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Study of Portfolio Performance Under Certain Restraint Comparison: Markowitz Model and Single Index Model on S&P 500

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ABSTRACT

The purpose of this study is to figure out what percentage of each stock should be in the optimal portfolio and minimal risk portfolio created by Markowitz Model and Sharpe's Single Index Model and to make a comparison of the portfolio model, as well as the return and risk differences between Markowitz Model and Single Index Model. The research is undertaken at the New York Stock Exchange on ten stocks which are from the S& P 500 index for the period May 2001 to May 2021, The data analysis technique is constrained optimization and regression analysis. The results show that the Single Index Model needs fewer estimators than Markowitz Model and simplifies the actual operation. But for some assets with the correlated residual return, the Markowitz model performs better than the Single Index Model. The optimal portfolio constructed with the Single Index Model has a higher return and risk than the optimal portfolio constructed with Markowitz Model. But for the minimal risk portfolio, the portfolio based on Markowitz Model performs better. And the portfolio including Stock Index has lower systemic risk.

Keywords: *Markowitz Model, Single Index Model, Optimal portfolio, Minimal risk portfolio*

1. INTRODUCTION

The S&P 500, NASDAQ and other market index all plunged as the COVID-19 chain reaction continued to spread across the global capital market. Under the turbulent stock market, investors' urgent demand for portfolio investment gradually appears. Portfolio investment is different from direct investment. In its true definition, it does not need to own a significant share in a target company but invests in a wide range of asset classes and sectors depending on the investor's risk appetite, the investment amount, investment duration, etc. Investing in a portfolio is favored by most investors because of its ability to reduce risk and optimize profit. To design an excellent portfolio, investors need to make important decisions that can affect the performance of the portfolio. This study reveals the factors that need to be considered when constructing the portfolio and the different performances when using the classical investment analysis model to construct the portfolio.

Putra and Dana used quantitative information of stocks included in the LQ45 index for the period February 2017 to January 2020 to investigate the stock composition and the percentage of different securities in the optimal portfolio under the Markowitz model and the Single Index model. Putra and Dana also put forward the hypothesis that the average return utilizing the single index approach versus the Markowitz model differs significantly. The Wilcoxon-Mann-Whitney test revealed that the optimal portfolio model utilizing the single index model outperforms the Markowitz model, but there is no statistically significant difference in the average return between the single-index model and the Markowitz model [1]. Varghese and Joseph focused on the importance of portfolio investment analysis and highlighted the Markowitz model and Sharpe's model which are two well-known portfolios, analysis models. By describing the two models from the perspective of formulas and parameters. The findings of the study revealed significant parallels and variations between the two models that could influence investment portfolio decisions. [2]. Mangram presented a simplified view of Markowitz's achievements of Modern Portfolio Theory, eschewing a detailed presentation of the complex mathematical models typically. Based on the discussion of the theoretical framework and key concepts of MPT and the comparative analysis of early MPT works and current economists' theories, the review summarizes the limitations of the analysis and prospects for future development: although MPT has many shortcomings, including too complex mathematical thinking and dependence on often overturned theoretical assumptions, in any case, using modern financial methods, Markowitz's contribution to portfolio selection for the MPT model can be simplified and done more efficiently (such as Microsoft Excel) [3]. Sen and Fattawat wanted to gain an understanding of the concept behind Sharpe's Single Index Model (SIM), as well as create an optimal portfolio utilizing Sharpe's Single Index Model and determine the return and risk of the optimal portfolio. With the calculations of return and risk based on the information of the website (www.Bseindia.com), the study discovered that the overall risk of the optimal portfolio is estimated using two different mechanisms revealed in SIM and Markowitz's models differs significantly. In comparison to Markowitz's Mean-Variance Model, Sharpe's Single Index Model provides an easy mechanism for a rational investor to design an optimal stock portfolio. The research found that utilizing Sharpe's Single Index Model to design an optimal portfolio investment is considerably simpler than using Markowitz's Mean-Variance Model [4].

