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# Review of Research on Markowitz Model in Portfolios

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**ABSTRACT**
In modern portfolio theory, the Markowitz model is also known as the mean-variance model, which is the core of modern portfolio theory. The Markowitz model explains well the process of calculating the optimal portfolio solution and its principles. In addition to this, it is clear that the model also describes various situations in a portfolio by constructing the horizontal axis as the variance and the vertical axis as the mean. It is also explained in the Markowitz model that this uses the variance to represent the risk of the portfolio and the mean to represent the expected return of the portfolio. This paper aims to analyze the study of the Markowitz model in the context of investment portfolios. Through analysis, it is found that Markowitz though includes advantages like the ones just mentioned, the model may still have some disadvantages as well as limitations. For example, the model is only valid if the financial markets are strong-form efficient. Further, strong-form efficiency implies that financial markets are free from insider information and irrational investors and investment behavior. Finally, based on this model, the paper will conclude with future perspectives and related recommendations.

***Keywords:*** *Markowitz Models, Mean-variance Models, Optimal Combinations, Efficient Frontier*

## 1. INTRODUCTION

The Markowitz model, which can also be defined as a mean-variance model, is at the heart of portfolio theory. Although central to many financial valuation models in modern portfolio theory, the Markowitz model is nevertheless often challenged by scholars. For example, the preconditions of the Markowitz model include strong efficient ground for financial markets in the efficient market hypothesis. Insider trading and irrational investment behavior in real financial markets can disrupt otherwise stable markets, thus making the Markowitz model not as useful as described in the theory. Therefore, this paper will study and summarize the Markowitz model through previous academic research on the model.

In this paper, previous scholarly research will be presented and reviewed firstly, Besides, it will illustrate the practical application and implications of the Markowitz model in the context of investment portfolios. Finally, this paper will summarize the strengths and limitations of Markowitz and discuss them, concluding with some suggestions for the future of the model. Through analysis, this paper will improve the Markowitz model by analysing the model itself, and help people understand the implications of the Markowitz model for the future of modern investment theory.

## 2. INTRODUCTION TO THE MARKOWITZ MODEL

Regarding the mean-variance model, specifically, the mean and variance represent the expected return and risk of a portfolio respectively. By adjusting the weights between different securities in a portfolio, different means and variances can be calculated. In addition, the portfolios with different weights can be represented graphically, forming an oval scatter chart. In addition, Markowitz defines one of the lines as the efficient frontier. The efficient frontier can be defined as the set of various portfolio scenarios under which maximum returns can be achieved with minimum risk under a return-risk constraint. As shown in Figure 1, the vertical axis represents the expected rate of return, and the horizontal axis represents the Standard Deviation( $\sigma$ ). At the same time, the standard deviation will be used here to represent the risk of the portfolio. Specifically, the risk here refers mainly to non-systematic risk, and since the model is analyzed under the premise that the market is strong form efficiency, systematic risk is neglected. The curve exhibits a monotonically increasing outwardly convex shape, which is not possible for portfolios located to the left of the efficient frontier and, under the same circumstances, portfolios on the right are not as efficient compared to those on the efficient frontier. Specifically, a portfolio on the efficient frontier has a higher expected return for the same risk compared to a portfolio on the right-hand side. Similarly, portfolios on the right-hand side of the efficient frontier are not less risky than portfolios on the efficient frontier for the same expected return.

In addition to this, the leftmost point on the efficient frontier has the smallest variance, the smallest risk, and this point is known as the minimum variance point (MVP). Secondly, regarding Markowitz's hypothetical preconditions. As mentioned earlier, the model holds when financial markets are strongly efficient. Specifically, strong efficiency comes from one of the cases of the efficient market hypothesis. The efficient market hypothesis classifies markets into three forms, the first being strongly efficient, the second being semi-strongly efficient and the third being weakly efficient. However, financial markets are generally weakly or semi-strongly efficient because there are still many abnormal trading behaviors in financial markets that cannot be fully explained by theory. This situation can be explained by the following points. The first point is the existence of factors such as insider trading in financial markets, which affects the stability of financial markets. The second point is that irrational investors cause financial markets to move towards trends that cannot be determined by theory. Thirdly, there is a herding effect among investors. This effect causes rational investors to make wrong judgments and consequently wrong investment transactions due to the trading of lots of irrational investors, resulting in anomalies in the financial markets.

