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The Application of Markowitz Model and Index Model on Portfolio Optimization

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ABSTRACT
With the progress of the global economy and science and technology, as well as the development of stock portfolio theory, sophisticated investors are considering return and risk management as their priority. In this article, we analyze different stocks from 10 renowned companies, representing the risk-free rate and the one-month federal funds rate. We calculated all the appropriate optimization inputs for the entire Markowitz model and the Index model, which means that we will need five different additional constraints on the allowed portfolio areas. For the output, we present the report as a combination of tables and graphs to conduct comparison between the constraint sets for each optimization problem and between the two models. By the same token, the final objective for this report would set in paving the way for future portfolio optimization and lay the theoretical foundation for investors' future choices.

Keywords: *Index model, Markowitz model, risk avoidance, Portfolio Optimization.*

1. INTRODUCTION

Portfolio theory can be divided into two dimensions. It can be simply recognize as the model created by Markowitz, as well as Index model (a innovation stem from previous effort). In this paper, we utilize mathematical analysis methods to apply Markowitz Mean-Variance Analysis Model (MM) and Index Model (IM) in the sector of stock in U.S which help to prove investors' returns and reduce investment risks. After all, the introduction of companies which includes data analysis of them, the constraints under MM and IM model as well as analysis of the results and a conclusion would be all involved in the following elaborate report analysis.

The foundations of Modern Portfolio Theory ("MPT") were established by Harry Markowitz in his 1952 PhD thesis in statistics. [1]. The conclusions of his dissertation, entitled "Portfolio Selection" [2], were first published in the Journal of Finance. Subsequently, these findings were greatly expanded with the publication of his book, "Portfolio Selection. Effective Diversification [3]. In 1958, economist James Tobin published an article in the Review of Economic Studies, "Liquidity Preference as Risk Behavior," which derived the concepts of "efficient frontier" and "capital market line" based on the work of Markowitz. [4] Tobin's model suggests that market investors, regardless of their risk tolerance, will maintain the same proportion of their stock portfolio as long as they "maintain the same expectations about the future."[5] Sharp [6] significantly advanced the concept of efficient frontier and capital market line in the derivation of CAPM. A year later, Lintner[7] derived the CAPM from the perspective of a firm issuing stocks.

Technically, modern portfolio theory ("MPT") consists of Markowitz's portfolio selection theory and William Sharpe's contribution to the theory of financial asset price formation, later known as the capital asset pricing model ("CAPM") [8]. In essence, MPT is an investment framework for selecting and constructing portfolios based on maximizing expected portfolio returns and minimizing simultaneous investment risk [9]. Diversification is actually the core concept of MPT and directly relies on the conventional wisdom of "never put all your eggs in one basket" [10].

The purpose of this article is to consider the allowable portfolio boundaries under different circumstances, taking into account the available historical data. For the research methods, we mainly analyze MM and IM. We conducted in-depth analysis through two models through 5 constraint conditions. Besides, we combined figures and tables for a better comparison of the two models.

For the overall arrangement, we first introduce the overall situation of 10 companies and conduct data analysis in section 2. Then, section 3 is the introduction

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of our method (MM &IM) with formulas and consumption involved, besides, constraints are also included in this part. In section 4 we conduct result analysis based on two models (MM&IM) and make the comparison within them. Finally, for the conclusion, we present an overview of our findings and summarize the shortcomings of this paper for future development

2. FIRM DESCRIPTION

In this section, 10 various corporations are included for a elaborate analysis on their background, stock price change within a certain period of time, so as conducting more specific data analysis on its risk & return.

2.1 Background information of 10 companies

NVIDIA

NVIDIA (NASDAQ: NVDA) is a world-renowned leading technology group that produces sound and graphics cards and various integrated parts, and the company is also a leader in artificial intelligence and the development of modern technology.

