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Optimal Metaverse Stock Portfolio Through Markowitz Model and Full Index Model

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ABSTRACT

The main purpose of this paper is to understand the different results of using the Markowitz Model and Full Index Model to derive the return and risk level of the optimal metaverse portfolios at the side of traders, and how different constraints affect the return and risk portfolios. The method used is Markowitz Model and the Full Index Model. In the paper, the detailed calculation methods are shown, and the results are generated by Excel. If you prefer to use a computer language other than Excel. The finding is Full Index Model always gave a lower rate of return at the same level of risk, and constraints always brought negative effects to the return of portfolios. Therefore, it suggests using the Full index Model to derive optimal portfolios and invest without any constraint.

Keywords: *Optimal portfolio, Markowitz Model, Full Index Model*

1. INTRODUCTION

On November 6th, 2021, the US Congress passed the Bipartisan Infrastructure Deal. One content of the deal is that to ensure every American has access to reliable high-speed internet, the US government will deliver $65 billion. The legislation will also help lower prices for internet services and help close the digital divide so that more Americans can afford internet access [1]. Also, with the development of techniques, Danisio, Burns, and Gilbert indicate the metaverse will be ubiquitous in the real world in the future [2]. Therefore, the metaverse stock may be better developed in the future.

Markowitz indicates the second stage of the process of selecting a portfolio starts with the relevant beliefs about the future performance and ends with the choices of the portfolio. Also, Markowitz considers the rule that investor does or should maximize discounted expected, or anticipated, return, and then uses the "expected returns – variance of return" rule to illustrate geometrically relations between beliefs and choices of portfolios [3]. Cochrane talks about how to calculate the risk and return of a portfolio and the method to find an optimal risky portfolio through asset allocation. Then he introduces the main background, factors, and the method of calculation of the Markowitz Optimization Model. The main stages of the model are asset allocation and security selection. Also, he gives the calculation methods of some basic factors, such as expected rate of returns, variance and standard deviation, etc [4].

Cochrane introduces the Index model, which contains background information, key factors, and calculation methods. Index model is an asset pricing model to measure both risk and the return of stock with the factor-alpha, beta, and epsilon. Also, Cochrane illustrates how to use the Index model to find optimization of the portfolio [5]. To optimize a portfolio is to maximize its security return and minimize its risk. Traditionally, the expected security return is a random variable, while Huang indicates that expected security returns are influenced by experts' judgment and estimations rather than the historical data. Therefore, Huang introduces a new risk index into the original Index model to develop a mean-risk index model, which is to select the uncertain portfolio and judge the portfolio investment [6]. Sharpe illustrates the upsides of utilizing a specific model of the relationships among protections for reasonable utilizations of the Markowitz portfolio investigation method. A PC program has been created to take full promotion vantage of the model: 2,000 protections can be examined for an incredibly minimal price just 2% of that related with standard quadratic supportive of codes. Besides, starter proof proposes that the somewhat couple of boundaries utilized by the model can prompt practically similar outcomes obtained with a lot bigger arrangements of connections among protections. The possibility of minimal expense investigation combined with a probability that a generally modest quantity of data need be forfeited makes the model an alluring candidate for beginning pragmatic uses of the Markowitz method [7]. On November 6th, 2021, the US Congress passed the Bipartisan Infrastructure Deal. One content of the deal is that to ensure every American has access to reliable high-speed internet, the US government will deliver $65 billion. The legislation will also help lower prices for internet services and help close the digital divide so that more Americans can afford internet access [8].

Danisio, Burns, and Gilbert talk about the current status and future possibilities of the 3D virtual world and the metaverse. They introduce the evolution of the virtual worlds and the future features of the metaverse. Overall, with the development of the technology, they indicate the metaverse will be ubiquitous in the real world in the future [9]. Markowitz broadens the procedures of direct programming to foster the basic line algorithm. The basic line calculation recognizes all doable portfolios that limit hazard (as estimated by change or standard deviation) for a given degree of anticipated return furthermore amplify anticipated return for a given degree of hazard. When charted in standard deviation versus expected return space, these portfolios structure the efficient frontier. The efficient frontier addresses the compromise between hazard what's more expected return looked by a financial backer while shaping portfolio. Most of the efficient frontier addresses very much broadened portfolios. This is because expansion is a strong method for accomplishing hazard decrease. Accordingly, the mean-variance examination gives exact numerical significance to the aphorism "Don't set up of your resources in one place" [10].

