

# Are Ideas Getting Harder to Find in the UK?

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## ABSTRACT

To deal with the question that whether idea production is decreasing return to scale during 1996 to 2017 in the United Kingdom, the writer of the paper derived a formula from Solow Growth Model and Romer Growth Model. The detailed derivation of the main formula is to be explained in the context. By quantifying data related to ideas production, which includes the national output, labor input of research of development sector, changes in capital stock, and patents applied during our timeframe, this work can calculate the capital stock of each year from 1996-2017. By using these data, calculation of the increase rate of  $A_t$  (the capital stock of each year) between every two consecutive years would be applicable. And furthermore, one would be able to calculate the value of  $z$ , which will be elaborated in the following passage. And for the  $z$  being negative, a conclusion may be drawn that during 1996 to 2017, ideas production is indeed decreasing return to scale in the United Kingdom.

**Keywords:** *Idea production, Solow Growth Model, the UK, Decreasing return to scale*

## 1. INTRODUCTION

In the field of macroeconomics, or to say, “growth economics”, researchers generally focus on the growth of Gross Domestic Product, household income, labour quantity, etc. However, when the field of knowledge production, or “new ideas” is taken into consideration, one would expect that this sector is lacking research and analysis. In the research of production - especially of new ideas, which are produced by research and development and research, these are costly - the most important point needing analysis is the return to scale. The return to scale directly affects the level of production of knowledge, the creativity and technology level of a society or a country. This important point needs research in order to begin its journey. In the work, the authors have seen the issue, and have gone on to solve it at least in the UK field. The authors analyzed multiple data, used two main formula or equations based on the Solow and Romer growth model, used the data to calculate important variables, and at last functioned a trend displaying diagram or curve in order to prove the return to scale of knowledge or new ideas production. It is hoped that this research result can be considered, and the UK government may react to the knowledge production field to ensure creativity and technology.

### 1.1 Theoretical analysis

Solow's neoclassical growth theory is the cornerstone of modern growth theory. Solow's model describes a perfectly competitive economy that assumes a production function with constant returns to scale, growth in capital and labor inputs causing growth in output, and a neoclassical production function that determines the diminishing marginal output of capital when labor supply is constant. This production function, combined with the assumptions of constant savings rate, constant population growth rate, and constant technological progress forms a complete general dynamic equilibrium model. The Solow model emphasizes the scarcity of resources and the limit of growth due to the accumulation of physical capital alone, and the steady-state zero growth with constant population growth rate and constant technological progress are the embodiments of this idea. The Cobb-Douglas function takes the form of labour-augmenting technological progress. The function at time  $t$  is, therefore, given by:

$$Y[t] = K[t]^{\alpha}(A[t]L[t])^{1-\alpha}. \quad 0 < \alpha < 1 \quad (1)$$

where  $Y$ ,  $K$ ,  $A$ , and  $L$  denote output, capital, the level of technology, and labour, respectively. The Solow model assumes that population growth rates and technological

growth rates are exogenously determined.

$$\begin{aligned}
 y &= F(k, 1) \\
 \Delta K &= Sy - (\delta + n)k \\
 y &= C + i \\
 c &= (1 - s)y \quad (2)
 \end{aligned}$$

Y refers to the output, it is calculated by the variables—k (capital input), and L (labor input). K and L are put into the F (production function), also, the s comes to saving rate while the  $\delta$  is calculated as depreciation.

The per capita savings curve  $f(k)$  lies below the per capita income curve  $f(k)$ , since savings are only a part of income. Point A is the equilibrium point, i.e., the deepening of capital at point A is equal to zero, and all per capita savings are used to equip the new population with capital (also known as the generalization of capital), and the economy reaches equilibrium. So, the main conclusion of the solow model is that all other exogenous variables being equal, an economy with low per capita capital has a faster increase in per capita capital and an economy with low per capita income has a higher growth rate; the increase in per capita output  $\left(\frac{Y}{L}\right)$  comes from the per capita capital stock and technological progress, but only technological progress can lead to a permanent increase in per capita output. Solow's model argues that when exogenous technology grows at a fixed rate, the economy will grow on an equilibrium growth path, while when the level of exogenous technology is fixed, the economy will tend to stagnate, investment will only compensate for depreciation of fixed assets and equipping new workers, and technological progress is the main driver of economic growth

So Solow only affirms the importance of idea production in the development of the economy, and does not answer some questions about technology advance (which is idea production in economic area) After realizing how important the idea creations (Economic growth arises from people creating ideas). Economists can never stop their research there, to measure economic growth arises from people creating ideas, one can decompose the long-run growth rate into the product of two terms: effective numbers of researchers and their research productivity. And the r&d-based models in the endogenous growth literature by romer.