Sarker used the Markowitz model to construct an optimal portfolio using the monthly closing prices of 164 companies listed on the Dhaka Stock Exchange (DSE) and the DSE all-share price index for the period July 2007 to June 2012, satisfying the constriction that the sum of percentages invested in the assets equals one. The study's findings revealed that the research's data coverage is moreover adequate for making an investment decision, as it covers 69.79 percent of the data, and 23 of the 164 companies give a lesser return than the risk-free rate. As a result, it can be argued that the Markowitz Model works effectively in both the Dhaka Stock Exchange and the Bangladesh Stock Exchange [5]. Grover and Lavin detailed a user-friendly Excel optimization tool that mutual fund investors can use to maximize their wealth because many investors do not have access to portfolio optimization tools or the background to comprehend mathematical models that provide perspective into efficient portfolio management. When the single-index model is acknowledged as the best way to forecast the covariance structure of returns, the method offered and proven in the research provided the optimum procedure for selecting a portfolio [6]. Amu and Millegard explained some key concepts like expected value and variance of the return of a portfolio to make an understandable and correct explanation of Markowitz's portfolio theory. According to the findings, the Markowitz model for minimizing risk under a given expected return is an optimization problem, and investors should determine the best number of weights to invest in each asset. The model is solved by using the Lagrange multiplier. Finally, the research showed that the Markowitz model is only a tool. Investors need to clearly understand asset attributes and risk factors and choose carefully because there is still a potential correlation between different assets [7].

Yuwono and Remdhani provided a superior choice in the decision-making process in picking the optimal portfolio of stocks included in the Jakarta Islamic Index in the Indonesia Stock Exchange, using the new theory of portfolio formation Markowitz model and single index mode. The Wilcoxon test with statistical software was utilized in the study's analysis. The findings revealed that there is no significant difference in the level of return obtained using the Markowitz model and the single-index model and that the level of return earned using the Markowitz model and single-index model is not greater than the risk-free asset return [8]. Clarke and De Silva focused their study on the analytic form and parameter values of individual securities weights in minimal variance portfolios. In portfolio mathematics, constrained solutions are usually intractable, but assuming a single-factor risk model provides for a straightforward and intuitive expression for the best weights. The study compared single factor analytics to quantitative optimizations on a very generalized covariance matrix and discovered that accounting for non-market sources of security correlation hardly slightly changes the analytically obtained optimal weights [9]. Kan and Zhou calculated analytically the expected loss function associated with employing sample means and the covariance matrix of returns to determine the best portfolio. The study also demonstrated that when parameter uncertainty exists, owning the sample tangency portfolio and the riskless asset is rarely the best option. Kan and Zhou argued that in the standard mean-variance framework, alternative sample estimates outperform the usual maximum likelihood estimate of optimal portfolio weights. They also showed that two-fund solution which is implemented by holding the sample tangency portfolio and the riskless asset is not preferable because a three-fund portfolio rule obtained by combining the usual two funds and the sample global minimum-variance portfolio can improve expected returns [10].

This study aims at comparing the similarities and differences between the Markowitz Model and Single Index Model to explore their respective advantages and disadvantages in the construction of portfolios and provide investors with a choice of model criteria. Based on ten stocks from S&P 500 and five additional restraints, this study constructs the optimal portfolio and the minimal risk portfolio by utilizing the above two models to compare their performance across different investment environments. Finally, this study also explores whether the inclusion of stock index futures on the impact of portfolio risk.

2. METHODOLOGY

The goal of the study is to provide some information regarding portfolio analysis using Harry Markowitz's mean-variance model and William Sharpe's single index model. It compares the performance of the portfolio with certain component securities, as determined by two alternative models.

2.1. Markowitz Mean-Variance Model

Most investors understand that deciding how to divide your money between stocks, bonds, and Treasury Bills to maximize profits while limiting risk is a vital decision. To achieve this balance between the highest possible return and the lowest possible volatility, Markowitz proposed the mean-variance model in 1952. Markowitz Model assumes that investors consider their options based on the probability distribution of the return on the security at a given time, and that their decisions depend only on the risk and return of the security. As per the model, the portfolio's expected return is the weighted average expected return of its security, where the weight represents the percentage of each constituent stock in the portfolio's current value. In general, the following equation can be used to compute the expected rate of return of a portfolio of 'n' securities:

$$E(r_p) = \sum_{i=1}^{n} \omega_i E(r_i) \quad (1)$$

Where, $E(r_p)$ =expected return of the portfolio, ω_i =proportion of investment in each component securities (security i), $E(r_i)$=expected return of each component security (security i), n= total numbers of securities in the portfolio.