The main object of the paper is to help investors better understand how to do portfolio investments on metaverse stocks. First, to help the investors to find a proper model to calculate the portfolio return and risk level, the differences of two classic models (Markowitz Model and Full Index model) are compared. Also, it calculated return and risk levels for the optimal portfolios according to the past data, which was aimed to help the investors see whether the return and risk level for metaverse stock portfolios has achieved their expectations. Lastly, since there are many limits during the investment, it simulated several constraints to see if there are any effects on the return or risk level to the portfolio.

2. METHODS

2.1 Data

This paper chose the recent 20 years of historical daily total return data (from 2001/05/11 to 2021/05/2) for ten stocks (NVDA, CSCO, INTC, GS, USB, TD CN, ALL, PG, JNJ, CL), one (S&P 500) equity index (a total of eleven risky assets) and a proxy for risk-free rate (1-month Fed Funds rate). To reduce the non-Gaussian effects, I aggregated the daily data to the monthly observations.

2.2 Markowitz Model

Markowitz Model is a portfolio optimization model, which is assists in the selection of the most efficient portfolio by analyzing various possible portfolios of the given securities. The model helps investors know how to reduce their risks. Then, two steps are needed.

2.2.1 Efficient Frontier

The first step of the Markowitz model approach is to determine the efficient set. A portfolio that gives the maximum return for a given risk, or a minimum risk for a given return is efficient. Investors prefer to have a higher return and lower risk because they are rational and risk-averse. In Markowitz Model, when graphed in standard deviation versus expected return space, these portfolios form the efficient frontier [9]. To find the efficient frontier, the expected return and variance of portfolios are needed. The expected return of the portfolio equation is:

$$r_p = w_1 r_1 + w_2 r_2 + \cdots + w_n r_n = \sum w_i r_i \tag{1}$$

where, p represents portfolio, r_p = the return of the portfolio, w_i = the weight of the stock i, r_i = the annualized average return of stock i. The variance of the portfolio equation is:

$$\mathrm{Var}(r_p) = \sum w_i^2 \,\mathrm{Var}(r_i) + 2\sum\sum w_i\, w_j \mathrm{Cov}(r_i, r_j) \tag{2}$$

where, $\mathrm{Var}(r_p)$= the variance of the portfolio, w_i = the weight of the stock i, w_j = the weight of the stock j, r_i = the annualized average return of stock i, r_j = the annualized average return of stock j, $\mathrm{Cov}(r_i,r_j)$ = the covariance between the annualized average return of stock i and stock j. The covariance is the correlation between two stocks. The standard deviation of the portfolio equation is:

$$\mathrm{StDev}(r_p) = \sqrt{\mathrm{Var}(r_p)} \tag{3}$$

where, $\mathrm{Stdev}(r_p)$= the standard deviation of the portfolio, $\mathrm{Var}(r_p)$= the variance of the portfolio. All the portfolios that lie on the efficient frontier are efficient.

2.2.2 Capital Allocation Line (CAL)

The second step of the Markowitz model approach is to determine the optimal risky portfolio. As before, the capital allocation line with the highest reward-to-volatility (Sharpe) ratio is needed. The Sharpe ratio is:

$$\text{Sharpe ratio} = r_p/\mathrm{StDev}(r_p) \tag{4}$$

The optimal risky portfolio corresponds to the tangent CAL to the efficient frontier [1]. The CML equation is:

$Rp = IRF+(R_M-IRF)\sigma_P/\sigma_M$ (5)

where, R_p = expected return of portfolio, R_M = return on the market portfolio, IRF = risk-free rate of interest, σ_M = standard deviation of market portfolio, σ_P = standard deviation of portfolio.