It is important to note that the Markowitz theory is based on the idea that with the optimal results of the model, the investor does not need to make subsequent profits, but only needs to buy the portfolio according to the calculated portfolio weights, and ideally, the expected return will be close to the calculated results. In addition to this, the Markowitz model promotes diversification, which allows for a less risky portfolio. Furthermore, when the investor sets the constraint that the downward convexity of the undifferentiated curve and the upward convexity of the efficient curve make the most portfolio is unique, it is the intersection of the two curves which is unique.

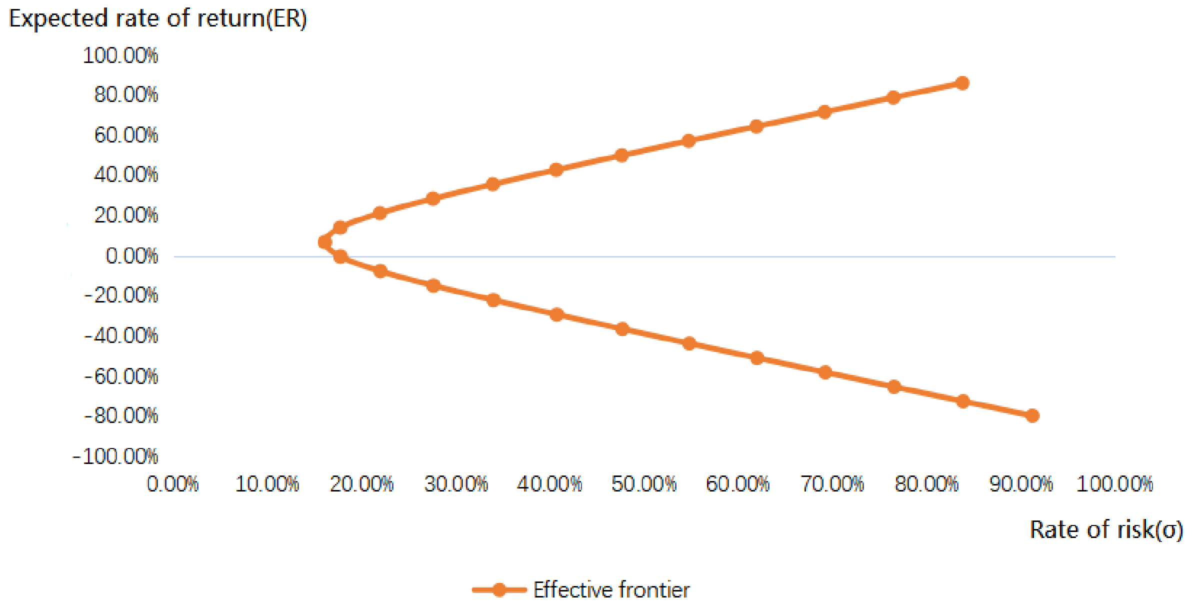


**Figure 1.** Mean-variance models

## 3. REVIEW OF PREVIOUS STUDIES

Many scholars have given different views about Markowitz's theory. Since Markowitz proposed the theory in 1952, Markowitz's mean-variance model has been one of the most important research topics in portfolios, and this also created the optimal portfolio in modern portfolio theory.

Zymler et al [1] state that portfolios need to identify constraints to address uncertainty in investment. However, those constraints are established if they satisfy investor preferences, such as expected return, minimum risk and accuracy.

Leung et al [2] demonstrate that the Markowitz model can be used to value a portfolio. Leung et al [2] suggest that the bias(the difference between the predicted and actual number of values from the model) may be due to the large sample size and data overload. Further, the predicted outcome of the expected return is usually higher than the actual outcome. In addition, the model does not have a closed-form valuation of the portfolio, which means that investors must add constraints according to their investment preferences. However, differences in constraints may also lead to different results when applying the mean-variance model. For example, when stocks can be shorted and when they cannot be shorted. What is clear is that the weightings calculated for the portfolio are usually different. Therefore, if the investor is not able to choose the constraints reasonably well, this may lead to inaccurate calculations and even poor decisions. Specifically, when there are too many samples and data, the complexity of the calculation leads to inaccurate calculations. This is also demonstrated by Witt and Dobbins [3], who point out that the Markowitz model is not able to address the situation when there are too many data and samples through empirical research. And when there are too many samples and data, the Markowitz model is unable to accurately calculate the expected return and the associated risk.