Romer's endogenous growth model is characterized by an emphasis on understanding the mechanisms of economic growth and development based on ideas or knowledge goods, beginning with the endogenous nature of technology. According to Romer's endogenous growth idea, in order to achieve long-term economic growth in developing countries, it is important to have a mechanism that enables new designs or ideas to be generated and used, which requires government policies to be

formulated with an emphasis on educational development and investment in science and technology, incentives and protection of innovation.

Production of output (we abstract from capital for now)

$$Y_t = A_t L_{yt}$$

New ideas are produced using existing stock of ideas  $A_t$  and researchers  $L_{at}$

$$\Delta A_t + 1 = z A_t L_{at}$$

The change in the stock of ideas is given by  $\Delta A_t + 1 \equiv A_t + 1 - A_t$ .

The stock of ideas that the economy starts out with is  $A_0$

In addition to the two factors of production included in Romer's model, capital and labor, there are two other factors, which are: human capital and technology level. Labor included in the model refers to unskilled labor, while human capital refers to skilled labor, and human capital is expressed in terms of the length of education, such as formal education and on-the-job training, so that the role of knowledge or education level in economic growth is considered.

Regarding the element of technological level included in the model, Romer assumes that it is embodied in material goods, such as new equipment, new raw materials, etc., which denote the results of technological innovation. In other words, the advancement of knowledge is reflected in two aspects: on the one hand, it is the level of proficiency embodied in workers, which is represented by human capital in the model; on the other hand, it is the technological advancement embodied in material goods such as new equipment, new raw materials, etc., which is represented by the level of technology in the model.

Although these factors exist, we mainly using production function with  $L, K$  to calculate the technology change rate in order to see whether the technology growth level is constant returns to scale or not.

Romer acknowledges that investment promotes knowledge accumulation, which in turn stimulates investment, creating a virtuous circle that leads to the conclusion that sustained growth in investment permanently increases growth rates - a view that had been rejected by traditional theory.

Romer's model analyzes the role of knowledge and technology in economic growth in a more systematic way, and he highlights the fact that the contribution of research and development to economic growth has a real value, which is consistent with the facts. But its main shortcoming is that it does not examine the initial human capital situation and the assumption of a constant total human capital.

**2. DATA COLLECTION AND ANALYSIS (RANGE 1995-2017)**

**2.1 Equation 1: the calculation of “ $A_t$ ”**

$$Y_t = F(K, L, A) = A_t K_t^{\frac{1}{3}} L_t^{\frac{2}{3}} \quad (3)$$

The first formula to be used - the “Formula 3” above - is going to be the formula of Solow’s growth model. The sole purpose of this research is to use this formula to calculate the storage of knowledge and new ideas in the economy, which is to calculate “ $A_t$ ”. In order to calculate this value, three other values must be acquired primarily as the foundations.

“ $Y_t$ ” represents the total national income of the society, which is often presented by the value of real GDP at a fixed price.

“ $L_t$ ” represents the economy’s total labor usage or working hours. This is not the total employment of the society, but the working hours of workers.

“ $K_t$ ” represents the society’s real capital stock, with a fixed price.