2.3 Index Model

Markowitz Model requires a large number of estimates of expected return (risk premiums), variances, and covariances, and it tells nothing on how to produce those estimates. The only guess that one gets is to use the historical sample averages. But the past returns may be unreliable. To eliminate some drawbacks of the Markowitz model, I chose Index Model to measure risks.

2.3.1 Excess Return

The equation of the excess return for each stock is:

$$R_i = r_i - r_f \quad (6)$$

where, R_i = excess return of stock i, R_i = the return of stock i, r_f = risk-free asset return, in this paper, I assume it as the return of New Frontier in Research Fund (NFRF).

Then, finding the excess return of the market index (S&P 500), which is denoted as M. The excess return of market index equation is:

$$R_M = r_m - r_f \quad (7)$$

where, R_M = the excess return of the market index.

2.3.2 The Factor Beta (β) and Alpha (α)

The factor Beta (β) and Alpha (α) can be estimated using the linear regression model between observations of Ri and RM:

$$R_i = \alpha_i + \beta_i \cdot R_M + e_i \quad (8)$$

where e_i is the unexpected component or return due to the unexpected events that are relevant only to the security (firm-specific).

2.3.3 Residual Standard deviation

Residual standard deviation is a statistical term to describe the difference in standard deviations of observed return versus expected returns. The results are used to measure the error of the regression line model. The residual standard deviation equation is:

$$\text{Residual}_i = \frac{\sqrt{(r_i - \beta_i \cdot r_m - \alpha_i)}}{\sqrt{n}} \quad (9)$$

where, Residual_i = Residual Standard deviation of stock i, n = the number of observations.

2.3.4 Return and Standard deviation in Index Model

The return of the portfolio equation is

$$R_p = w_i \sum_{i=1}^{n}(\alpha_i + \beta_i \cdot R_M + e_i) = w_i \sum_{i=1}^{n} \alpha_i + (w_i \sum_{i=1}^{n} \beta_i) \cdot R_M + w_i \sum_{i=1}^{n} e_i \quad (10)$$

From which follows:

$$\beta_p = w_i \sum_{i=1}^{n} \beta_i,\ \alpha_p = w_i \sum_{i=1}^{n} \alpha_i,\ e_p = w_i \sum_{i=1}^{n} e_i.$$

The portfolio's standard deviation is:

$$\sigma_p = \sqrt{\beta_p^2 \sigma_M^2 + \sigma^2 e_p} \quad (11)$$

where, $\sigma^2 e_p$ is a firm-specific risk, which can be diversifiable as:

$$\sigma^2 e_p = \sum_{i=1}^{n} w_i^2\, \sigma^2(e_i) \quad (12)$$

And the Sharpe ratio is the same as the one in Markowitz Model.

3. RESULTS

The data was processed with the above two models within the time range from May 2002 to May 2021. Since there are various situations during the investment, I added five additional optimization constraints for the two models.

Constraint 1: $\sum_{i=11}^{11} |w_i| \leq 2$ (13)

Constraint 2: $|w_i| \leq 1$, for $\forall_i$ (14)

Constraint 3: None

Constraint 4: $w_i \geq 0$, for $\forall_i$ (15)

Constraint 5: $w_1 = 0$ (16)

The fitting results for five additional optimization constraints within two models are derived respectively.

Table 1. Results for Constraint 1

Model	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
MinVariance	7.508%	10.930%	69%	7.508%	10.293%	73%
maxSharpe	14.008%	13.921%	101%	14.008%	14.706%	95%

Table 2. Results for Constraint 2

Model	Markowitz Model	Index Model

	Return	StDev	Sharpe	Return	StDev	Sharpe
MinVariance	7.508%	10.930%	69%	7.508%	10.293%	73%
maxSharpe	16.557%	16.031%	103%	16.557%	17.514%	95%