Cisco Systems, Inc.;

Cisco Systems (NASDAQ: CSCO) is a specialist in the provision of networking solutions. As an American company, Cisco Group has strong economic and technical capabilities and has been ranked among the top 500 companies in the world for many years.

Intel Corporation

Intel Corporation (NASDAQ: INTC) a global leader in the semiconductor industry and computing innovation, is now striving to shift to a data-centric company with a mission to drive a smart, connected world

Goldman Sachs

Goldman Sachs, (NASDAQ: GS) Founded in 1869, Goldman Sachs Group is one of the world's most established investment banks, headquartered in New York.

Us Bancorp

Us Bancorp (NASDAQ: USB) is a financial services holding company, ranked as the 5th largest commercial bank in the United States.

Td Bank

The TD Banking (NASDAQ: TD CN) ranks among the top online financial services companies in the world and will be Canada's most valuable brand by 2020.

Allstate

Allstate, (NASDAQ: ALL) a leading corporation in the personal lines business and has grown to become the third most recognized life insurance company.

Procter & Gamble

Founded in 1837, P&G (NASDAQ: PG) is one of the world's leading household commodity companies and the tenth most highly regarded company in the Fortune 500.

Johnson & Johnson

Johnson & Johnson (NASDAQ: JNJ) is the world's leading and most diversified healthcare and consumer care products company.

Colgate-Palolive

Colgate-palmolive (NASDAQ:CL), is a renowned brand in daily hygiene, and is traditionally strong in areas such as oral care and personal care.

2.2 Figure Analysis

2.2.1 Stock Price of 10 Companies

As the chart above shows, the share prices of most companies are concentrated below 10 and remain stable, slowly rising. The only difference, however, is the orange curve, which represents Nvidia, which is not only highly volatile but also far more volatile than other companies, able to reach over 50.

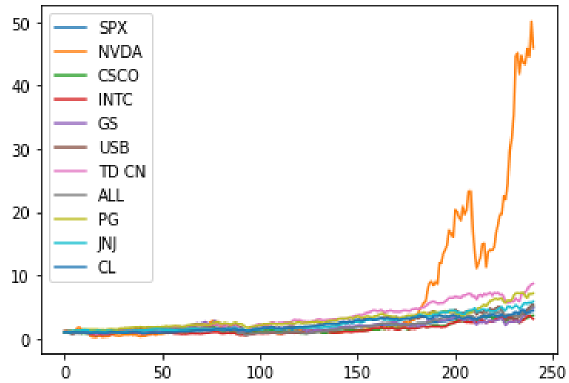

Figure 1: Stock Price of 10 Companies

2.2.2 Various Data Analysis

After calculating the stock price, we fully analyzed several related indicators, they are annualized average return, annualized StDev, beta, annualized alpha, annualized residual StDev. The maximum for each of these components are all Nvidia. For the minimum, in order, they are CL(7.105%), PG(14.587%), PG(0.405118), INTC(-0.00052), JNJ(12.423%) respectively.

2.2.3 Correlation Coefficient of 10 Companies

The correlation of each group as the below table 3 shows the degree of connection between the two companies, and again we yellow the maximum and minimum values.

The Goldman Sachs Group and SPX have the highest correlation coefficient.(0.708092) Intel Corporation and Colgate-Palmolive Company have the lowest correlation coefficient.(0.110064)

Table 1. Various Data Analysis of 10 Companies

	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
Annualized Average Return	7.54%	32.80%	9.71%	8.91%	10.83%	9.88%	11.01%	10.08%	9.44%	8.46%	7.11%
Annuazlied StDev	14.85 %	55.77%	30.81%	30.50%	29.57%	23.68%	18.13%	24.88%	14.59%	14.79%	15.35%
Beta	1.000	1.9788	1.3206	1.1875	1.4100	0.9712	0.7870	1.0562	0.4051	0.5398	0.4544
Annuazlied Alpha	0.000	0.1788	-0.0025	-0.0005	0.0019	0.0255	0.0507	0.0211	0.0638	0.0439	0.0368
Annuazlied Residual StDev	0.00%	47.41%	23.76%	24.89%	20.88%	18.78%	13.87%	19.32%	13.29%	12.42%	13.79%

