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Analysis of Ten Stock Portfolios Using Markowitz and Single Index Models

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ABSTRACT

Reasonable choice of modern portfolio model is a decisive step when making strategy during the investing in capital market or money market. In this paper, the Markowitz Model and Single-Index Model are selected for comparison to show which one could provide better performance. Meanwhile, the Capital Asset Pricing Model (CAPM) is used to support the research. Totally 20 years of historical daily total return data for ten famous companies (Microsoft Corporation, Akamai Technologies, etc.) are collected in this research dealt with a set of complicated mathematical methods. In the analysis step, Excel Slover is used to managing the optimization problem under five different constraints for each model. According to the paper, Markowitz Model performs better in the high-risk portfolio and when faced with a low-risk investment project, utilizing Index Model would be a better choice for the investor.

Keywords: *Markowitz Model, Single-Index Model, Constraints, Portfolios.*

1. INTRODUCTION

Asset allocation is an important investment strategy to balance risk and reward by apportioning a portfolio's assets according to goals individually, risk tolerance, and investment horizon. The research of how to manage the asset properly has been started at an early age. For instance, the Markowitz Portfolio Selection Model was published by Harry Markowitz in 1952 which has been very successful theoretically. However, another model may perform better in different ways when investing in daily life. The swift spread of the COVID-19 has caused unpredictable fluctuations in financial markets, learning and making a comparison in different modern portfolio models to find the better strategy when investing has become a more significant subject for both private investors and portfolio managers. Two portfolio models are chosen for making inferences and the Capital Asset Pricing Model (CAMP) which shows the relationship between the expected rate of return on assets and risky assets is used as extra support.

Siruk and Kren explored how Markowitz Portfolio Theory (MPT) can provide investment portfolios. The authors show that individual securities' weights can be calculated using the derivation from MPT's capital marketing pricing model (CAPM). The calculations can result in different portfolios, for example, a portfolio having high and low beta coefficients. In addition, a random portfolio can be derived, and a capital asset marketing model can be used to create a reference from the same portfolio. The sample selected to examine various classes of assets only selected securities for this study. The authors applied a set of rules to group stock according to their portfolio. Dow Jones Industrial Average Index was used as a reference for selecting a stock used in this study. The levels of risk and return for each portfolio were used to compare the profiles. The outcome of this research is expected to give suggestions on the best approach for choosing stock belonging to a particular investor [1]. Vasilieva and Natalia propose that the food shortage and climate change crisis can be addressed by adopting better methods for the cultivation of oilseeds and cereals. The authors believe that using MPT can be applied in championing the move towards realizing green energy usage. The study proposes that better cereal and oilseeds farming practices will help to increase food production. Vasiliev and Natalia observe the natural phenomena and marketing problems resulting in poor yields and fluctuating prices of cereals such as barley, soy seeds, wheat, and sunflower. The authors propose the application of the Markowitz Portfolio Model to address the named problems. The performance of the world's best cereal and oilseeds farmers can be examined by basing arguments from the findings on Markowitz-mean-variance indicators research. The study on MPT can be applied nationally to examine the compromise between risks and possible incomes from a given cereal. The study examined how farmers in Ukraine can position themselves in the world market by selecting the appropriate farms for the cultivation of oilseeds and cereals [2]. Way et al. examined how experience curves can be followed in technologies by applying theories on portfolios. Optimal investment options can be issued to competing technologies by considering the Markowitz Portfolio Theory. The standard mean-variance method is used to allocate the stock investment options for the competing technologies. The authors assume the relationship between cost and availability following Wright's Law. The learning curve, otherwise called Wright's Law, proposes that the price of a given commodity decreases when the product's availability declines. The theory, however, introduces complexity in the portfolio problem since it establishes a relationship between investment and cost. Therefore, investors are faced with a dilemma to choose between investing in many projects to reduce the risk of failure or maximizing profits by focusing on only a single project. The study made its case by comparing two different technologies. The two technologies were examined in light of the rate of progress and discount, diversity, risk mitigation measures, and the starting investment and experience. The portfolios of the technologies are visualized using the frontier framework. The rate of discount is established by studying uncertainty and possible risks from two different periods [3].