Table 3. Results for Constraint 3

Model	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
MinVariance	7.508%	10.930%	69%	7.508%	10.293%	73%
maxSharpe	16.988%	16.442%	103%	16.988%	18.034%	94%

Table 4. Results for Constraint 4

Model	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
MinVariance	8.876%	11.243%	79%	8.876%	10.495%	85%
maxSharpe	12.057%	13.092%	92%	12.057%	13.419%	90%

Table 5. Results for Constraint 5

Model	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
MinVariance	8.706%	11.154%	78%	8.706%	10.419%	84%
maxSharpe	13.058%	13.660%	96%	13.058%	14.444%	90%

The comparison of minimum return portfolio and maximum Sharpe ratio portfolio charts for five constraints between two models illustrate below respectively.

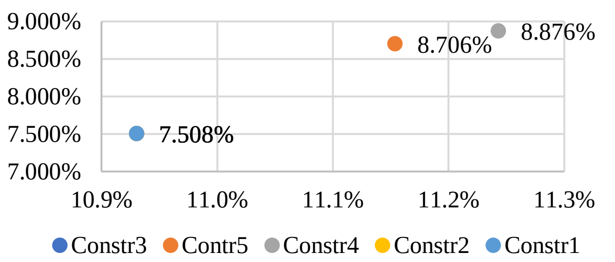

Figure 1 Optimal portfolio under each constraint to minimize variance for Markowitz Model.

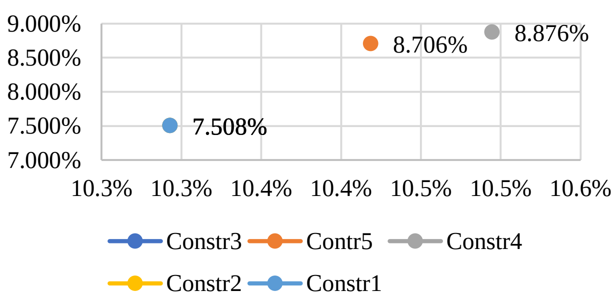

Figure 2 Optimal portfolio under each constraint to minimize variance for Full Index Model.

The portfolios with minimum risk under constraints 1,2 and 3 were the same, which were all give the lowest return and risk. While under the constraints 5 and 4 gave higher returns and more risky portfolios. Moreover, the Full Index model Markowitz Model performed better than the Markowitz Model because, at the same level of return, the Full Index Model gives a smaller risk level.

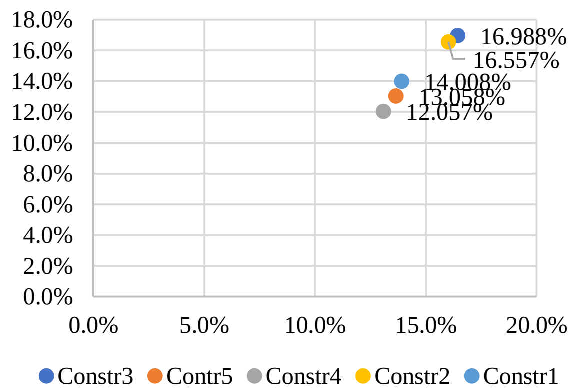

Figure 3 Optimal portfolio under each constraint to maximize Sharpe ratio for Markowitz Model.

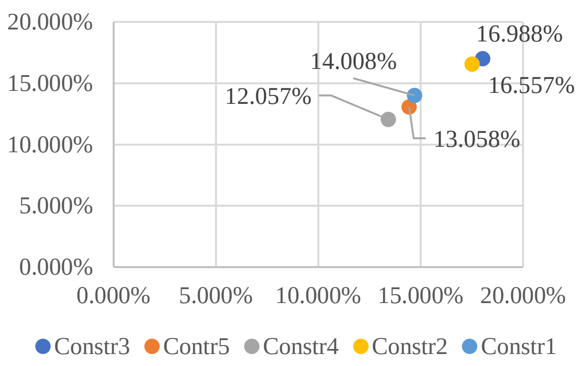

Figure 4 Optimal portfolio under each constraint to maximize Sharpe ratio for Full Index Model.