As per the model, the total risk of a portfolio differs from the total return. It cannot be expressed linearly by the risk of each asset. Because there is a potential correlation between the different securities, except for the risk of each security, mutual risk between different bonds should also be considered. The standard deviation σ_i or variance σ_i^2 can be used to calculate the risk of each security (security i). σ_i or σ_i^2 reflects the amount of variance between the predicted and actual return. The mutual risk can be measured by covariance σ_{ij} or $Cov(r_i, r_j)$. Therefore, the total risk of a portfolio can be defined as the sum of a weighted average of risk of individual securities and mutual risk. In general, the variance of a portfolio including 'n' securities can be calculated by the following equation:

$$\begin{aligned} \sigma_p^2 &= \sum_{i=1}^{n} \omega_i \sigma_i^2 + \sum_{i\neq j}^{n} \omega_i \, \omega_j \, Cov(r_i, r_j) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \, \omega_j \, Cov(r_i, r_j) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \, \omega_j \, \rho_{ij} \sigma_i \sigma_j \end{aligned} \quad (2)$$

Where, σ_p^2 =portfolio variance, ω_i =proportion of each component securities (security i), σ_i^2=variance of each component securities (security i), $Cov(r_i, r_j)$= the covariance between security i and security j, r_i=return of security i, r_j=return of security j, ρ_{ij}= the correlation coefficient between security i and security j ($\rho_{ij} = \frac{Cov(r_i, r_j)}{\sigma_i \sigma_j}$). Calculating the square root of the portfolio variance yields the portfolio standard deviation σ_p.

The Markowitz model reveals that if a portfolio has 'n' securities, the number of estimates of returns is n. the number of non -repetitive elements in the covariance matrix is $\frac{n\times(n+1)}{2}$, which is composed of: n diagonal terms, individual assets squared standard deviations, and $\frac{n\times(n-1)}{2}$ off-diagonal terms, cross-covariances. Thus, the total number of estimates needed is: $n + \frac{n\times(n+1)}{2}$, which for eleven securities portfolios is equal to seventy-seven.

2.2. Single Index Model

Markowitz's model requires many estimates, and it tells nothing on how to produce those estimates. Therefore, William Sharpe discovered the single index model, which simplifies the estimate of covariance matrix problems, improves security anticipated returns analysis, and allows to assess these risk components for securities and portfolios. Sharpe's Single Index Model assumes that stock returns are affected by a common macro factor, which is expressed in terms of the return on the market index, and that the residual uncertainty of stock return caused by other factors besides this common factor is unique to the company. In other words, the covariance between different stock returns depends on the common factor only, but there is no correlation between the stock residual returns, and the expectation of these residual returns is also zero.

As per this model, the portfolio's expected return is the weighted mean of the return of the market-related component stocks and the non-market-related component stocks. It can be calculated as follows for a portfolio with 'n' securities:

$$E(R_p) = \sum_{i=1}^{n} \omega_i \, \beta_i E(R_m) + \sum_{i=1}^{n} \omega_i \alpha_i \quad (3)$$

Where, $E(R_p)$ =expected return of the portfolio, ω_i =proportion of investment in each component securities (security i), $E(R_m)$ =expected of a market index, β_i= the regression coefficient between stock i's returns and the market index's returns. It can be calculated as: $\beta_i = \frac{Cov(r_i, r_m)}{\sigma_m^2} = \rho_{i,m} \frac{\sigma_i}{\sigma_m}$ (σ_m^2=variance of a market index, $\rho_{i,m}$=the correlation coefficient between stock i and market index, α_i= return of security that is unsystematic or unrelated to the market i.

As per the model, the total risk of the portfolio is decomposed into systematic risk and firm-specific components. Therefore, the total risk of the portfolio can be expressed as a weighted average of the systemic risk and the specific risk of the assets in the portfolio. In general, the variance of a portfolio consisting of 'n' securities can be calculated by the following equation:

$$\sigma_p^2 = \sum_{i=1}^n \omega_i^2 (\beta_i \sigma_m)^2 + \sum_{i=1}^n \omega_i^2 \sigma^2(e_i) \quad (4)$$

Where, σ_p^2 =portfolio variance, ω_i =proportion of investment in each component securities (security i), β_i=the regression coefficient of between the returns of stock i and the market index returns, σ_m =standard deviation of a market index, $\sigma^2(e_i)$= variance of security i's return not related to the market index.