Kumar establishes profits and risks as the two sides of any investment made by an individual or company. The study claims that every investor expects to maximize returns when the levels of risk are kept to a minimum. In other words, for a given amount of risk endured for any investment, a given level of profit is expected. The author asserts that the MPT model proposed by Harry Markowitz can be used to find a compromise between expected returns and possible risks for investing in the stocks of a given portfolio. The MPT helps in the selection of the best security portfolio for maximum returns and minimum risks by comparing various portfolios. Kumar agrees that modern portfolio theory derived its hypothesis from the Markowitz theory on portfolios. The study findings establish that MPT demonstrates a relationship between risk and returns. The author further finds out that risk can be minimized by applying Harry Markowitz's theory. The paper's focus is on the contributions of Markowitz's work on portfolio theory on the analysis of stocks [4]. Bilbao et al. examine portfolio selection grounded on a single index model .in; their discussion’s expert predictions about future betas of every financial asset has been put into the account in the model. Bilbao et al. suggest that to get an optimal portfolio, a Goal Programming model includes precise investors admirations, including assets proportion of both high- and low-level risk assets. The semantics of these targets is inlined with the fuzzy theory. To illustrate the model's working, a presentation of the actual portfolio selection is undertaken [5]. Roll shows serious doubts while testing the capital asset pricing model with investments. The insofar as proxies according to the study were used for the stock market portfolio. The study did not use and test the Sharpe Lintner theory. Furthermore, Rolls made emphasis on the regression test and probably became of low power and grouped lower. The findings and testing of this empirical study made an odd state. The explanations given above indicate the single-factor capital asset pricing model and it does not build the other factors towards asset returns [6]. Ünlü and Xanthopoulos explore the reasons behind the popularity of consensus learning. This study emphasizes the advantages presented by ensemble learning. The paper identifies the efficiency of combining multiple solutions into one. The authors identify algorithm schemes such as the voting algorithm as one of the existing angles of viewing ensemble learning. The clustering accuracy determines how different algorithms for clustering are modified to fit their purpose. This paper explores algorithms that produce varied performances upon multiple runs to integrate weights. The Markowitz theory on portfolios inspired the researchers. The author proposes an acceptable level of risk that can be tolerated to gain a certain amount of profit from investing in securities of given portfolios. The authors suggest an approach that results in stability and an acceptable level of fluctuation [7].

Maier-Paape and Zhu propose that an investment's success is often characterized by risk and utility. The authors propose that any investment requires a certain compromise level between utility and risk. Plotting utility and risk on a simple chart produces a convex curve. The general framework presented by this paper results in Markowitz theory and capital market pricing index, when the standard deviation is equated as risk, and mapping of identity as a utility. The authors explore the different theories that can be demonstrated using the general format of the utility function. The general scheme presented by the authors provides a common ground for various existing portfolio theories. This study, however, goes beyond just establishing the relationship between these existing theories; it introduces a bit of complexity [8]. Chao et al. examine how Markowitz Portfolio theory can be applied when making decisions on securities investment. The paper asserts that investors can use several tools for making intelligent, informed decisions before investing in securities. Every week's average interest rate of all available stock options can be tracked. Calculations involving a variety of portfolios can also be carried out using statistical tools such as Matlab. The authors established the value of portfolios selected using the Markowitz model on the A-share market [9]. Dumrongpokaphan and Kreinovich give an analogy of medicine used to investigate the effect of spreading risks in securities investments. The authors argue that as much it is a good practice to invest in securities of different portfolios, it still poses a risk of even more significant losses in case of any mishap. The study gives an interesting example of using different types of medication as increasing the possibilities of side effects from the drugs used. The authors propose that applying the Markowitz Portfolio theory will help an investor make an investment decision involving the right combination of portfolios that will result in profits [10].

2. METHODS

2.1 Markowitz Model

Markowitz believes that investors are risk evaders, and they are unwilling to bear the additional risks that are not compensated by corresponding expected returns. Investors can use the diversified portfolio to minimize the deviation of expected returns, so Markowitz solves the risk problem in diversified portfolio assets according to a set of complicated mathematical methods.

