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Optimal Pricing Strategy of SaaS Providers Charge by Usage

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ABSTRACT
This paper studies the renewal cost, discount rate and free trial time when SaaS providers provide free trial services, and discusses how these factors will affect the pricing strategy of SaaS providers by constructing a two-stage game model. This paper finds that before users perceive the real quality of SaaS services, The optimal profit increases with the increase of free trial time and user perception rate. When SaaS providers charge by usage, in any case, the hybrid price discrimination strategy cannot make SaaS providers achieve the optimal profit, and when the transaction cost increases, the user's utility and SaaS service provider's profit will also decrease. When the transaction cost is low, SaaS providers will adopt intertemporal price discrimination strategy. With the growth of transaction cost, the profit of behavior-based price discrimination strategy is higher than intertemporal price discrimination.

Keywords： *SaaS service; charge by usage; Free trial strategy*

1. INTRODUCTION

With the development of Internet technology and the maturity of application software, software as a service (SaaS) is a software application model rising in the 21st century. According to iimedia research, the global cloud computing market will reach US $224.5 billion in 2020 and is expected to reach US $265.4 billion in 2021. SaaS market has always been one of the fastest-growing cloud computing industry segments and the third largest part of the whole industry. Gartner predicts that the SaaS market will reach $122.6 billion in 2021. At present, the office software used in all walks of life (such as financial software, supply chain management system, human resources software and customer relationship management system) are gradually adopting SaaS mode. Under the SaaS service mode, the software service provider uses the cloud server to uniformly manage the products. Users need to pay the corresponding service fees to the service provider before they can use the corresponding services of the software through the Internet. Charging by usage is the main charging method of SaaS providers, such as the video marketing platform Wistia, which adopts the strategy that charge by usage method to charge for the number of videos applied by users, and the charging standard is the same in each cycle. So what pricing strategy should SaaS providers choose to maximize profits under different circumstances?

At present, SaaS providers adopt the optimal pricing problem of charging by usage, which has attracted extensive attention of scholars at home and abroad. Altmann [1] used the method of empirical research to propose a pricing strategy that is beneficial to both network service providers and market users. Sundarajan [2] studied the nonlinear pricing method of pay per use, and pointed out that the fixed price strategy of low price was adopted to penetrate the market in the early stage of the market. Karlapalem [3] used the utility function model and found that when the network effect is strong enough, software providers should adopt free trial services. Fishburn [4] research shows that under the market competition, the duopoly SaaS providers adopt the market equilibrium of charging by stage and charging by use, which is the result of price war, all competitive providers seize the market through low price. Chen [5] used the extended model and studied the selection of pricing indicators (variables) and corresponding pricing schemes based on the general differences between pricing schemes of data services. Cheng [6] found that providing free trial is more profitable than segmenting the market. The above literature does not study the optimal pricing strategy when SaaS providers provide free trial services and charge by usage.

Based on the above, this paper constructs a two-stage model of three price discrimination strategies when SaaS providers provide free trial services and charge by usage. Discussing how the trial time, trial perception rate and discount rate of SaaS services affect the pricing strategy of SaaS services in the context of product upgrading, which provides a theoretical reference for SaaS providers to make optimal decisions.

2. PROBLEM DESCRIPTION AND MODEL ASSUMPTIONS

This paper assumes that the free trial time provided by SaaS provider to users is t . and $0<t<1$. SaaS providers will sell their SaaS services by usage. The quality of SaaS services in phase 1 is v_1. The price of SaaS service in stage 1 is P_{s1}. The quality of SaaS services in stage 2 is v_2. The price of SaaS service in stage 1 is P_{s2}. For old users, the price of phase 2 is P_{s0}, usually $P_{s0} \le P_{s2}$. Users have a basic perception of the quality of SaaS providers in phase 1 and phase 2: v_1、v_2, and $v_2 > v_1 = \lambda v_2$, $0<\lambda<1$. During the trial of SaaS service, the increase rate of users' perception of its quality is β , The overall perception of SaaS services is $(1+t\beta)v$. The true quality of SaaS services is v_t. and $(1+t\beta)v \le v_t$. Considering that users will consider both current and future benefits when making decisions, for future benefits, the discount rate is δ , and $0<\delta<\lambda<1$. Making users more likely to purchase SaaS services in phase 1. There are differences in users' usage, and users' usage can reflect the frequency of users' use of SaaS services. d represents users' usage, which is evenly distributed in [0,1]. The greater d, the more frequent users use. This paper does not consider the situation that users change their purchase behavior due to the change of usage, that is, users' usage will not change with the change of stage and product price. The pricing method of paying by usage will produce a certain transaction cost for users, and the unit cost is T . this cost mainly comes from the psychological utility loss caused by users' continuous payment, as well as the inconvenient cost caused by continuous recording and control of software usage.