Table 2. Correlation Coefficient of 10 Companies

Correlation	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
SPX	1.0000	0.5269	0.6365	0.5781	0.7081	0.6091	0.6445	0.6304	0.4124	0.5422	0.4396
NVDA	0.5269	1.0000	0.4872	0.5238	0.3431	0.1598	0.3380	0.1569	0.0596	0.1653	0.0695
CSCO	0.6365	0.4872	1.0000	0.6142	0.4875	0.3281	0.4101	0.2973	0.2202	0.2388	0.1650
INTC	0.5781	0.5238	0.6142	1.0000	0.4107	0.2796	0.4115	0.2857	0.1364	0.3249	0.1101
GS	0.7081	0.3431	0.4875	0.4107	1.0000	0.4717	0.4938	0.4174	0.1731	0.2955	0.2031
USB	0.6091	0.1598	0.3281	0.2796	0.4717	1.0000	0.5392	0.5401	0.3359	0.2341	0.2178
TD CN	0.6445	0.3380	0.4101	0.4115	0.4938	0.5392	1.0000	0.4167	0.2310	0.2727	0.2117
ALL	0.6304	0.1569	0.2973	0.2857	0.4174	0.5401	0.4167	1.0000	0.3463	0.4518	0.4066
PG	0.4124	0.0596	0.2202	0.1364	0.1731	0.3359	0.2310	0.3463	1.0000	0.4937	0.4833
JNJ	0.5422	0.1653	0.2388	0.3249	0.2955	0.2341	0.2727	0.4518	0.4937	1.0000	0.5268
CL	0.4396	0.0695	0.1650	0.1101	0.2031	0.2178	0.2117	0.4066	0.4833	0.5268	1.0000

3. METHOD

In this section, I will introduce the Markowitz model and the exponential model, respectively, from the history of the two models, the presuppositions, the formulas, and the restriction intervals in detail. At the same time, I will take a comparative approach to the presentation of the two models to more objectively evaluate the two portfolio models.

3.1. Markowitz Mean-Variance model

When an investor is looking for the best risk-return combination among possible portfolios, two steps are required.We need to perform a comprehensive reasoning analysis combining target return, risk and investor's investment preferences, with the aim of harvesting the optimal portfolio, i.e., the maximum return with the minimum risk.

3.1.1.Assumptions in Markowitz Model

(1)The securities market is efficient

(2)Investors are risk-averse

(3)Investors are insatiable

(4)Multiple securities returns are correlated

3.1.2. Formulas

After introducing the four underlying assumptions, we will further analyze the formulas, the following two formulas can adequately calculate the average return as well as the variance of the investors.

$$Mean: r_p = \sum x_i * r_i$$

For this formula, it demonstrate the relationship within the total return and various return of stocks within the whole portfolio.

$$Variance: min\sigma^2(r_p) = \sum\sum x_i * r_i * Cov(r_i, \ r_j)$$

This formula reveals the relationship among the variance and covariance between each stock.

Its practical significance resides in the fact that the investor can ascertain the expected return in advance and autonomous allocation of the amount or proportion of investments in stocks with different risks and returns, with different minimum variance combinations for different expected returns, and these combinations constitute the minimum variance set.

3.2. Index Model

The single index model (SIM), also known as the index model, is a simple asset pricing model commonly used in the financial industry to assess the risk and return of stocks. The single index model can greatly simplify the calculation process, which is a drawback of the Markowitz model, and has demonstrated that this kind of portfolio analysis model is not inferior to the Markowitz model in terms of scientific accuracy and precision.