Assuming that there are n different risk assets, the actual rate of return of the I-type risk asset in the T year is Rit, and the actual average rate of return in the N years is recorded as Ri, $Ri = \sum_{i=1}^{n} Rit$， the investment ratio of type I risk assets in the portfolio is Xi, and $\sum_{i=1}^{n} Xi = 1$，$Xi \geq 0$.

The investment ratio of type I risk assets in the portfolio is Xi, and $Rp = \sum_{i=1}^{n} XiRi$； Assuming that through combination, the expected target of yield is R, that is, the conditions are met $r = \sum_{i=1}^{n} XiRi$.

The standard deviation of the risk-return ratio of the portfolio, and the variance is the square of the standard deviation, that is:

$$\delta^2 = \sum_{ij=1}^{n} XiXj\sigma ij = \sum_{ij=1}^{n} \sigma iXi \ \ \sum_{i,j=1}^{n} ijXiXj \qquad (1)$$

To sum up, Markowitz optimization model is

$$min\delta^2 = \sum_{ij=1}^{n} \sigma iXi + \sum_{i,j=1.i=j}^{n} ijXiXj \qquad (2)$$

Among them, the conditions are met：

$$\sum_{i=1}^{n} XiRi = r \qquad (3)$$

$$\sum_{i=1}^{n} Xi = 1\ ,\ \mathrm{Xi} \geq 0, \mathrm{i} = 1,2, \ldots\ldots \mathrm{n} \qquad (4)$$

2.2 CAPM

As a prediction model based on the equilibrium of expected return of risk assets, CAPM describes the Formulation of market equilibrium using Markowitz theory. It indicates the expected return and expected risk as a linear relation. In other words, there is a positive relationship between the expected return of assets and beta value which shows the risk of assets.

$$ER_i = R_f + \beta_i \times (ER_m - R_f) \qquad (5)$$

Where ER_i is the expected return of asset i , R_f is the risk-free rate of the capital, ER_m is the expected market rate of return, The beta coefficient shows how sensitive the return on an asset is to market changes, and $(ER_m - R_f)$ is market risk premium.

2.3 Single-Index Model

The single-index model (SIM) is a simple asset pricing model, which is usually used in the financial industry to evaluate the risk and return of a stock.

$$r_{it} - r_f = \alpha_i + \beta_i(r_{mt} - r_f) + \epsilon_{it} \qquad \epsilon_{it} \sim N(0, \sigma_i) \qquad (6)$$

r_{it} is returned to stock i in period t, r_f is the risk-free rate, r_{mt} is the return to the market portfolio in period t, α_i is the stock's alpha or abnormal return, β_i is the stocks' beta or responsiveness to the market return, $r_{it} - r_f$ is called the excess return on the stock, $r_{mt} - r_f$ the excess return on the market, ϵ_{it} is the residual (random) return, which is assumed normally distributed with mean zero and standard deviation σ_i.

Table 1. Stocks and constraints

	SPX	QCOM	AKAM	ORCL	CVX	XOM	IMO	PEP	MSFT	KO	MCD	return	stdav	sharps
Constr1	-50.5%	6.1%	9.0%	2.8%	14.9%	8.8%	-18.3%	12.0%	5.3%	22.2%	50.0%	13.5%	15.4%	0.88
Constr2	-91.8%	-91.8%	11.2%	6.4%	22.6%	21.6%	-29.0%	15.9%	12.5%	32.0%	50.0%	15.9%	17.9%	0.89
Constr3	-91.8%	9.9%	11.2%	6.4%	22.6%	21.6%	-29.1%	15.9%	12.5%	32.0%	50%	15.9%	17.9%	0.89
Constr4	0.0%	3.1%	11.0%	0.0%	10.7%	0.0%	0.0%	7.1%	1.0%	23.8%	50%	16.0%	19.7%	0.81
Constr5	0.0%	5.1%	10.6%	-0.3%	13.4%	15.2%	-43.6%	16.8%	6.4%	28.1%	50%	16.9%	20.0%	0.85

3. RESULT

3.1 Markowitz Model

Table 1 shows the optimal portfolio of each of the five constraints under the Markowitz model for ten stocks and the S&P500 index.