**2.2 The collection of  $Y_t$**

Collection of  $Y_t$ :

**Table 1:** the Gross domestic production data of the UK from 1995 to 2017 Gross domestic product at market prices YBGB Index, base year 2016 = 100 [1]

PreUnit	£
Unit	m
1995	1322839
1996	1355798
1997	1423254
1998	1476025
1999	1524662
2000	1578121
2001	1621208
2002	1656531
2003	1711559
2004	1750690
2005	1802435
2006	1850989
2007	1894682
2008	1889401
2009	1811672
2010	1849247
2011	1872838
2012	1899626
2013	1941155
2014	1996725
2015	2043909
2016	2079113
2017	2115296

**2.3 Collection of  $K_t$ :**

**Table 2:** the real capital stock of the UK from 1995 to 2017 [2]

PreUnit	£
Unit	m
1995	7315548
1996	7471049
1997	7604100
1998	7761846
1999	7918813
2000	8080496
2001	8223229
2002	8371089
2003	8516802
2004	8664854
2005	8829615
2006	8999683
2007	9187809
2008	9336387
2009	9411605
2010	9502740
2011	9601110
2012	9704910
2013	9818383
2014	9960483
2015	10113322
2016	10270868
2017	10439211

As shown in *table 1*, “ $Y_t$ ” is the UK’s real GDP at a fixed price from 1995 to 2017. In order to get official UK data, the UK’s official government data website: ons.gov.uk have been used. The base year of the  $Y_t$  value collected is the year 2016.

As shown in *table 2*, “ $K_t$ ” is the UK’s real capital stock at a fixed price from 1995 to 2017. The “rnnn” value in the Penn World Table was used as the  $K_t$  value. The “rnnn” value is based on a constant price level (the 2011 UK national price), so using it will be more appropriate and accurate.

2.4 The collection of  $L_t$ : two parts

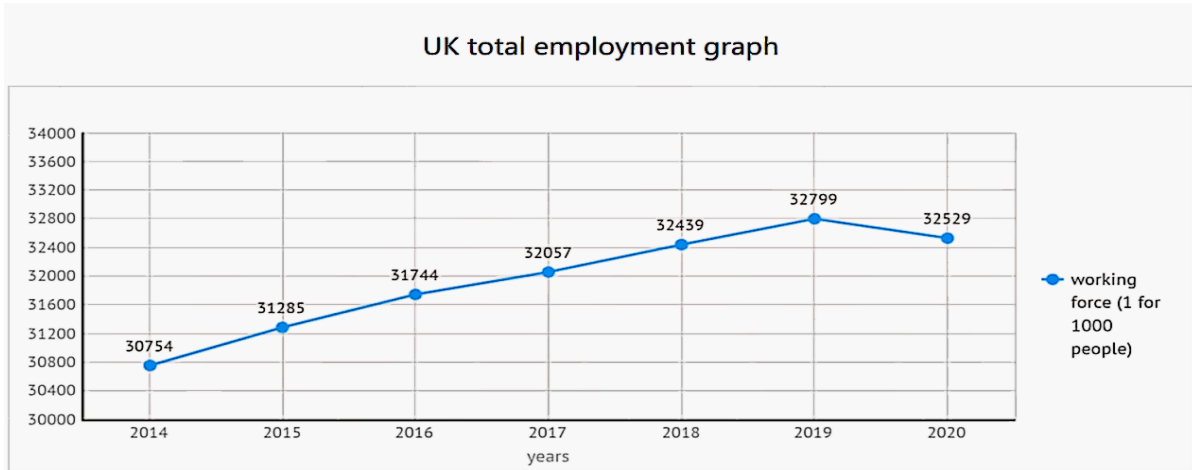


Figure 1:UK total employment graph. [3]