SaaS providers can adopt three different pricing strategies: hybrid price discrimination strategy, intertemporal price discrimination strategy and behavior-based price discrimination strategy. Under the hybrid price discrimination strategy, the supplier cannot ensure that the price of each period in the future is consistent. Under the intertemporal price discrimination strategy, the supplier makes pricing according to the time stage, and the price in the first stage is P_{u1}, the price of phase 2 is P_{u2}. Under the behavior-based price discrimination strategy, suppliers collect users' previous purchase information, so as to charge different purchase fees for new and old customers. Suppliers promise to set the same purchase price for all new customers, that is, $P_{u1} = P_{u2}$ and $P_{u0} = P_{u2}$.

3. SAAS PROVIDERS CHARGE BU USAGE

Table 1. Consumer utility

Symbol	Purchase options	Consumer surplus
SS	Purchase in phase 1 and purchase in phase 2	$U_{SS} = (1+t\beta)tv_1d + \delta(1+t\beta)tv_2d + (1+t\beta)v_1d - P_{s1}d + \delta((1+t\beta)v_2d - P_{s0}d)$
SN	Purchase in phase 1 and not purchase in phase 2	$U_{SN} = t(1+t\beta)v_1d + \delta tv_2d + (1+t\beta)v_1d - P_{s1}d$
NS	Not Purchase in phase 1 and purchase in phase 2	$U_{NS} = \delta((1+t\beta)v_2d - P_{s2}d - Td) + tv_1d + \delta t(1+t\beta)v_2d$
NN	Neither phase was purchased	$U_{NN} = tv_1d + \delta tv_2d$

3.1 Hybrid Price Discrimination

Assuming that the no difference point between the purchase strategy and the no purchase strategy adopted by the user in the whole cycle is v_0. the no difference point of the user is $v_0 = \frac{\lambda(T+P_{s2})}{(1+t\beta+t^2\beta)}$. When $U_{SN} > U_{NS}$, the user chooses to purchase SaaS services in the first phase, not in the second phase, but not in the first phase, and the purchase strategy in the second phase point is $v_1 = \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)}$. When $U_{SS} > U_{SN}$, the user chooses to purchase SaaS services in both cycles rather than just in the first phase. The difference is $v_2 = \frac{\lambda(T+P_{s0})}{1+t\beta+t^2\beta}$. the demand in the three cases are $D_{u2} = 1 - \frac{\lambda(T+P_{s0})}{1+t\beta+t^2\beta}$, $D_{u1} = \frac{\lambda(T+P_{s0})}{1+t\beta+t^2\beta} - \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)}$, $D_{u0} = \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)} - \frac{\lambda(T+P_{s2})}{(1+t\beta+t^2\beta)}$. This paper assumes that $0<\delta<\lambda<1$. the indifference point v_1 must be smaller than the no difference point v_2. The optimal profit of SaaS service provider is $\Pi_{u1} = P_{u1}D_{u1} + \delta\Pi_{u2}$, where $\Pi_{u2} = P_{u0}D_{u2} + P_{u2}D_{u0}$. Through corresponding calculation, the corresponding optimal profit of SaaS service provider is shown in lemma 1.

Lemma 1: when SaaS providers adopt the hybrid price discrimination strategy by charging by usage under the free trial strategy, the equilibrium price and profit are as follows.

When $P_{s1} = -T + T\lambda + (\lambda - \delta)P_{s0} + \delta P_{s2}$. the market segmentation structure is [NN, NS, SS] , and $P_{u1} = \frac{-\delta + T\delta - t\beta\delta - t^2\beta\delta + \lambda - 2T\lambda + t\beta\lambda + t^2\beta\lambda + T\lambda^2}{2\lambda - \delta}$, $P_{u0} = \frac{1 + t\beta + t^2\beta - T\lambda}{2\lambda}$, $P_{u2} = \frac{-\delta - t\beta\delta - t^2\beta\delta + \lambda + t\beta\lambda + t^2\beta\lambda + T\delta\lambda - T\lambda^2}{2\lambda(-\delta + 2\lambda)}$. the optimal profit of SaaS providers is $\Pi_{u1} = \frac{(1+t(1+t)\beta - T\lambda)\left(4(1+t(1+t)\beta + T(-2+\lambda))\lambda^3 + \delta^3(1+t(1+t)\beta - T\lambda) + \delta^2\lambda(-3-3t(1+t)\beta + T(-2+5\lambda)) - \delta\lambda^2(1+t(1+t)\beta + T(-8+7\lambda))\right)}{4(\delta - 2\lambda)^2(\lambda + t(1+t)\beta\lambda)}$.