Under the constrain 1, SPX, QCOM, AKAM, ORCL, CVX, XOM, IMO, PEP, MSFT, KO are -50.5%, 6.1%, 9.0%, 2.8%, 14.9%, 8.8%, -18.3%, 12%, 5.3%, 22.2% respectively. The relationship function between the return and standard deviation of this portfolio and their CAL curves are shown in Figure 1.

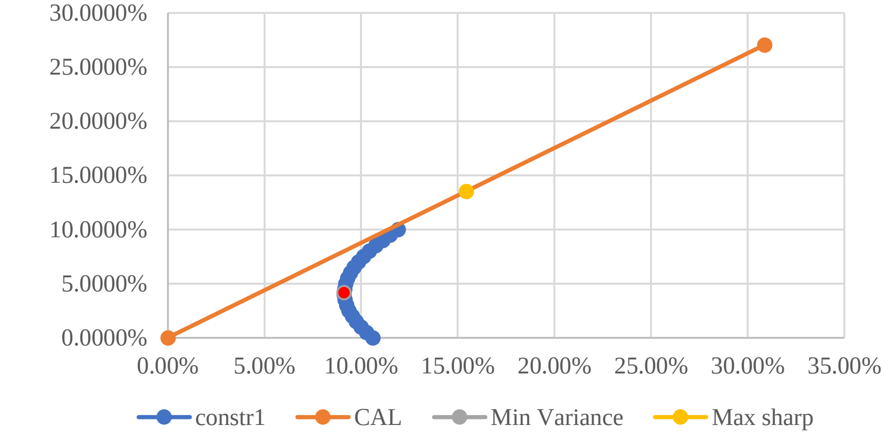

Figure 1 Constraints 1 $\sum_{i=1}^{11} | W_i | \leq 2$.

Under the constrain 2, SPX, QCOM, AKAM, ORCL, CVX, XOM, IMO, PEP, MSFT, KO are -91.8%, -91.8%,11.2%, 6.4%, 22.6%, 21.6%,-29.0%, 15.9%, 12.5%, 32.0% respectively. The relationship function between the return and standard deviation of this portfolio and their CAL curves are shown in Figure 2.

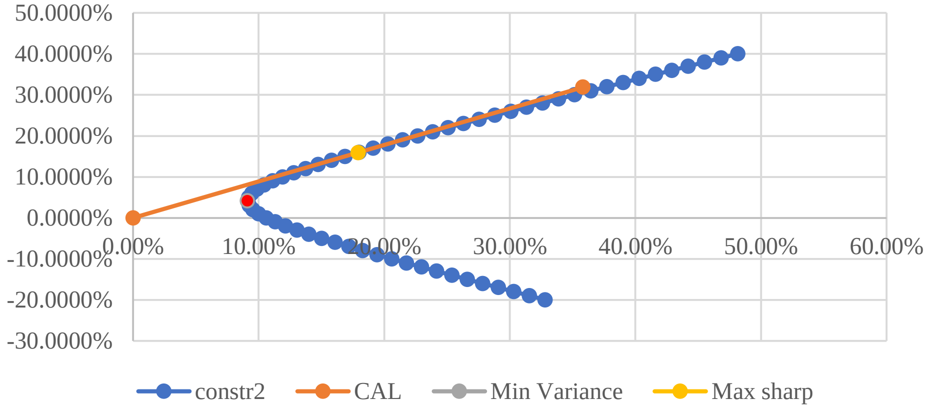

Figure 2 Constraints 2 $|W_i| \leq 1$, for ∀ i.

Under the constrain 3, SPX, QCOM, AKAM, ORCL, CVX, XOM, IMO, PEP, MSFT, KO are -91.8%, 9.9%,11.2%, 6.4%, 22.6%, 21.6%, -29.1%, 15.9%, 12.5%, 32.0% respectively. The relationship function between the return and standard deviation of this portfolio and their CAL curves are shown in Figure 3.