If a portfolio contains 'n' securities, the total estimates are n estimates of the expected excess returns, α_i, n estimates of the sensitivity coefficients, β_i , n estimates of the firm-specific variances, $\sigma^2(e_i)$, 1 estimate of the market risk premium, $E(R_m)$, 1 estimate of the variance of the common macroeconomic factor, σ_m^2. Then, using these (3n+2) estimates, the optimization technique can be prepared. The number of estimations required for a portfolio of eleven securities is thirty-five.

3. RESULTS

This study selects two technology stocks: AAPLE and CITRIX SYSTEMS, three financial service stocks: The Progressive Corporation, JPMorgan, and Berkshire Hathaway, four industrial stocks: United Parcel, Service, FedEx Corporation, JB. Hunt Transport Services and Landstar System, a cyclical consumer socks: AMAZON.COM and an equity index (S&P 500) to construct portfolios using two models by solving constrained optimization problems and compare their performance under five constraints. Through recent twenty years of historical daily total return for ten stocks and a proxy for risk-free rate (1-month Fed Funds rate), the optimization inputs for the Markowitz Model and Single Index Model were calculated in two tables, as follows:

Table 1. Calculations about optimization inputs of eleven risky assets

Item	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
Annualized average return	7.5%	33.8%	34.0%	15.6%	11.9%	9.0%	15.4%	9.9%	13.0%	22.5%	17.4%
Annualized Standard deviation	14.9%	41.4%	34.4%	41.5%	29.0%	16.2%	21.1%	21.4%	26.7%	30.7%	23.9%
Beta	1	1.35	1.26	1.22	1.36	0.57	0.71	0.83	1.10	1.08	0.80
Annualized alpha	0.00	0.24	0.25	0.06	0.02	0.05	0.10	0.04	0.05	0.14	0.11
Residual Standard deviation	0.0%	36.2%	29.0%	37.3%	20.8%	13.8%	18.2%	17.5%	21.1%	26.2%	20.8%

To reduce the non-Gauss effects, the daily data was aggregated to monthly observations and based on them, the annualized average return and annualized standard deviation were calculated and shown in Table 1. Using regression analysis, this paper also obtains the beta coefficient and intercept alpha of eleven assets relative to the market index. According to $e_i = r_i - \beta_i * r_m - \alpha_i$, lastly, the study finds the Residual Returns of the corresponding stock every month and multiply the standard deviation to get the Residual standard deviation.

Table 2. Correlation coefficients between eleven assets.

ρ	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
SPX	1.000	0.485	0.542	0.437	0.697	0.523	0.502	0.575	0.614	0.521	0.495
AMZN	0.485	1.000	0.377	0.217	0.252	0.118	0.200	0.296	0.280	0.308	0.256
AAPL	0.542	0.377	1.000	0.332	0.244	0.173	0.240	0.231	0.330	0.268	0.287
CTXS	0.437	0.217	0.332	1.000	0.324	0.181	0.271	0.264	0.331	0.290	0.252
JPM	0.697	0.252	0.244	0.324	1.000	0.452	0.393	0.361	0.440	0.442	0.375
BRK/A	0.523	0.118	0.173	0.181	0.452	1.000	0.264	0.404	0.385	0.239	0.234
PGR	0.502	0.200	0.240	0.271	0.393	0.264	1.000	0.392	0.365	0.280	0.289

UPS	0.575	0.296	0.231	0.264	0.361	0.404	0.392	1.000	0.675	0.459	0.441
FDX	0.614	0.280	0.330	0.331	0.440	0.385	0.365	0.675	1.000	0.537	0.482
JBHT	0.521	0.308	0.268	0.290	0.442	0.239	0.280	0.459	0.537	1.000	0.590
LSTR	0.495	0.256	0.287	0.252	0.375	0.234	0.289	0.441	0.482	0.590	1.000

As shown in Table 2. The correlation coefficients between eleven assets were calculated from the above return and standard deviation data and they were used to calculate the portfolio total risk in Markowitz Model. This study considers the following five constraint cases and uses the constraint optimization tool to find the minimum risk portfolio and the optimal portfolio (Max Sharpe ratio) under each constraint. Portfolio analysis based on the Markowitz model and the Index model is compared as shown in the following five tables.