It can be concluded from lemma 1 that when SaaS providers adopt the hybrid price discrimination strategy by charging by usage under the free trial strategy, the market segmentation structure is that when old users purchase SaaS services in phase 2, their cost is higher than that of new users ($P_{u0} > P_{u2}$), and users will not purchase SaaS services alone in phase 1, It is often delayed to purchase SaaS services in phase 2, and some users who purchase SaaS services in phase 1 will continue to purchase SaaS services in phase 2 because of the high quality of SaaS services in phase 2.

3.2. Intertemporal Price Discrimination

When adopting the intertemporal price discrimination strategy, the SaaS service provider will provide the same price for new and old users in the second stage ($P_{u0} = P_{u2}$) . All users will decide whether to adopt the SaaS service strategy in each stage. The indifference pint between the strategy of purchasing SaaS services in the second stage and the strategy of not purchasing SaaS services is v_0 . And the indifference point between NN and NS is the same as that between Sn and SS, that is, $v_2 = v_0 = \frac{\lambda(T + P_{u2})}{1 + t\beta + t^2\beta}$, Easy to find that $\Pi_{u2} = P_{u2}(1 - v_2)$, the indifference point between buying behavior and not buying behavior in stage 1 is v_1. Demand $D_{u1} = 1 - v_1$. It can be obtained from the no difference between NN strategy and Sn strategy or between SS strategy and NS strategy, $v_1 = \frac{\lambda(T - T\delta + P_{u1} - \delta P_{u2})}{(1 + t\beta + t^2\beta)(\lambda - \delta)}$. The optimal profit of SaaS providers is $\Pi_{u1} = P_{u1}D_{u1} + \delta\Pi_{u2}$, where $\Pi_{u2} = P_{u2}D_{u0}$. Through corresponding calculation, the optimal profit of SaaS service provider is shown in lemma 2.

Lemma 2: when SaaS providers adopt the intertemporal price discrimination strategy by charging by usage under the free trial strategy, the equilibrium price and profit are as follows.

(1) When $T < \frac{\delta + t\beta\delta + t^2\beta\delta}{2\lambda + \lambda\delta - 2\lambda^2}$. the market segmentation structure is $[NN, SN, SS]$, $P_{u1} = \frac{1}{4}(T(-2+\delta) + \frac{(1+t(1+t)\beta)(2\lambda - \delta)}{\lambda}$, $P_{u0} = P_{u2} = P_{u2} = \frac{1 + t\beta + t^2\beta - T\lambda}{2\lambda}$. The optimal profit of SaaS providers is $\Pi_{u1} = \frac{4(1 - T + t(1+t)\beta)^2\lambda^2 - 4T\delta(-1+\lambda)\lambda(1+t(1+t)\beta - T\lambda) - 3\delta^2(1+t(1+t)\beta - T\lambda)^2}{16(1+t(1+t)\beta)(\lambda - \delta)\lambda}$.

(2) When $T > \frac{\delta + t\beta\delta + t^2\beta\delta}{2\lambda + \lambda\delta - 2\lambda^2}$. the market segmentation structure is [NN, SS], and $P_{u1} = \frac{1}{2}(1 + t(1+t)\beta + T(-2+\lambda)) and P_{u0} = P_{u2} = \frac{1 + t\beta + t^2\beta - T\lambda}{2\lambda}$, The optimal profit of SaaS providers is $\Pi_{u2} = \frac{(1 + t(1+t)\beta - T\lambda)(\delta + t(1+t)\beta\delta + \lambda + t(1+t)\beta\lambda - T\delta\lambda + T(-2+\lambda)\lambda)}{4(\lambda + t(1+t)\beta\lambda)}$.