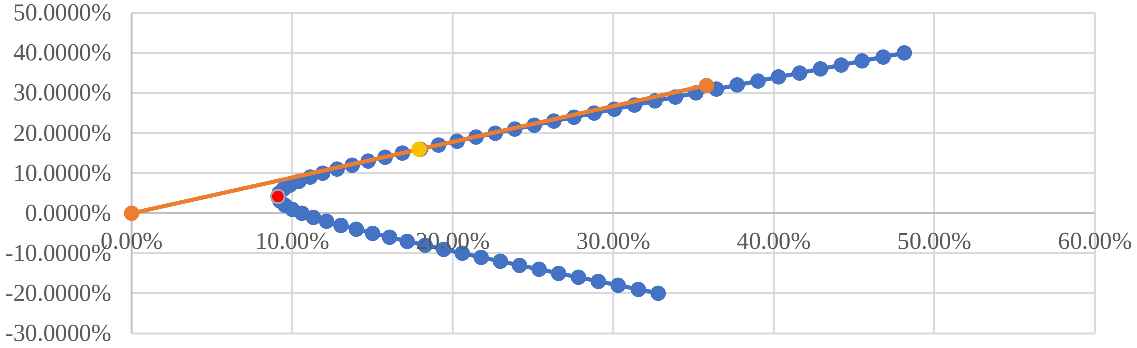

Figure 3 Results of no constraints.

Under the constrain 4, SPX, QCOM, AKAM, ORCL, CVX, XOM, IMO, PEP, MSFT, KO are 0.0%, 3.1%, 11.0%, 0.0%,10.7%, 0.0%, 0.0%, 7.1%, 1.0%, 23.8%respectively. The relationship function between the return and standard deviation of this portfolio and their CAL curves are shown in Figure 4.

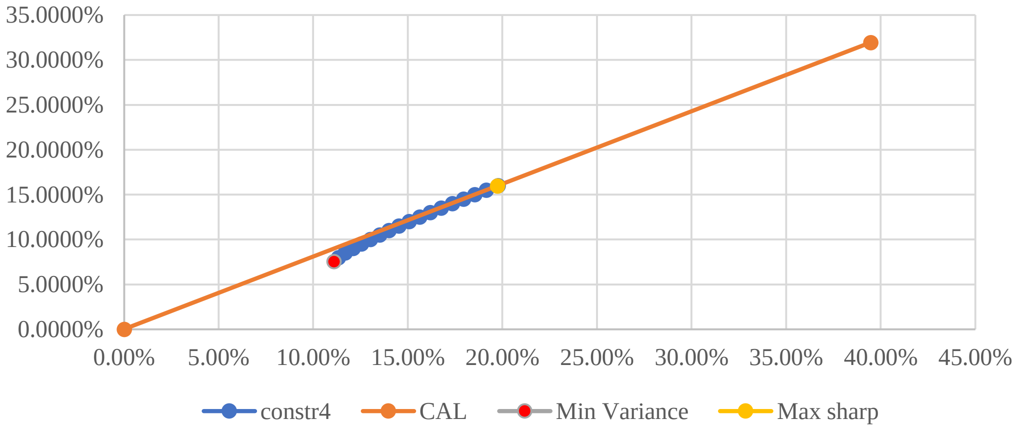

Figure 4 Constrain 4 $W_i \geq 0$, for ∀ i.

Under the constrain 5, SPX, QCOM, AKAM, ORCL, CVX, XOM, IMO, PEP, MSFT, KO are 0.0%, 5.1%, 10.6%, -0.3%, 13.4%, 15.2%, -43.6%, 16.8%, 6.4%, 28.1%respectively. The relationship function between the return and standard deviation of this portfolio and their CAL curves are shown in Figure 5.