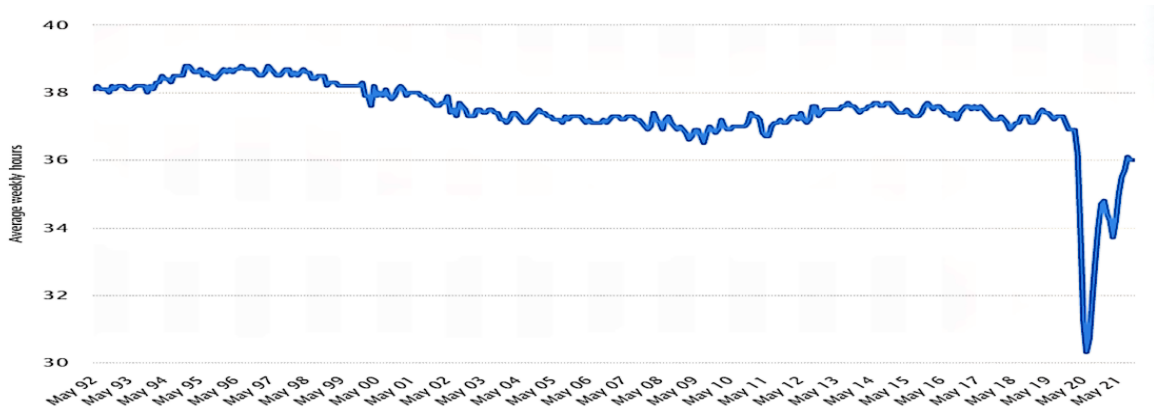


Figure 2. “Average weekly hours of work for full-time workers in the United Kingdom from May 1992 to July 2021” [4]

As show in *Figure 1*, in order to calculate the total labor usage by the UK, the total employment data per year for the people in the UK has been acquired. The graph is 1 for 1000 people, but as long as the ratio of one against another stays the same, the true value is not quite important.

As shown in *Figure 2*, the working hours of workers in the United Kingdom should be considered throughout the time limit stated above. The average hours used is driven from the figure above and is calculated for full time workers in the industry, in order to minimize inaccuracy.

Above are representations of the data collected. The full value of data will be presented when the value of “ $L_t$ ” is calculated.

Table 3. Data set for the calculation of “ $L_t$ ”. [3,4]

year	Employment (1/1000)	Working hours/ week	Multiplication
1995	25818	38.7	999156.6
1996	26060	38.7	1008522

1997	26526	38.6	1023903.6
1998	26795	38.5	1031607.5
1999	27168	38.3	1040534.4
2000	27484	37.9	1041643.6
2001	27712	38.0	1053056
2002	27944	37.9	1059077.6
2003	28221	37.4	1055465.4
2004	28530	37.3	1064169
2005	28850	37.3	1076105
2006	29138	37.2	1083933.6
2007	29378	37.2	1092861.6
2008	29628	37.1	1099198.8
2009	29156	36.8	1072940.8
2010	29228	37.0	1081436
2011	29378	36.9	1084048.2
2012	29697	37.2	1104728.4
2013	30043	37.5	1126612.5
2014	30754	37.6	1156350.4
2015	31285	37.4	1170059
2016	31744	37.5	1190400
2017	32057	37.4	1198931.8
2018	32439	37.1	1203486.9

As shown in Table 3, The multiplication gives the result of “ $L_t$ ” in hours per week.

The value needed is per year, but since there are 52 weeks in approximately every year, constants can be deducted after the calculation, just like the “1 for 1000 people” previously mentioned.

The following is Table 4, a collection of the work done above, and a calculation fully of the value needed:  $A_t$ .

2.5 The calculation of “ $A_t$ ”

Table 4. The full calculation process for the value “ $A_t$ ”.

	$Y_t$	$K_t$	$L_t$	$A_t$
PreUnit	£		hours	( $\alpha=1/3$ )
Unit	$m$		$k$	value= $\frac{Y_t}{K_t^\alpha [L_t^{1-\alpha}]}$
1995	1322839	7315548	999156.6	0.681818835
1996	1355798	7471049	1008522	0.689621633
1997	1423254	7604099.5	1023903.6	0.712459917
1998	1476025	7761845.5	1031607.5	0.730178523
1999	1524662	7918812.5	1040534.4	0.744930795
2000	1578121	8080496	1041643.6	0.76532904
2001	1621208	8223228.5	1053056	0.775991518
2002	1656531	8371088.5	1059077.6	0.785212253
2003	1711559	8516802	1055465.4	0.808482003
2004	1750690	8664854	1064169	0.817739731
2005	1802435	8829615	1076105	0.830441906
2006	1850989	8999683	1083933.6	0.843321128
2007	1894682	9187809	1092861.6	0.852620098
2008	1889401	9336387	1099198.8	0.842455614
2009	1811672	9411605	1072940.8	0.818730875
2010	1849247	9502740	1081436	0.828663275
2011	1872838	9601110	1084048.2	0.835014472
2012	1899626	9704910	1104728.4	0.833362438
2013	1941155	9818383	1126612.5	0.837266757
2014	1996725	9960483	1156350.4	0.842361455
2015	2043909	10113322	1170059	0.851187238
2016	2079113	10270868	1190400	0.851557083
2017	2115296	10439211	1198931.8	0.857601641