Table 3. Portfolio analysis under constraint 1

Item	Markowitz Model		Single Index Model	
	Min Variance Weighted	Max Sharpe Weighted	Min Variance Weighted	Max Sharpe Weighted
SPX	72.24%	-48.25%	65.79%	-47.30%
AMZN	-2.35%	16.40%	-4.14%	17.84%
AAPL	-3.85%	30.02%	-4.74%	30.52%
CTXS	-1.04%	-0.10%	-2.44%	0.50%
JPM	-18.47%	-0.09%	-12.87%	-2.67%
BRK/A	36.21%	41.31%	34.51%	22.44%
PGR	13.91%	32.96%	13.42%	31.53%
UPS	3.43%	-0.02%	8.56%	1.17%
FDX	-10.28%	-1.47%	-3.61%	0.11%
JBHT	-0.55%	12.50%	-1.72%	19.20%
LSTR	10.75%	16.72%	7.24%	26.66%
Return	7.15%	26.42%	6.46%	28.57%
Standard deviation	12.24%	18.69%	12.43%	19.98%
Sharpe	0.584	1.413	0.520	1.430

This additional optimization constraint is designed to simulate Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity: $\sum_{i=1}^{11}|\omega_i| \leq 2$. As shown in Table 3, for minimal risk portfolio, Markowitz model has better performance: a higher return (7.15%) and less risk (12.24%), however, for optimal portfolio, Single Index model has a deeper Sharpe ratio.

Table 4. Portfolio analysis under constraint 2

Item	Markowitz Model		Single Index Model	
	Min Variance Weighted	Max Sharpe Weighted	Min Variance Weighted	Max Sharpe Weighted
SPX	72.24%	-100.00%	65.79%	-100.00%
AMZN	-2.35%	22.33%	-4.14%	21.85%
AAPL	-3.85%	39.76%	-4.74%	36.62%
CTXS	-1.04%	-1.20%	-2.44%	3.09%
JPM	-18.47%	-0.50%	-12.87%	-8.79%
BRK/A	36.21%	62.48%	34.51%	33.32%
PGR	13.91%	46.01%	13.42%	39.98%

UPS	3.43%	-3.11%	8.56%	9.52%
FDX	-10.28%	-10.56%	-3.61%	5.62%
JBHT	-0.55%	20.88%	-1.72%	24.97%
LSTR	10.75%	23.91%	7.24%	33.83%
Return	7.15%	33.18%	6.46%	34.07%
Standard deviation	12.24%	22.11%	12.43%	22.40%
Sharpe	0.584	1.501	0.520	1.521

This additional optimization constraint is designed to simulate some arbitrary "box" constraints on weights, which may be provided by the client: $|\omega_i| \leq 1, for \; \forall \; i$. As shown in Table 4, like the result of condition 1, the Markowitz model performs better in the minimal risk portfolio, while the single exponential model performs slightly better in the optimal portfolio.

Table 5. Portfolio analysis under constraint 3

Item	Markowitz Model		Single Index Model	
	Min Variance Weighted	Max Sharpe Weighted	Min Variance Weighted	Max Sharpe Weighted
SPX	72.54%	-237.52%	65.79%	-329.79%
AMZN	-2.34%	37.06%	-4.14%	43.02%
AAPL	-4.60%	65.40%	-4.74%	69.96%
CTXS	0.31%	0.42%	-2.44%	11.02%
JPM	-18.90%	17.30%	-12.87%	8.83%
BRK/A	36.82%	91.59%	34.51%	58.48%
PGR	14.08%	68.19%	13.42%	72.35%
UPS	1.52%	1.27%	8.56%	27.91%
FDX	-10.02%	-8.69%	-3.61%	24.94%
JBHT	-0.86%	30.97%	-1.72%	50.35%
LSTR	11.45%	34.02%	7.24%	62.93%
Return	7.05%	49.61%	6.46%	60.91%
Standard deviation	12.25%	32.25%	12.43%	38.16%
Sharpe	0.576	1.539	0.520	1.596

Considering a "free" problem, without any additional optimization constraints, constraint 3 aimed at illustrating how the area of permissible portfolios in general. As depicted in Table 5, compared with the former, the result of the minimum risk portfolio is still unchanged, but for the optimal portfolio constructed by the two models, the expected return is greatly increased, but it also faces a higher risk.