It can be concluded from lemma 2 that when SaaS providers adopt the intertemporal price discrimination strategy by charging by the usage under the free trial strategy. The market segmentation structure and the profit of SaaS providers depend on a variety of factors, including the update degree of the second phase SaaS service quality, discount rate, the transaction cost, the free trial time and the increase rate of users' perception of its quality. Under the intertemporal price discrimination strategy, SaaS providers have two cases of the optimal profit. When $T < \frac{\delta + t\beta\delta + t^2\beta\delta}{2\lambda + \lambda\delta - 2\lambda^2}$, users will not delay their purchase strategy because of the low transaction cost. Some users will purchase SaaS services in phase 2 for the high quality of phase 2. When $T > \frac{\delta + t\beta\delta + t^2\beta\delta}{2\lambda + \lambda\delta - 2\lambda^2}$, easy to find that $P_{u2} > P_{u1}$. SaaS providers launch a new generation of SaaS services, it will increase the price in phase 2. users who purchase in phase 1 will purchase SaaS services in phase 2.

3.3. Behavior-based Price Discrimination

When adopting the behavior based price discrimination strategy, SaaS providers charge the same price for new users who take the purchase behavior at any stage（$P_{u1} = P_{u2}$）, For users who take the purchase behavior in phase 1 and continue to purchase in phase 2, the purchase prices in phases 1 and 2 are respectively P_{s1}、P_{s0}. The no difference points corresponding to NN strategy, NS strategy, Sn strategy and SS strategy are: $v_0 = \frac{\lambda(T + P_{u1})}{1 + t\beta + t^2\beta}$, $v_1 = \frac{(1-\delta)\lambda(T + P_{u1})}{(1 + t(1+t)\beta)(\lambda - \delta)}$, $v_2 = \frac{\lambda(T + P_{u0})}{1 + t\beta + t^2\beta}$. In the first stage, users who do not purchase SaaS services and in the second stage, the demand for SaaS services is $D_{u2} = v_1 - v_0$,In phase 1, the demand for SaaS services is $D_{s1} = 1 - v_1$,The demand for SaaS services in both phases is $D_{u0} = 1 - v_2$, The optimal profit of SaaS providers is $\Pi_{u1} = P_{u1}D_{u1} + \delta\Pi_{u2}$,The profit of stage 2 can be expressed as: $\Pi_{u2} = P_{u1}D_{u2} + P_{u0}D_{u0}$ .Through the corresponding calculation, the optimal profit of SaaS providers is shown in proposition 3.

Lemma 3: when SaaS providers adopt behavior-based price discrimination strategy by charging by usage when providing free trial strategy, the equilibrium price and profit are as follows.

(1)when $\frac{1}{4}(-1 - 2\lambda + 4\lambda^2) + \frac{1}{4}\sqrt{1 + 12\lambda - 4\lambda^2 - 16\lambda^3 + 16\lambda^4} < \delta < \lambda$ or $0 < \delta < \frac{1}{4}(-1 - 2\lambda + 4\lambda^2) + \frac{1}{4}\sqrt{1 + 12\lambda - 4\lambda^2 - 16\lambda^3 + 16\lambda^4}$ and $\frac{\delta + t\beta\delta + t^2\beta\delta + 2\delta^2 + 2t\beta\delta^2 + 2t^2\beta\delta^2 - \lambda - t\beta\lambda - t^2\beta\lambda + 2\delta\lambda + 2t\beta\delta\lambda + 2t^2\beta\delta\lambda - 4\delta\lambda^2 - 4t\beta\delta\lambda^2 - 4t^2\beta\delta\lambda^2}{-\delta - \lambda + 2\delta\lambda - 2\delta^2\lambda - 2\delta\lambda^2 + 4\delta^2\lambda^2} < T < 1$, The market segmentation structure is $[NN, NS, SS]$, $P_{u1}=\frac{2\delta(1+t\beta+t^2\beta-T\lambda)}{1+4\delta\lambda}$, $P_{u0}=\frac{-T(1+2\delta(1+2\delta)\lambda)+(1+t(1+t)\beta)(1+2\delta(-1+2\delta+2\lambda))}{2\delta(1+4\delta\lambda)}$, $\Pi=\frac{(1-T+t(1+t)\beta)^2+\frac{4\delta^2(1+t(1+t)\beta-T\lambda)^2}{1+4\delta\lambda}}{4+4t(1+t)\beta}$.(2)when $0<\delta<\frac{1}{4}(-1-2\lambda+4\lambda^2)+\frac{1}{4}\sqrt{1+12\lambda-4\lambda^2-16\lambda^3+16\lambda^4}$ and $\frac{\delta+t\beta\delta+t^2\beta\delta+2\delta^2+2t\beta\delta^2+2t^2\beta\delta^2-\lambda-t\beta\lambda-t^2\beta\lambda+2\delta\lambda+2t\beta\delta\lambda+2t^2\beta\delta\lambda-4\delta\lambda^2-4t\beta\delta\lambda^2-4t^2\beta\delta\lambda^2}{-\delta-\lambda+2\delta\lambda-2\delta^2\lambda-2\delta\lambda^2+4\delta^2\lambda^2}>T$,The market segmentation structure is $[NN, SS]$, $P_{u0}=\frac{-1+t(1+t)\beta(-1+\delta)+\delta+T\delta+T\lambda-2T\delta\lambda}{-\lambda+\delta(-1+2\lambda)}$, $P_{u1}=\frac{(-1+T-t(1+t)\beta)\lambda+\delta(1+T+t(1+t)\beta-2T\lambda)}{-\lambda+\delta(-1+2\lambda)}$, $\Pi=\frac{\delta(1-\lambda)(-(1+t(1+t)\beta)(\delta^2+\delta\lambda-\lambda(1+\lambda))-T(1+\delta+\lambda)(-\lambda+\delta(-1+2\lambda)))}{(\delta+\lambda-2\delta\lambda)^2}$.