Equation 2: The calculation and analysis of “z”

$$\frac{\Delta A_{t+1}}{\Delta A} = zL_{at} \tag{4}$$

As shown in formula 3, the values shown on the left-hand side of the equation represents the growth rate of  $A_t$ , which is basically the growth rate of knowledge storage in the society. This is marked as the rate of change of output of research and development.

$L_{at}$  is not the total labor usage in the society, but the labor usage of the research and development sector in the society. This is marked as the input of research and development.

“Z” is the value calculated by divide the rate of change of output with the input of research and development. In the solow model, we assumed constant return to scale in the production of ideas. This indicates the rate will increase in the same percentage as the

increase of input, making “Z” a constant.

However, if the production of ideas is in fact decreasing return to scale, then the rate will increase in smaller percentage than the increase of input. Z will decrease as input increases over time. This is what we seek to prove.

2.6 The calculation of the growth rate of “ $A_t$ ”

Table 5. The table of  $A_t$  calculations.

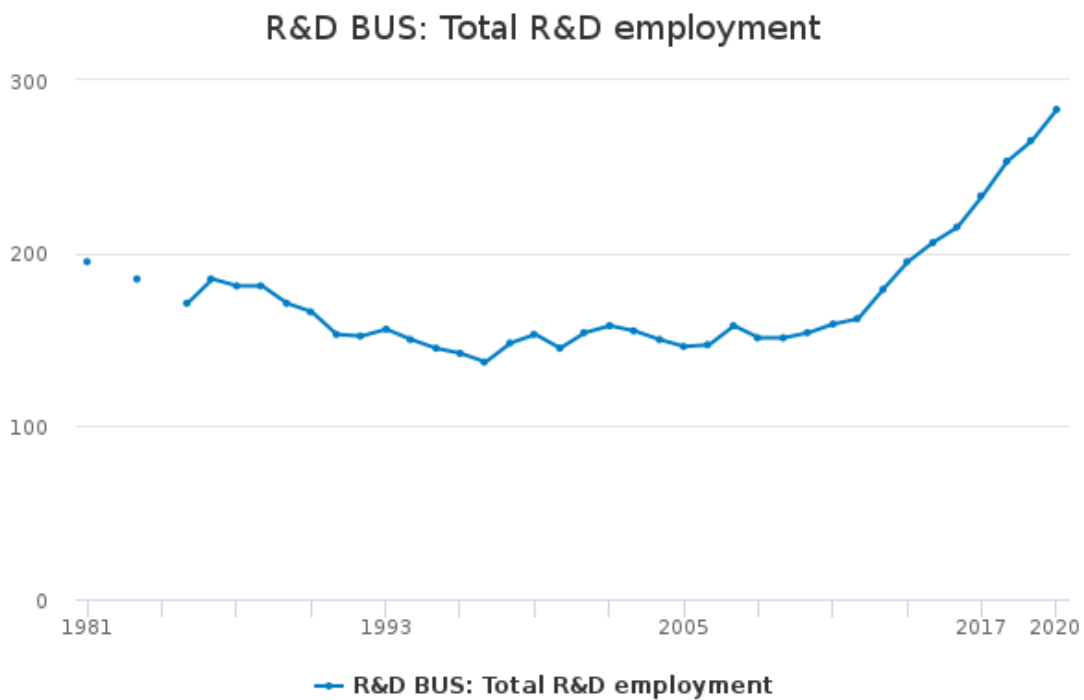
years	$A_t$	% Increase in $A_t$
	$(\alpha = \frac{1}{3})$	
	value $= \frac{Y_t}{K_t^\alpha [L_t^{1-\alpha}]}$	
1995	0.681818835	
1996	0.689621633	0.011444092