At the same time, it is also argued that individual asset returns do not correspond to variance. You and Daigler [4] point out that individual asset returns are driven by time, not risk, and that this accounts for the bias in the model's forecasts. This view is supported by Briec and Kerstens [5], who go further and argue that time value(the effect of time on some asset or some currency with respect to value) has some influence on the prediction results of the Markowitz model. It is clear that inflation or deflation is a common phenomenon in financial markets, both of which result in assets becoming more or less valuable over time. For example, if you have a hundred dollars in a bank account, a year later you may have a hundred dollars in your bank account and the interest on that hundred dollars. In general, the value of an asset changes over time. Also, the more pronounced the change in time, the more pronounced the change in the value of the asset that time acts upon, which is the time value.If the time value factor could be added to the Markowitz model, then the accuracy of the model could potentially be improved.

## 4. POTENTIAL PROBLEMS IN THE PRACTICAL APPLICATION OF THE MARKOWITZ MODEL

As for the practical application of the Markowitz model, firstly, it is worth noting that different constraints may cause the Markowitz model to predict different results. For example, when the weighting in the constrained portfolio is non-short, it is likely that a small proportion of the stocks will be heavily weighted, while the rest of the securities predicted by the Markowitz model to be less profitable will be as underweighted as possible. Therefore, the mean-variance model requires different constraints depending on the circumstances of the portfolio. In addition, as defined by Markowitz theory, when a portfolio is subject to a non-shorting constraint, it is possible that the portfolio may not appear on the efficient frontier in the Markowitz model, even though it has been presented with an optimal portfolio relative to that portfolio. As a result, the model may not have good utility in practical applications in financial decision-making.

Further, as mentioned before. Constraints may affect the accuracy of the prediction results by using the Markowitz model. However, in practical applications, the choice of constraints is usually complex when an investor uses the Markowitz model for portfolio analysis and the investor may affect the accuracy of the portfolio's prediction results by choosing the wrong constraints. Secondly, regarding the expected return. According to Markowitz theory, if the Markowitz model is used to analyze the expected rate of return of a portfolio, the model is likely to exhibit extremely large expected rates of return, which may be unrealistic. In detail, Markowitz's theory allows for a very large expected return, however, a very large expected return also means that the investor using the portfolio has to take a correspondingly large amount of risk. As a result, even though Markowitz’s theory demonstrates the right idea, the model is usually not as restrictive. Further, although the less restrictive aspects of Markowitz’s theory are applicable to different securities. However, at the same time this means that when investors have different portfolios, the theory is likely to fail to predict more accurate results, except when the investor adds the right constraints and the right optimizations. That is the constraint that makes the combination more realistic. For example, specifying that the return and the standard deviation representing the risk capacity are in the investor's risk appetite case, or that the Sharpe ratio is greater than 0. Specifically, the Sharpe ratio is equal to the expected return minus the risk-free rate, and then this difference is divided by the standard deviation. If the expected return is less than the risk-free rate, the portfolio is meaningless, so a Sharpe ratio greater than 0 is generally a required constraint

## 5. ADVANTAGES AND LIMITATIONS OF MARKOWITZ MODEL

### *5.1. Advantages of Markowitz model*

Markowitz's theory marked the beginning of modern portfolio theory. It is worth noting that the mean-variance model is valid in practice, which is the first advantage of Markowitz's theory. Markowitz [6] demonstrated that the mean-variance model is effective in reducing risk. Thus, Konno and Suzuki [7] argue that this may be due to the fact that the theoretical mean-variance analysis assumes that data returns are normal, and these scholars demonstrate the utility of the model in practical applications where returns are distributed asymmetrically. A second advantage is that, as noted earlier, Markowitz's theory is applicable to a wide range of portfolio situations. In detail, Markowitz's theory is less restrictive and can therefore be applied to different portfolios. By setting different constraints and optimizing the model in different ways, the model can be applied to different portfolio situations. A third advantage is that the model can clearly show the mean and variance of a portfolio under different scenarios, giving investors a clearer picture of their portfolios.