The two models gave the same result that under constraint 3, the portfolio with maximum Sharpe ratio had the highest return and standard deviation, and then are constraints 2, 1, 5, 4. Moreover, the Markowitz Model gave the higher return at the same level of risk compared to the Full Index Model. Overall, the Markowitz model performed better. Then, the comparison graphs between the minimum variance frontier for five additional optimization constraints processed within two models are illustrated below respectively.

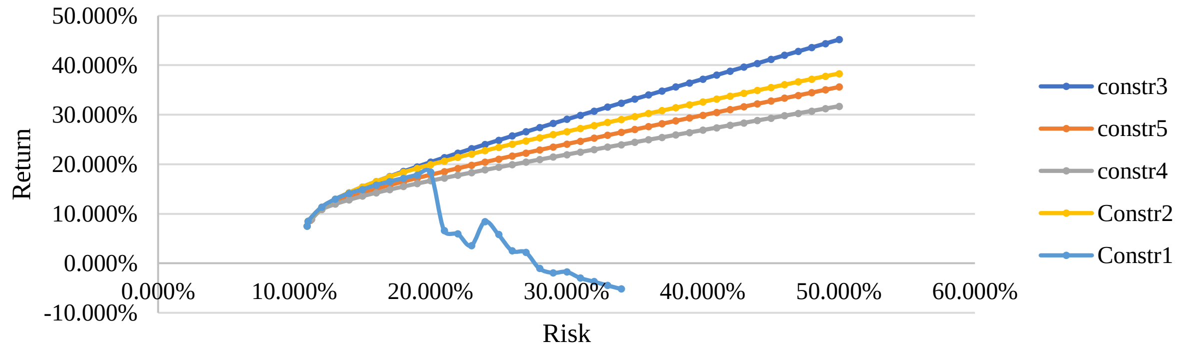

Figure 5 Comparison of the efficient frontier under five constraints within the Markowitz Model.

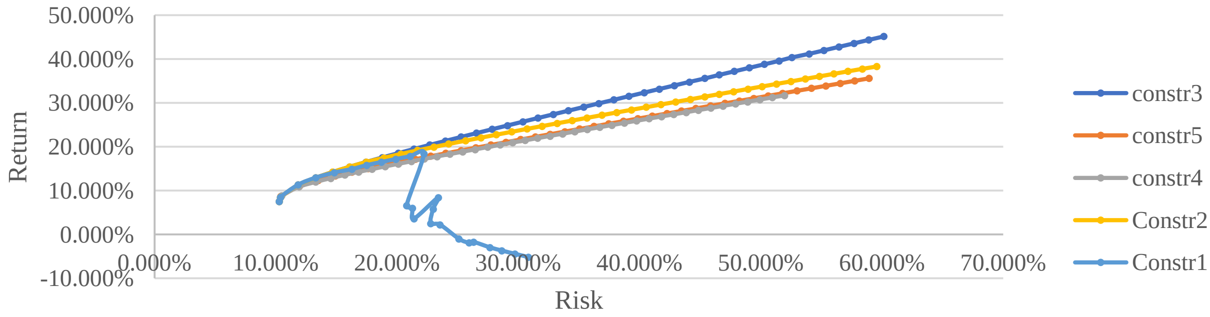

Figure 6 Comparison of the efficient frontier under five constraints with Full Index Model.

From the graphs, it was easy to see that all the constraint efficient frontiers had an increasing trend excluding the one under constraint 5. In other words, except under constraint 5, when investors want a higher return, they must afford a higher risk, vice versa. Moreover, the efficient frontier from the Full Index model is steeper than the ones from the Markowitz Model, which means at the same level of risk, the Markowitz Model gave a high rate of return.