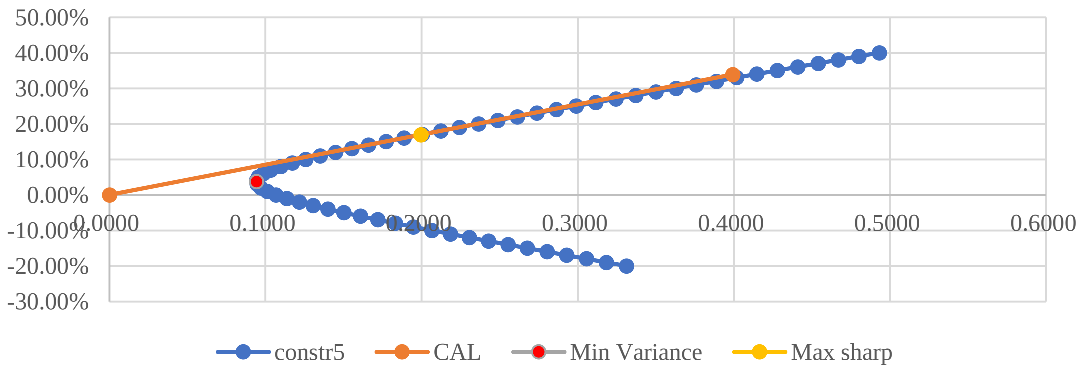

Figure 5 Constrain 5 $W_1 = 0$.

4. DISCUSSION

4.1 Markowitz Model

Compared with other constraints, the asset allocation of MM under constraint 4 is quite different, because in the case of $W_i \geq 0$, for ∀ i, the loan to invest is not allowed to exist, and it is greatly restricted by the principal, which leads to a great difference between the asset allocation and other combinations.

4.2 Index Model

In the stock indicators in the comparison of constraint 1, constraint 2, and constraint 3, it can be seen that almost all indicators change with the constraints, such as SPX, QCOM, and other indicators showing a trend of reduction. As shown by the analysis of the functional relationship between the portfolio return and the standard deviation and its CAL curves, Constraints 1:

$$\sum_{i=1}^{11} |W_i| \leq 2 \quad (7)$$

The purpose of this constraint is to simulate the range of rule T of FINRA. From the perspective of stock operation, this constraint will partly give stock agents more room to help clients profit from account equity. Constraints 2 is

$$|W_i| \leq 1, \text{ for } \forall \text{ i} \quad (8)$$

In this constraint, i as a free variable partly reflects the influence of the free constraint on the stock return client. According to the assumptions of the capital asset pricing model, it can be seen that the more internal funds, the lower the effective discount rate.

The capital asset pricing model assumes that there is no imperfect market situation and investors are rational. They pursue risk minimization and maximize utility. From the perspective of model constraints for capital assets, take the stock market as an example, assuming that investors invest in the stock market in the form of funds, the investors' expected return and risk-free interest rate directly affect the performance of the market risk premium. That means that investors take on the inseparable and unpredictable risks of the stock market. In the analysis coefficient, it can be seen that just washing measures the indispensable risk of assets. If given the coefficient value, investors can determine the correct discount rate of the asset limit. The discount rate is the expected rate of return on the same risk asset.

4.3 Markowitz Model & Index Model

From the optimal asset allocation obtained by MM and IM, we can see that the two models are highly consistent in determining the effective portfolio. However, the exponential model is simple and practical, and the assumptions are not very strict. However, Macovei's model is the basis of the single index model, and Macovei's model can accurately manage the risk of a small number of assets. At the same time, the practical significance of Macovei's model is to warn investors that appropriate investment diversification can reduce risks, but excessive investment types cannot continue to reduce risks. The single index model is an empirical improvement of Macovei's model, and it has incomparable advantages over Macovei's model in analyzing a large number of securities portfolios. And this model has been widely used in the effective markets of various countries. Macovei model and single index model are two extremes in the number of parameters, but these two models are the basis of the whole portfolio model, and still have some practicability.

5. CONCLUSION

Aiming to cover both asset pricing models, this article compares and contrasts ten stocks from each, each with a distinct portfolio, and each subject to five limitations. They did this after analyzing the best portfolio of 10 stocks under the Index model as well as under the Markowitz model. They then compared the most case portfolios under both the Index and the Markowitz models. Under the Markowitz model portfolio, the yield is 16 percent to 18 percent higher than 15 percent and below 15 percent lower than 15 percent. Under the Index model portfolio, the yield is 12 to 14 percent higher and lower than 15 percent. When building a stock portfolio, stock investors must select the most appropriate portfolio model for their particular assets as well as the current state of the stock market. When faced with high-risk investment projects, investors might utilize the Markowitz model to estimate their portfolio's performance. Similarly, when dealing with low-risk investment projects, investors can utilize the Index model to make predictions about their portfolios.

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