1997	0.712459917	0.033117123
1998	0.730178523	0.024869618
1999	0.744930795	0.02020365
2000	0.76532904	0.027382738
2001	0.775991518	0.013931887
2002	0.785212253	0.011882521
2003	0.808482003	0.029634981
2004	0.817739731	0.011450753
2005	0.830441906	0.015533274
2006	0.843321128	0.015508878
2007	0.852620098	0.011026606
2008	0.842455614	-
2009	0.818730875	-
		0.028161412

2010	0.828663275	0.012131459
2011	0.835014472	0.007664388
2012	0.833362438	-
		0.001978449
2013	0.837266757	0.004685019
2014	0.842361455	0.006084916
2015	0.851187238	0.010477429
2016	0.851557083	0.000434506
2017	0.857601641	0.007098241

As shown in Table 5, the growth rate of  $A_t$  is calculated via the values of  $A_t$  that have been calculated above. The data will serve as the “output value” in the final calculation.

**2.7 The calculation of “ $L_{at}$ ”: The input value**



**Figure 3.** The total R&D employment of the UK. [5]

Source:

**Table 6.** “R&D BUS: Total R&D employment” [5]

R&D BUS: Total R&D employment	
year	data
1995	145
1996	142
1997	137
1998	148
1999	153
2000	145
2001	154
2002	158

2003	155
2004	150
2005	146
2006	147
2007	158
2008	151
2009	151
2010	154
2011	159
2012	162
2013	179
2014	195
2015	206
2016	215
2017	233

As shown in *Figure 3* and *Table 6*, the input value “ $L_{at}$ ” is, in fact, the total working hours of research and development employees in the society. However, the R&D employment was solely used to be the input value, because the working hours data of R&D employees in the UK cannot be found via normal procedures. An assumption will be made that the hours are the same and

will not matter in the calculation. This would be a source of error and will be mentioned in the critical thinking part.

### 2.8 The calculation of “z”

**Table 7.** The calculation of “z”.

	% Increase in At	R&D input	TFP*10 <sup>2</sup> (z)
1995		145	
1996	0.011444092	142	0.008059219
1997	0.033117123	137	0.024173083
1998	0.024869618	148	0.016803796
1999	0.02020365	153	0.013205
2000	0.027382738	145	0.018884647
2001	0.013931887	154	0.00904668
2002	0.011882521	158	0.007520583
2003	0.029634981	155	0.019119342
2004	0.011450753	150	0.007633835
2005	0.015533274	146	0.010639229
2006	0.015508878	147	0.010550257
2007	0.011026606	158	0.006978864
2008	-0.011921469	151	-0.007895012
2009	-0.028161412	151	-0.018649942
2010	0.012131459	154	0.007877571
2011	0.007664388	159	0.00482037
2012	-0.001978449	162	-0.001221265
2013	0.004685019	179	0.002617329
2014	0.006084916	195	0.00312047

2015	0.010477429	206	0.005086131
2016	0.000434506	215	0.000202096
2017	0.007098241	233	0.003046455

As shown in Table 7, “z” is the value calculated by divide the rate of change of output with the input of research and development.

now a diagram representing its trend ought to be created, in order to show its real trend.