Table 6. Portfolio analysis under constraint 4

Item	Markowitz Model		Single Index Model	
	Min Variance Weighted	Max Sharpe Weighted	Min Variance Weighted	Max Sharpe Weighted
SPX	38.47%	0.00%	30.46%	0.00%
AMZN	0.00%	12.96%	0.00%	15.46%
AAPL	0.00%	25.21%	0.00%	27.12%
CTXS	0.00%	0.00%	0.00%	0.00%
JPM	0.00%	0.00%	0.00%	0.00%
BRK/A	38.49%	19.26%	37.65%	3.26%

PGR	14.23%	22.72%	14.64%	21.36%
UPS	0.72%	0.00%	9.34%	0.00%
FDX	0.00%	0.00%	0.00%	0.00%
JBHT	0.00%	8.81%	0.00%	13.98%
LSTR	8.09%	11.04%	7.90%	18.82%
Return	10.04%	22.09%	10.24%	24.46%
Standard deviation	13.09%	17.62%	12.99%	19.16%
Sharpe	0.767	1.254	0.788	1.277

For this condition, it is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions, $\omega_i \geq 0, for\ \forall i$. As shown in Table 6, the portfolio constructed by the Single Index model has better performance both in a minimal risk portfolio and an optimal portfolio.

Table 7. Portfolio analysis under constraint 5

Item	Markowitz Model		Single Index Model	
	Min Variance Weighted	Max Sharpe Weighted	Min Variance Weighted	Max Sharpe Weighted
SPX	0.00%	0.00%	0.00%	0.00%
AMZN	2.45%	14.65%	-1.55%	18.56%
AAPL	4.19%	26.73%	-0.81%	31.69%
CTXS	0.84%	-3.44%	-0.11%	-0.47%
JPM	-7.09%	-15.54%	-5.02%	-21.23%
BRK/A	56.31%	36.35%	47.99%	11.67%
PGR	23.52%	31.99%	21.65%	27.66%
UPS	11.41%	-12.17%	17.82%	-5.30%
FDX	-8.05%	-13.21%	3.45%	-5.40%
JBHT	0.60%	17.83%	2.82%	18.57%
LSTR	15.82%	16.81%	13.76%	24.25%
Return	13.20%	23.89%	11.47%	26.95%
Standard deviation	13.39%	18.02%	13.21%	20.24%
Sharpe	0.986	1.326	0.869	1.331

Lastly, this study considers an additional optimization constraint: $\omega_1 = 0$ to find if the inclusion of the board index into the portfolio has a positive or negative effect. As shown in Table 7, there is no significant difference between the two models under this constraint. To make more significant research on the problems mentioned in beginning, this paper makes regression analysis on the market index of the portfolio with SPX and the portfolio without SPX constructed by using Single Index Model, the results are shown in the following table.

Table 8. The regression result of optimal portfolio returns concerning the market return

Portfolio	Item	Coefficients	Standard Error	t Stat	P-value
With SPX	intercept	0.04562(α)	0.0077521	5.884244	1.34E-08
	X Variable	0.81776(β)	0.1792833	4.561291	8.13E-06
Without SPX	intercept	0.01675(α)	0.0030600	5.473598	1.11E-07
	X Variable	0.90785(β)	0.0707691	12.82837	5.2E-29

Based on the weight of the optimal portfolio constructed by the Single Index Model under restraints 3 and 5, this research constructs 241 new portfolios using the data of the stock monthly return from 2001-to 2021. Make regression analysis to its rate of return relative to the market rate of return, the result is as shown in Table 8.