According to the market equilibrium results, it is obvious that the market segmentation structure mainly depends on the discount rate in stage 2. When the discount rate is high, the SaaS service in phase 2 gives users higher utility. At this time, users may adopt the delayed purchase strategy, users will purchase SaaS services in phase 2 after purchasing SaaS Services in phase 1. when users do not take the second stage of the trial and the discount rate of the free service, it will be affected by the relatively high purchase cost, Users who purchase SaaS services in phase 1 will continue to use SaaS services at a more favorable price in phase 2, so users who purchase SaaS services will purchase in both phases.

4. NUMERICAL ANALYSIS

Based on lemma 1-3, comparing the optimal profits under three different price discrimination, the optimal pricing strategy of SaaS providers is as follows:

Lemma 4：When SaaS providers charge by usage under the free trial strategy, the profit of intertemporal price discrimination strategy is higher than that of mixed price discrimination strategy. However, comparing the optimal profit of the other two price discrimination strategies, the calculation is more complex and difficult to obtain. This paper will compare it through the method of numerical analysis order $\lambda=0.8$, $\delta=0.6$, $\beta=0.8$, This paper uses $t=0.1$ and $t=0.05$ to represent the impact of free trial time on the profits of SaaS providers.

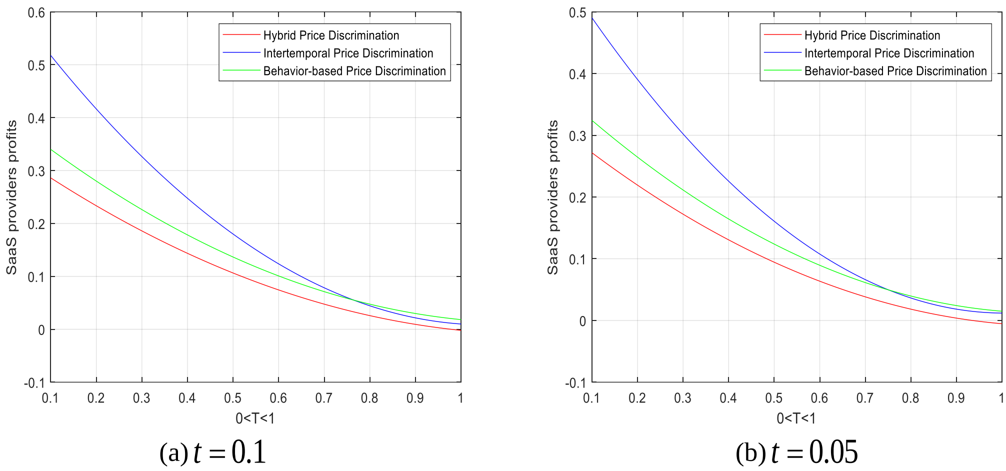

Figure 1 Changes of SaaS service provider's profit under different transaction costs

Figure 1 verifies the mathematical analysis of this paper. In any case, the hybrid price discrimination strategy cannot make the SaaS service provider achieve the optimal profit, and when the transaction cost increases, the user's utility and the SaaS service provider's profit will also decrease. When the T is low, SaaS providers will adopt the intertemporal price discrimination strategy, and users will not delay their purchase strategy. Users who buy SaaS services in phase 2 will also buy SaaS services in phase 1. Because the prices of new and old users are the same, it will attract more users to buy SaaS services in phase 1 to achieve the best profit; With the growth of T, $P_{s2}>P_{s1}$. Despite the price rise in phase 2, users who purchase SaaS services will purchase in both phases, and no users will only buy SaaS services in phase 1 or phase 2. At this time, the profit of behavior-based price discrimination strategy is higher than intertemporal price discrimination. Under the behavior-based price discrimination strategy, SaaS providers only make decisions based on consumers' past purchase strategies, $P_{so}<P_{s1}$, in stage 2, old users can buy SaaS services at a more favorable price, and users are more inclined to buy SaaS services in both phases due to the higher pricing in phase 2, It prevents some users from delaying purchase, so the price discrimination strategy based on behavior realizes the optimal profit.