Finally, the values of “z” have been calculated, and

### 2.9 The trend and analysis of “z”

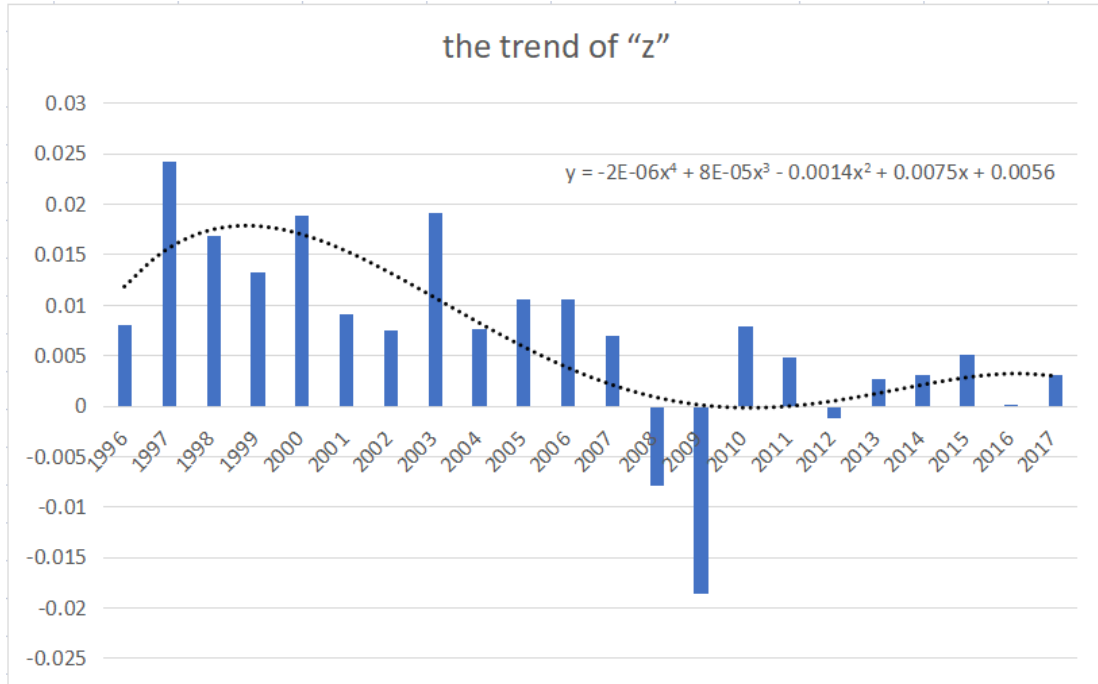


Figure 4. The bar chart of changes trend of “z” during 1996 -2017.

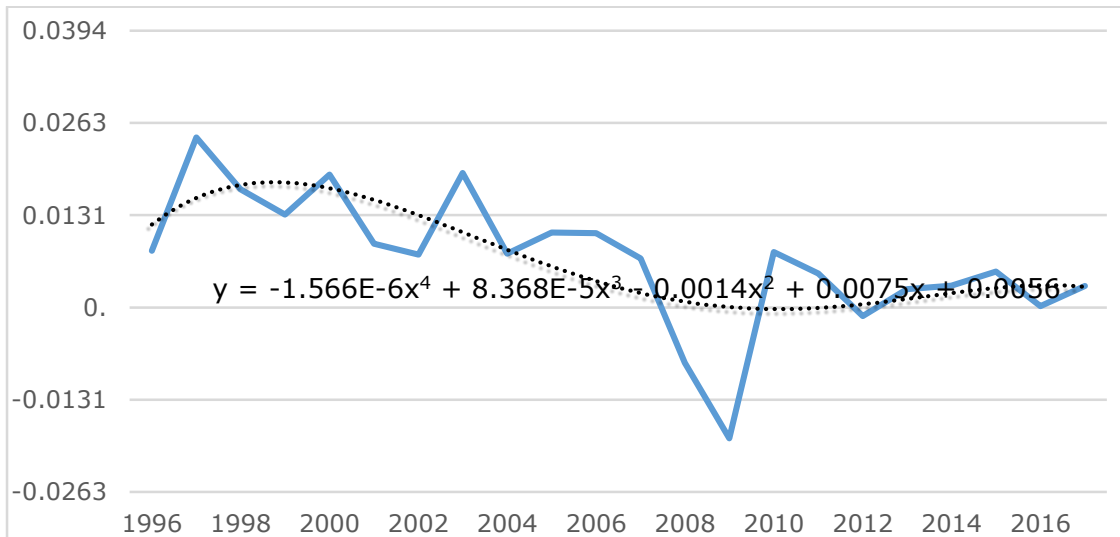


Figure 5. The developing trend of the value “z” in the UK society.

“The constant return to scale occurs when an increase in inputs (capital and labour) cause the same proportional increase in output.” [6] “The decreasing return to scale occurs when an increase in all inputs (labour/capital) leads to a less than proportional increase in output.” [6] “Z” is the value calculated by divide the rate of change of

output with the input of research and development. Since the value of inputs have increased daily, if “z” is to be constant, as the solow model have suggested, then the rate of change of output has increased at the same rate as the value of input. Exactly the definition of constant returns to scale.