4. DISCUSSION

4.1. Comparison between Markowitz Model and Single Index Model

As mentioned earlier, the Markowitz Model needs to calculate the covariance or correlation coefficient between each stock to determine the portfolio mutual risk. For a portfolio with 'n' securities, a total of $n + \frac{n\times(n+1)}{2}$ estimators need to be calculated to construct it. For the Single Index Model, the covariance between different stocks is influenced by a common factor: the market index. Therefore, the Single Index Model greatly simplifies the problem of estimating the covariance matrix. For a portfolio with 'n' stocks, a total of 3n+2 estimators need to be calculated. In other words, it reduces the number of required estimates from O(n^2) to O(n). Based on this point, the Single Index Model is preferred in practical investment management. However, the choice of the two models should be considered from a more detailed aspect. Markowitz Model shows that if there is a perfect negative correlation between assets, the total risk of the entire portfolio can be reduced to zero, while the single index model shows that as the diversification of assets increases, there is no longer a specific risk across the portfolio, but the systemic risk is not reduced by diversification. In this respect, it seems that portfolios constructed by Markowitz Model perform better. However, identifying securities with complete negative correlation is almost impossible. and sometimes the covariances are hard to estimate with any degree of confidence, at which point the benefits of the Markowitz Model seem illusory. On the other hand, the Single Index Model assumes that there is no correlation between the unexpected returns of different assets. But in fact, stocks of the same sector often have correlated residual returns, which leads to inferior portfolios than the full Markowitz Model. If the correlation of residual returns of two stocks is positive, then the Markowitz Model will give a smaller weight to both stocks, if it is negative, then the Single Index Model will give too little weight to both stocks resulting in higher than Markowitz variance.

4.2. Comparison between portfolios under five constraints

As the previous data results show, the two models lead to substantially different portfolios for small number of instruments. Regardless of the constraints, the optimal portfolio created by Single Index Model has a larger Sharpe ratio than the optimal portfolio created by Markowitz Model, and the minimal risk portfolio based on Markowitz Model performs better than the minimal risk portfolio based on Single Index Model. This shows that conservative investors who like to avoid risk (risk-averse) might invest in their equities using the Markowitz Model's minimal risk portfolio for higher returns and lower risk than the Single Index Model. For those investors who want to get higher returns by taking higher risks, it is more sensible to invest in their stocks with an optimal portfolio based on a Single Index Model. The results also show that the profitability of the portfolio is affected by different investment constraints and environments. In the unconstrained case (constraint 3), the returns of the optimal portfolios constructed by both models are significantly higher than in the other cases, while for the more stringent constraints (constraint 4), whatever the model is, the portfolio is limited in its ability to return, but it faces less risk. The portfolio established in this study still has a lot of limitations. The number of investment instruments is too small, and stocks of different sectors are not sufficient to fully comply with the assumption of the index model. For future researchers who study similar issues, it is suggested that more investment instruments and more stocks of different sectors should be used to construct the investment portfolio to obtain more convincing results and based on this research to obtain a fuller conclusion. Future researchers are also suggested to employ a variety of analytical techniques to gather more reliable data.

4.3. Comparison of the portfolio with and without SPX

As shown in Table 8, both regression models are significant. For the optimal portfolio with SPX, the beta coefficient (0.81776) is lower than that of the portfolio without SPX (0.90785). This shows that the optimal portfolio including stock index futures faces much less systemic risk than the optimal portfolio without stock index futures. Shorting stock index futures provides a risk-averse ability for a portfolio. Therefore, more and more investment managers choose to use stock index futures hedging market analysis to build market-neutral products with low correlation to the market. On this basis, investment managers focus more on the excess return part of the portfolio (α), which may place higher demands on many asset managers in the future.

5. CONCLUSION

This research selects ten stocks from the S&P 500 in four sectors and one equity index, then constructs the optimal portfolio and minimal risk portfolio with Markowitz Model and Single Index Model. This study aimed at comparing the similarities and differences between the two models and to find out the criteria for selecting the models. Given that investors may be in different to investment environments, this research adds five additional different investment restrictions. Through the above analysis, it can be concluded that the two models have different methods to calculate the total risk of the portfolio, but the Single Index Model is simpler. However, the Single Index Model may not do as well as the full Markowitz Model for the risk estimation of some assets with the relative residual return. In constructing the optimal portfolio, the Single Index Model shows higher return and higher risk, while the Markowitz Model shows higher return and lower risk in constructing the minimal risk portfolio. In addition, the study concludes that the portfolio including stock index futures shows higher risk resistance, which also puts forward higher requirements for investment managers. For future researchers who study a similar problem, it is suggested to add more different stocks and investment tools in the portfolio to make the portfolio fit the model hypothesis better. The investment analysis tools used in this study are also limited. Future researchers are expected to use more investment tools to get more accurate results.

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