optimal Portfolio Formation: The Evidence from LQ-45 during the Covid-19, *Review of International Geographical Education Online*, vol.11, no. 6, pp. 121-131, 2021.
- [5] R. Nalini, Optimal Portfolio construction using Sharpe's Single Index Model-A study of selected stocks from BSE, *International Journal of Advanced Research in Management and Social Sciences*, vol.3, no.12, pp. 72-93, 2014.
- [6] J. F. Mary, G. Rathika, The single index model and the construction of optimal portfolio with cnxpharma scrip, *International Journal of Management*, vol.6, no.1, pp.87-96, 2015.
- [7] K. Lam, H. M. Mok, I. Cheung, H. C. Yam, Family groupings on performance of portfolio selection in the Hong Kong stock market, *Journal of banking & finance*, vol.18, no.4, pp.725-742, 1994. DOI: [https://doi.org/10.1016/0378-4266\(94\)00017-4](https://doi.org/10.1016/0378-4266(94)00017-4)
- [8] J. Varghese, A. Joseph, A Comparative Study on Markowitz Mean-Variance Model and Sharpe's Single Index Model in the Context of Portfolio Investment, *PESQUISA*, vol.3, no.2, pp.36-41, 2018.
- [9] A. J. Du Plessis, M. Ward, A note on applying the Markowitz portfolio selection model as a passive investment strategy on the JSE, *Investment Analysts Journal*, vol.38, no.69, pp.39-45, 2009. DOI:<https://doi.org/10.1080/10293523.2009.11082508>
- [10] E. J. Elton, M. J. Gruber, M. W. Padberg, Simple criteria for optimal portfolio selection, *The Journal of Finance*, vol.31, no. 5, pp. 1341-1357, 1976. DOI: <https://doi.org/10.2307/2326684>
- [11] S. N. Chen, S. J. Brown, Estimation risk and simple rules for optimal portfolio selection, *The Journal of Finance*, vol.38, no.4, pp.1087-1093, 1983. DOI: <https://doi.org/10.2307/2328013>
- [12] K. H. Cao, Q. Li, Y. Liu, C. K. Woo, Covid-19's adverse effects on a stock market index, *Applied Economics Letters*, vol.28.no.14, pp.1157-1161, 2021. DOI:<https://doi.org/10.1080/13504851.2020.1803481>
- [13] K. Qu, Y. Tian, J. Xu, J. Zhang, Comparison of the Applicability of Markowitz Model and Index Model Under 5 Real-World Constraints for Diverse Investors, In 2021 3rd International Conference on Economic Management and Cultural Industry (ICEMCI 2021), pp.1323-1332, 2021. DOI: <https://doi.org/10.2991/assehr.k.211209.216>
- [14] Z. Bodie, A. Kane, A. J. Marcus, *Investments*, 2013. DOI: 10.1002/9783527627691.ch11