3.2.1. Assumptions in Index Model

This is a method that facilitates the analysis by assuming that there is only one macro factor such as market returns that affects the return risk of the entire equity portfolio. The following formulas are also based on this assumption. In fact it is a systematic risk such as changes in important national indicators, international policy changes, or major disasters or wars, while the non-systematic risk is by default diversifiable or insignificant through the portfolio. Most stocks have positive covariance and Professor Sharp introduced the coefficient beta to reflect the magnitude of sensitivity. This total sensitivity, or covariance, is simply the concatenation of the beta coefficients of the different stocks.

3.2.2. Formulas

$$r_{it} - r_f = \alpha_i + \beta_i(r_{mt} - r_f) + \varepsilon_{it}$$

This formula reflects the relationship between the composite return of a security and a single index. The left-hand formula reflects the difference between the security's return and the risk-free return, while the right-hand formula represents the degree of volatility with beta as the slope plus alpha as the intercept (independent of market volatility) and the total containing the residuals.

3.3. Constraint

In this simulation analysis, we set five different constraints for each model. By utilizing these five constraints, we can simulate the most realistic policies and regulations in economic markets and corporations around the world.

1. This additional optimization constraint is designed to allows broker-dealers to permit their customers to have positions, 50% or more of which are funded by the customer’s account equity:

$$\sum_{i=1}^{11} |w_i| \le 2$$

2. This additional optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which may be provided by the client

$$|w_i| \le 1, \text{for } \forall i$$

3. A “free” problem, without any additional optimization constraints.

4. This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry

$$w_i \ge 0, \text{for } \forall i$$

5. Lastly, we would like to consider an additional optimization constraint for broad index:

$$w_1 = 0$$

4. RESULTS ANALYSIS

In this case, we must first convert the data to obtain some useful data from numerous numbers. Excess returns and surpluses help us solve further problems. So, for each stock, we implicitly use the current month's number minus the previous month's number to calculate the return for each stock. Then we need an indicator called NRFR.

Following with NRFR, we still need to get NRFR increase rate by this formula. In this formula, risk- free rate is equivalent to the federal rate of n+1 divided by the federal rate of n which comprise of a coefficient and then minus one.

$$\Delta NRFR = (FEDR01_{n+1}/FEDR01_n) - 1$$

Now we can take the NRFR per month minus the return per month to get the excess return. Then we took the average return and standard deviation of each stock for later analysis. Covariance is also needed. Therefore, we make a correlation matrix to construct the co-variances of all stocks. We separately weighted 10 different stocks for both models as well as the market returns and the results are shown in Table 3 below

From Table 4, we can roughly see that the Sharpe ratio, return, and standard deviation obtained by the two models are approximately the same, which lays the groundwork for a more in-depth analysis later on .

Table 3. Weights of SPX index

	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
Weight	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.5

Table 4. Results in MM & IM

Return	StDev	Sharpe	Return	StDev	Sharpe
9.49%	13.40%	0.707750199	9.49%	13.70%	0.692340779

Table 5. Minimum variance portfolio and maximum shape portfolio

Stock	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
Max Sharpe	-0.3045	1.2893	-0.0120	-0.0014	-0.0687	-0.0530	0.0114	-0.0584	0.1993	-0.0021	0.0000
Minimal Variance Frontier	0.0949	0.0000	0.0000	0.0000	0.0000	0.0000	0.1985	0.0000	0.2891	0.2062	0.2113
ABS	0.3045	1.2893	0.0120	0.0014	0.0687	0.0530	0.0114	0.0584	0.1993	0.0021	0.0000

Table 6. Results in MM & IM

Markowitz Model			Index Model		
Return	StDev	Sharpe	Return	StDev	Sharpe
40.00%	68.75%	0.581789125	40.00%	68.75%	0.581789768
8.88%	11.27%	0.787806394	8.88%	10.51%	0.844590952

Then, we need to figure out the minimum variance combination and the maximum shape combination. The results obtained from the Markowitz model's portfolio and the Index model's portfolio are shown in Table 5 and Table 6. In addition, the point dispersion range of the portfolio in the two models is almost the same under different constraints.