5. CONCLUSION

In recent years, the product pricing strategy for SaaS providers has shown an increasing trend. SaaS providers can ensure the quality of their services through real-time updates. Users will adopt different purchase strategies according to their actual needs and the commitments of SaaS providers. Based on the relevant background, this paper constructs a two-stage game model to explore how these factors (free trial time, discount rate, etc.) will affect the pricing strategy of SaaS providers. We found that the optimal profit increases with the free trial time t and the user perception rate β before the user perception reaches the real quality of SaaS service. When SaaS providers charge by usage, in any case, the hybrid price discrimination strategy cannot make SaaS providers achieve the optimal profit, and when the transaction cost increases, the user's utility and SaaS service provider's profit will also decrease. When the transaction cost is low, SaaS providers will adopt intertemporal price discrimination strategy. With the growth of transaction cost, the profit of behavior-based price discrimination strategy is higher than intertemporal price discrimination.

The results show that enterprises should seriously consider the usage, renewal level and discount rate of SaaS providers when deciding the pricing strategy. These findings can explain why different types of SaaS providers have different pricing strategies. Wistia, a video hosting platform, allows users to host videos they publish through Wistia, which can optimize video clips and content, and promote users' videos through a series of marketing means, with similar charges per cycle. It provides users with a free package, which can upload three videos, with a traffic limit of 200g per month. Wistia adopts the intertemporal price discrimination strategy under the pricing method based on usage, because the number of videos released by users is limited and the transaction cost is low.

This paper provides theoretical support for the pricing strategy adopted by SaaS providers when providing free strategy, which is helpful for SaaS providers to obtain the optimal profit through the pricing strategy. The model in this paper mainly analyzes the pricing strategy of oligopoly SaaS providers who provide free trial service, Without considering the network effect and the competition among multiple oligopoly SaaS providers, these problems can be further studied in the future.

APPENDIX:

Proof of lemma 1

a. When $v_0 < v_1 < v_2$, the indifference points are $v_0 = \frac{\lambda(\mathrm{T}+P_{s2})}{(1+t\beta+t^2\beta)}, v_1 = \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)}, v_2 = \frac{\lambda(T+P_{s0})}{1+t\beta+t^2\beta}$. The optimal profit is

$$\Pi_{s1} = P_{s1}D_{s1} + \delta\Pi_{s2},$$

$\Pi_{s2} = P_{s0}D_{s2} + P_{s2}D_{s0} = P_{s0}(1-v_2) + P_{s2}(v_1 - v_0)$. $P_{s0} = \frac{1+t\beta+t^2\beta-T\lambda}{2\lambda}$ $P_{s2} = \frac{T-T\lambda+P_{s1}}{2\lambda}$, $\frac{\partial\Pi_1}{\partial P_{s1}} = 0$, $P_{s1} = \frac{2(1-T+t(1+t)\beta)(\lambda-\delta)}{4\lambda-3\delta}$, $P_{s2} = -\frac{T(-1+\lambda)}{2\lambda} + \frac{2(1-T+t(1+t)\beta)(-\delta+\lambda)}{-3\delta+4\lambda}$, contrast constraints $-T+T\lambda+\lambda P_{s2} < P_{s1} < -T+T\lambda+(\lambda-\delta)P_{s0}+\delta P_{s2}$, it does not satisfy , so

$$P_{s1} = -T+T\lambda+(\lambda-\delta)P_{s0}+\delta P_{s2},$$

$P_{s1} = \frac{-\delta+T\delta-t\beta\delta-t^2\beta\delta+\lambda-2T\lambda+t\beta\lambda+t^2\beta\lambda+T\lambda^2}{2\lambda-\delta}$, $P_{s0} = \frac{1+t\beta+t^2\beta-T\lambda}{2\lambda}$, $P_{s2} = \frac{-\delta-t\beta\delta-t^2\beta\delta+\lambda+t\beta\lambda+t^2\beta\lambda+T\delta\lambda-T\lambda^2}{2\lambda(-\delta+2\lambda)}$, the optimal profit is