### *5.2. Limitations of Markowitz model*

This model also exists some limitations. Firstly, the mean-variance model does not apply to non-normally distributed data in real life. As a result, the predictions of the Markowitz model are somewhat skewed. This is because, in real financial markets, the sample data of portfolios are often not non-normally distributed. Jondeau and Rockinger [8] demonstrate that the portfolio of returns in the model exhibits non-normal distribution and time-varying characteristics. Furthermore, they demonstrate that the accuracy of the theory is significantly improved when optimizing for these cases. Secondly, Markowitz's theory does not consider the time discounting of the portfolio. Specifically, Markowitz's theory is unable to predict the correct time discounted value of the portfolio and it is clear that the theory does not take this factor into account. In addition, this factor contributes to the theory's inability to accurately predict the portfolio. Briec, Kerstens [5] states that if time discounting is added to the mean-variance model, the efficiency of the model in the practical application will be improved. The third point is that the model is based on the premise that financial markets should be strong form efficient. However, most current financial markets are weakly formally efficient, as most of them are still dominated by irrational investors and insider information. These irrational investment behaviors also lead to irrational directions in financial markets. Obviously, the mean-variance model does not take these circumstances into account.

## 6. FUTURE OUTLOOK AND RECOMMENDATIONS

Because of the shortcomings and limitations mentioned before, the model needs to be optimized for these drawbacks and limitations. Therefore, the author presents the outlook and recommendations for the future. Since the constraints need to be chosen by the investor depending on the different portfolios, the first thing to note is that the model needs to use the constraints correctly depending on the different portfolios. For example, in a high-return, low-risk portfolio, the investor can relax the risk constraint to further increase returns. Leung et al [2] demonstrate that if some constraint improvements are added to the constraints that apply to different portfolios, this can serve the purpose of improving the accuracy of the model. For example, the addition of Markowitz model constraints, transaction costs, liquidity constraints, turnover constraints and budget constraints to the constraints would improve the accuracy of the model's prediction results. Bai et al [9] argue that Markowitz may be more applicable to high-return, low-risk portfolios, and in such portfolios, this means that the accuracy of the model will be improved. Furthermore, the model's forecasts are biased, and forecasts of expected returns should be reduced in the portfolio. Bai et al [9] demonstrate that this bias affects the accuracy of forecasts when using mean-variance models.

Secondly, the model needs to consider the unusual trading behavior of financial markets and the mean-variance model needs to be suitably improved by behavioral finance theory. Specifically, the expected return of a portfolio in a truly weak form efficient financial market should be appropriately reduced compared to a strong form efficient financial market, and appropriately adjusted for abnormal trading behavior in financial markets.

Thirdly, to address the forecast bias caused by large sample data and the complexity of the calculation process, some scholars argue that the weights among securities in the portfolio should be appropriately adjusted and controlled, which can lead to more diversification and risk reduction. Bai et al [10] point out that there is a Y-fold deviation from the theory, and if the forecast results are optimized by this method, it can improve the accuracy of Markowitz the model. Therefore, investors should adjust the Markowitz model according to different portfolio situations, and the portfolio should try to satisfy the preconditions of Markowitz theory to make the prediction results more accurate. In addition to this, Yu et al [11] proposed a new optimization method, these scholars converted the single cycle valuation in the Markowitz model into a multi-cycle valuation, which can control the risk more precisely. In addition, the concept of maximum absolute deviation has been added as an optimization method to the Markowitz model.

## 7. CONCLUSION

Through analysis, the author finds that although Markowitz model is applicable to a wide range of portfolio situations, it still has several flaws. For example, the wrong choice of constraints may lead to inaccurate forecasting results, as well as the fact that the model applies to strong form efficient rather than weak form efficient in real financial markets. In addition, the Markowitz model usually predicts higher results than those in real financial markets regarding expected returns. The paper also gives some suggestions, such as controlling the proportion of investments between individual securities in a portfolio when choosing to use the Markowitz model, which can reduce risk and achieve risk diversification. Secondly, regarding the model not being applicable to the actual financial market sample belonging to a situation that does not conform to a normal distribution, appropriate adjustments need to be made to the calculation method of the model. For example, Bai et al [10] demonstrate that there is a Y-fold deviation between realistic results and theoretical estimates, and this deviation can be applied to the Markowitz model to eliminate this error for the purpose of improving accuracy. This paper does not use empirical evidence to prove the ideas in that paper, so in the future study, the author will use empirical evidence to further demonstrate the problems with the Markowitz model in practical applications.

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