But if “z” has decreased or showed a decreasing trend over time, then the rate of change of output has increased in a slower rate as the value of input, proving that the production of new ideas or knowledge is indeed having a decreasing return to scale. Back to the graphs, the two trends have both shown a decreasing return to scale, with the value “z” decreased since 1999 until the year 2007. Figure 4 is a trend shown in bars, and figure 5 is a trend shown in lines, both are trend-showing graphs. Even if the 2008 subprime mortgage crisis is considered as a variable, the decreasing return to scale trend have shown itself before the year 2008 as shown in the graph, from 1999 to 2007. After the subprime mortgage crisis, the storage of knowledge in the society have been deducted considerably, so that the society is in a sense of recovery, and the value have gone through a small increase until the year 2016, and just started to decrease again in the year 2017, right after we stopped our research. A conclusion can thus be made, through careful collection, calculation and analysis of data, that the production of new ideas or knowledge is indeed facing a decreasing return to scale in the UK during the given time period.

### 3. CRITICAL THINKING PART

#### 3.1 The Value of $L_{at}$

In data collection process, difficulties have been faced when acquiring all data completely fit in our plan. Some data could easily be found on the official website such as the Office of National Statistics (ONS) and the Trade Union Congress (TUC), but sometimes there is no trace of any macro data for area. For example, the authors have planned to get digital values of average weekly work hours of full-time researchers. However, the data about average working time of all kinds of workers is obtained instead, provided from the official website of TUC. According to formula  $\frac{\Delta A_{t+1}}{\Delta A} = zL_{at}$  and  $L_{at} = \text{number of researchers} \times \text{weekly average working time}$  A decision has been made to only use number of researchers to be  $L_{at}$ . The replacement of  $L_{at}$  can undoubtedly affect the accuracy of z.

#### 3.2 Without considering capital factor effects.

When calculating the z value and utilizing input value, labor usage was considered solely as the input of research, researchers have ignored to take account of capital usages and its effects on z value. In one research, for example, researchers always have necessary needs towards buildings and facilities by which they can conduct their own study project. Obviously, cost of these capitals should be counted into our input. The lack of consideration for capital factor could, therefore, also be a flaw of the research.

#### 3.3 Data source

In the data collection process, accuracy of data sources is always a constant topic. Most of the data collected in the research are from the ONS and rest of them are from TUC and PWT (Penn World Table). Considering about the fact that the ONS is a website of British government, there might be an inevitable tendency and a potential intention that the government would show their achievement or accomplishment on developing country and improving the living quality of the public, especially when most of crowd believe that GDP is digital version of indication of citizen’s living level. Therefore, the GDP value collected may have some bias in them, which influences our result of  $A_t$  value (growth rate). For TUC, it may have the same potential problem as ONS, since they need to make their average working time higher to maintain a powerful bargaining counter in the negotiation with government. Accuracy of PWT’s value may directly relate to models what they use to estimate all kinds of value. Different models could reach different answers, so there could be an extent of small deviation of data.

#### 3.4 In addition

In the study, value of  $K_t$  and Y have been used to calculate growth rate, but the value of  $K_t$  is based on the currency value of 2011 and Y value is based on another year - 2016. In most of the time one would expect these two values to have great incompatibility, since the year they based on is different. However, in this study, because of the need to calculate growth rate  $A_t$ , effect of the difference is offset and deducted, thus result still can show the change of growth rate.

### 4. CONCLUSION

The work proves that idea production of the UK has decreasing return to scale during 1995 to 2017 through utilizing the knowledge of Solow model and formula of Romer's model. All required values of the work are acquired from reliable sources, such as ONS (Office for national statistics) or Penn world table. By analyzing these data, the work confidently concludes a fact of declination of idea production of the UK within 1995-2017. But the study ignores some factors that may potentially affect the precise digital result, because of limitation of data collection and nature of formula itself.

Moreover, the result helps people to better understand how the input of research and development influence the development of one country. There is a possibility that the Romer model may be applied to measure the potential of development of certain country in other works.

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