$$\frac{(1+t(1+t)\beta-T\lambda)\left(4(1+t(1+t)\beta+T(-2+\lambda))\lambda^3+\delta^3(1+t(1+t)\beta-T\lambda)+\delta^2\lambda(-3-3t(1+t)\beta+T(-2+5\lambda))-\delta\lambda^2(1+t(1+t)\beta+T(-8+7\lambda))\right)}{4(\delta-2\lambda)^2(\lambda+t(1+t)\beta\lambda)}.$$

b. When $v_1 > v_2 > v_0$, the indifference points are $v_0 = \frac{\lambda(\mathrm{T}+P_{s2})}{(1+t\beta+t^2\beta)}$,

$v_1 = v_2 = \frac{T+T\delta-\mathrm{T}\delta+\delta P_{s0}+P_{s1}-\delta P_{s2}}{(1+t\beta+t^2\beta)} \Pi_{s2} = P_{s0}D_{s2}+P_{s2}D_{s0} = P_{s0}(1-v_1)+P_{s2}(v_1-v_0)$.

$P_{s0} = \frac{T-T\lambda+P_{s1}}{2\lambda}$, $P_{s2} = \frac{T\delta-\lambda+T\lambda-t\beta\lambda-t^2\beta\lambda-T\delta\lambda+\delta P_{s1}+\lambda P_{s1}}{2\delta\lambda}$, and $\frac{\partial\Pi_2}{\partial P_{s1}} = 0$. it does not satisfy the constraint. So $P_{s1} = -T+T\lambda-\delta P_{s0}+\lambda P_{s0}+\delta P_{s2}$, At this time, it is consistent with a.

Proof of lemma 2

a. When $v_1 \le v_0$,the indifference points are $v_2 = v_0 = \frac{\lambda(T+P_{s2})}{1+t\beta+t^2\beta}$, $v_1 = \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)}$,the optimal profit is $\Pi_{s2} = P_{s2}\left(1-\frac{\lambda(T+P_{s2})}{1+t\beta+t^2\beta}\right)$, $P_{s2} = \frac{1+t\beta+t^2\beta-T\lambda}{2\lambda}$,Optimal pricing can be obtained by derivation $P_{s1} = \frac{1}{4}(T(-2+\delta)+\frac{(1+t(1+t)\beta)(2\lambda-\delta)}{\lambda}$,Consider constraints $P_{s1} = \lambda P_{s2}$, when $T < \frac{\delta+t\beta\delta+t^2\beta\delta}{2\lambda+\lambda\delta-2\lambda^2}$ .it satisfies,

$$\Pi_{s1} = \frac{4(1-T+t(1+t)\beta)^2\lambda^2-4T\delta(-1+\lambda)\lambda(1+t(1+t)\beta-T\lambda)-3\delta^2(1+t(1+t)\beta-T\lambda)^2}{16(1+t(1+t)\beta)(\lambda-\delta)\lambda}.$$

When $T > \frac{\delta+t\beta\delta+t^2\beta\delta}{2\lambda+\lambda\delta-2\lambda^2}, P_{s1} = -T+T\lambda+\lambda P_{s2}$, $P_{s2} = \frac{1+t\beta+t^2\beta-T\lambda}{2\lambda}$, $P_{s1} = \frac{1}{2}(1+t(1+t)\beta+T(-2+\lambda))$. The optimal profit is $\Pi_{s2} = \frac{(1+t(1+t)\beta-T\lambda)(\delta+t(1+t)\beta\delta+\lambda+t(1+t)\beta\lambda-T\delta\lambda+T(-2+\lambda)\lambda)}{4(\lambda+t(1+t)\beta\lambda)}$.

b. When $v_1 > v_0$, the indifference points are $v_2 = v_0 = \frac{\lambda(T+P_{s2})}{1+t\beta+t^2\beta}$, $v_1 = \frac{\lambda(T-T\delta+P_{s1}-\delta P_{s2})}{(1+t\beta+t^2\beta)(\lambda-\delta)}$,At this time, the market equilibrium result is consistent with the above, so we won't repeat it again.