Constraint 1: The minimum variance combinations of the two models are almost identical. Other factors are all indicating a large figure for Markowitz model than Index model.

Constraint 2: The minimum value of the minimum variance frontier of the Markowitz model is smaller than that of the Index model. Minimum variance portfolio, the effective frontier and minimum profit frontier of the two models are almost the same. The figures of other two ingredients are more in Markowitz than in index model.

Constraint 3: The maximum value of Cal lines in the Markowitz model is greater than that in the Index model. While other indicators are relatively same for both models.

Constraint 4: The maximum value of Cal lines in the Markowitz model is greater than that in the Index model. The maximum Sharpe portfolio of the Markowitz model is larger than that of the index model. While other factors reveals a relatively same figure.

Constraint 5: all these indicators are almost the same in the two models.

The minimum variance combination and maximum Sharpe combination on the effective frontier are calculated by using the five constraints of the two models. We find that both the model estimate the risk and return of 10 stocks with fairly low accuracy. The five minimum variance boundaries of the two models are shaped like bullets.

5. CONCLUSION

In this paper, we want to use data analysis to get a portfolio. In the beginning, we collected the data of 10 stocks and analyzed their share prices and correlation, supplemented by charts for more explicit analysis. Then, we use tools in Excel for data analysis and chart drawing of MM model and exponential model respectively. After obtaining two models with five constraints, we compare their similarities and differences. Finally, we came up with what we thought was the optimal portfolio.

In general, the two models draw nearly identical graphs for investors to determine the portfolio frontier of minimum variance, usually stocks and an SPX index. This means that back into reality, if an investor wants to make an investment, whichever method he chooses, it is perfectly adaptable, which fully validates our assumption that the two methods of analysis are approximately the same. In conclusion, For ordinary investors like us rather than professional investment scientists, a single index model seems to be more suitable for us. The simplification of covariance calculation helps to reduce the need for estimators in exponential models.

This portfolio analysis project has two main drawbacks. Firstly, after experimenting and evaluating the data, I have only proposed a basic portfolio. It can indeed be used as a reference for investment in my opinion. However, I should have compiled more valid data for a more accurate analysis and reduce the operational error, therefore, such analysis is not accurate enough.

Secondly, my analysis steps and the application of formulas are relatively correct, but this does not lead to a completely reliable portfolio strategy back into reality, because there are still many objective factors that affect the analysis results simultaneously. As we all know, the world economy has become more vulnerable than ever due to the COVID-19 epidemic. Therefore, an eligible portfolio needs to be based on the combination of practical situation more rather than merely figures and previous or historic data. After take all these into consideration, I would dedicate myself in updating the content of this article to be more in line with the actual economic conditions and industry development prospects.

REFERENCES

[1] Megginson, W. (1996). A historical overview of research in finance. Journal of Finance, 39(2), 323-346.

[2] Markowitz, H. (1952). Portfolio Selection. The Journal of Finance 7(1), pp. 77-91University Press.

[3] Markowitz, H. (1959). Portfolio Selection. New Haven, Connecticut: Yale

[4] Mossin, J. (1966). Equilibrium in a capital asset market. Econometrica, 34, 768-783.

[5] Tobin, J. (1958). Liquidity preference as behavior toward risk. Review of Economic Studies, 65-85

[6] Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19, 425-442.

[7] Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. Journal of Finance, 20, 587-615.

[8] Veneeya, V. (2006, July). Analysis of modern portfolio theory. Coursework4you.

[9] Fabozzi, F., Gupta, F., & Markowitz, H. (2002, Fall). The legacy of modern portfolio theory. Journal of Investing, 7-22.

[10] McClure, B. (2010) Modern portfolio theory: Why it's still hip. Retrieved on 12/10/11 from http://www.investopedia.com/articles/06/MPT.asp#axzz1g3JQY7nY