4. METAVERSE FUTURE AND DISCUSSION

4.1 Additional Optimization Constraint

The first additional optimization constraint is the sum of the absolute value of each stock weight is less than or equal to 2 ($\sum_{i=11}^{11}|w_i| \leq 2$). It is designed to simulate Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity. The second additional optimization constraint is the absolute value of each stock weight is all less than or equal to 1 ($|w_i| \leq 1$, for $\forall_i$). This additional optimization constraint is designed to simulate some arbitrary "box" constraints on weights, which may be provided by the client. The third one is a "free" problem without any additional optimization constraints. The fourth additional optimization constraint is each of the stock weights should be more than or equal to zero ($w_i \geq 0$, for $\forall_i$). This constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry because a U.S. open-ended mutual fund is not allowed to have any short positions. The last additional optimization constraint is the weight of S&P should be zero ($w_1 = 0$) since I wanted to see if the inclusion of the broad index into the portfolio has a positive or negative effect.

Under the first constraint, investors cannot leverage more than twice. Therefore, from the graph, they can't get a rate of return of more than 20%. In other words, if they do not know the results, and blindly pursue higher returns (over 20%), they will afford an unnecessarily high risk. However, from the past data, this constraint gives the lowest risk portfolio no matter from Markowitz Model or Full Index Model. Overall, under Regulation T by FINRA, investors can try to be risk-averse to choose low risk and low return portfolios to avoid unnecessary risk. From Constraints 3 and 5, the results convinced that having inclusion of the broad index into the portfolio has a positive effect to increase return or reduce risk.

4.2 Minimum Risk and Maximum Sharpe Ratio

The portfolio theory of Markowitz assumes that investors are risk-averse and try to minimize risk and maximize return [3]. Therefore, the minimum variance and Sharpe ratio are important indexes.

Under the constraint 1, 2, and 3, both models give the same minimum variance portfolio, which all gave the lowest return and risk. While under the constraints 5 and 4 gave higher returns and more risky portfolios. Therefore, only long positions or not having inclusion of the broad index into the portfolio will increase the return but also increase the risk because hedging and long a risk-free stock are both methods to reduce the risk. In this case, S&P was more secure than a hedge.

The Sharpe ratio measures the performance of an investment such as security or portfolio compared to a risk-free asset, after adjusting for its risk. It is defined as the difference between the returns of the investment and the risk-free return, divided by the standard deviation of the investment returns. It represents the additional amount of return that an investor receives per unit of increase in risk [6]. A high Sharpe ratio is good when compared to similar portfolios or funds with lower returns. The two models gave the same result that under constraint 3, the portfolio with maximum Sharpe ratio had the highest return and standard deviation, and then are constraints 2, 1, 5, 4. Therefore, when there is no restriction for investment, investors can get the most return for one additional unit of risk. Constraint 4 brings the lowest maximum Sharpe ratio, which means only expecting the stock price increase is not a good idea for investment in this case.

4.3 Efficient Frontier

The efficient frontier comprises investment portfolios that offer the highest expected return for a specific level of risk. Under the first constraint, investors cannot leverage more than twice. Therefore, from the graph, they can't get a rate of return of more than 20%. In other words, if they do not know the results, and blindly pursue higher returns (over 20%), they will afford an unnecessarily high risk. However, from the past data, this constraint gives the lowest risk portfolio no matter from Markowitz Model or Full Index Model. Overall, under Regulation T by FINRA, investors can try to be risk-averse to choose low risk and low return portfolios to avoid unnecessary risk. From Constraints 3 and 5, the results convinced that having inclusion of the broad index into the portfolio has a positive effect to increase return or reduce risk.

5. CONCLUSION

This paper used Markowitz Model and Full index model to derive optimal portfolios under different conditions according to the past 20 years' metaverse stock data. The first thing to notice is at the same level of risk, the Markowitz Model always gives a higher return than the Full Index Model, so to be on the safe side, it should recommend the Full Index Model. Also, as mentioned before, there is still more limitation to Markowitz Model. Another point needed to be noticed is that constraints have negative effects on the return of portfolios, especially the higher rate of return and a higher level of risk, the more obvious the negative effects. Therefore, it suggests that if possible, choosing investment with free constraint, so that to get a better return at the same level of risk.

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