Proof of lemma 3

a. When $v_1 \ge v_2$, the indifference points are $v_0 = \frac{\lambda(T+P_{s1})}{1+t\beta+t^2\beta}$, $v_1 = v_2 = \frac{T+\delta P_{s0}+P_{s1}-\delta P_{s1}}{1+t\beta+t^2\beta}$,the optimal profit is $\Pi_{s1} = P_{s1}D_{s1}+\delta\Pi_{s2}$,Optimal pricing can be obtained by derivation $P_{s0} = \frac{1-T+t\beta+t^2\beta-P_{s1}+2\delta P_{s1}}{2\delta}$,, $P_{s0} = \frac{-T(1+2\delta(1+2\delta)\lambda)+(1+t(1+t)\beta)(1+2\delta(-1+2\delta+2\lambda))}{2\delta(1+4\delta\lambda)}$. Consider constraints, if and only if $\frac{1}{4}(-1-2\lambda+4\lambda^2)+\frac{1}{4}\sqrt{1+12\lambda-4\lambda^2-16\lambda^3+16\lambda^4} < \delta$ or $0 < \delta < \frac{1}{4}(-1-2\lambda+4\lambda^2)+\frac{1}{4}\sqrt{1+12\lambda-4\lambda^2-16\lambda^3+16\lambda^4}$ and

$$\frac{\delta+t\beta\delta+t^2\beta\delta+2\delta^2+2t\beta\delta^2+2t^2\beta\delta^2-\lambda-t\beta\lambda-t^2\beta\lambda+2\delta\lambda+2t\beta\delta\lambda+2t^2\beta\delta\lambda-4\delta\lambda^2-4t\beta\delta\lambda^2-4t^2\beta\delta\lambda^2}{-\delta-\lambda+2\delta\lambda-2\delta^2\lambda-2\delta\lambda^2+4\delta^2\lambda^2} < T < 1,$$

Meet the conditions, $\Pi_1 = \frac{(1-T+t(1+t)\beta)^2+\frac{4\delta^2(1+t(1+t)\beta-T\lambda)^2}{1+4\delta\lambda}}{4+4t(1+t)\beta}$. It does not satisfy, and $P_{s1} = \frac{T(\lambda-1)+(\lambda-\delta)P_{s0}}{1-\delta}$, $\frac{\partial\Pi_2}{\partial P_{s0}} = 0$, $P_{s0} = \frac{-1+t(1+t)\beta(-1+\delta)+\delta+T\delta+T\lambda-2T\delta\lambda}{-\lambda+\delta(-1+2\lambda)}$, $P_{s1} = \frac{(-1+T-t(1+t)\beta)\lambda+\delta(1+T+t(1+t)\beta-2T\lambda)}{-\lambda+\delta(-1+2\lambda)}$,

$$\Pi_2 = \frac{\delta(-1+\lambda)\left(-(1+t(1+t)\beta)(\delta^2+\delta\lambda-\lambda(1+\lambda))+T(1+\delta+\lambda)(-\lambda+\delta(-1+2\lambda))\right)}{(\delta+\lambda-2\delta\lambda)^2}.$$

b. When $v_1 < v_2$, the indifference points are $v_0 = \frac{\lambda(T+P_{s1})}{1+t\beta+t^2\beta}$. , $v_1 = \frac{(1-\delta)\lambda(T+P_{s1})}{(1+t(1+t)\beta)(\lambda-\delta)}$, $v_2 = \frac{\lambda(T+P_{s0})}{1+t\beta+t^2\beta}$. the optimal profit is $\Pi_{s2} = P_{s1}D_{s2} + P_{s0}D_{s0}$, $P_{s0} = \frac{1+t\beta+t^2\beta-T\lambda}{2\lambda}$, $\frac{\lambda-T\lambda+t(1+t)\beta\lambda-\delta\left(1+t(1+t)\beta+T(-2+\lambda)\lambda\right)}{2\left(\lambda+\delta(-2+\lambda)\lambda\right)}$,it does not satisfy. So $P_{s1} = \frac{T(\lambda-1)+(\lambda-\delta)P_{s0}}{1-\delta}$, At this time, the equilibrium result is the same as a.

Proof of lemma 4

When $T < \frac{\delta+t\beta\delta+t^2\beta\delta}{2\lambda+\lambda\delta-2\lambda^2}$,the profit of SaaS providers $\Pi_h - \Pi_c < 0$ .When $T < \frac{\delta+t\beta\delta+t^2\beta\delta}{2\lambda+\lambda\delta-2\lambda^2}$, $\Pi_h - \Pi_c =< 0$.

We can find that, SaaS providers adopt hybrid price discrimination strategy, and their profits are lower than intertemporal price discrimination strategy at any time.

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