Stability Analysis of Biological Wastewater Treatment in Facultative Stabilization Ponds: Mathematical Model

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ABSTRACT

Wastewater stabilization pond is a very large pond where wastewater is stored for a long time, in that pond there is a biological process of organic matter in the sludge produced from primary deposition, and it's resulting in stable biological treatment of wastewater. Wastewater treatment is used to reduce organic matter and remove pathogens to a level that complies with water quality standards. Many biological processes are carried out using microorganisms such as bacteria. The purpose of this paper is to develop a simple mathematical model of a system nonlinear differential equations in biological wastewater treatment by considering the inflow and outflow during the treatment process. The system of equations is built using the basic Monod equations. Then linearize the system by finding the Jacobian matrix. Stability of the system differential equations around the equilibrium point is analyzed by looking for characteristic equations, eigenvalues and phase portraits and its shows that the model is asymptotically stable. Numerical simulations to determine the changes that occur at each concentration over time were carried out using the Fourth-order Runge Kutta method. The data were obtained from the facultative stabilization pond in Sewon, Bantul, Yogyakarta. The simulation shows that the concentration of DO (dissolved oxygen) has increased to stable, while the concentration of OM (organic matter) and B (bacteria) has decreased to stable. Computational calculations and visualization system to get the equilibrium point using Maple 18 and finding the solution using the Matlab (R2015a).

Keywords: *Wastewater Treatment, Facultative Stabilization Pond, Jacobian Matrix, Stability Analysis, System of Non-Linear Differential Equations.*

1. INTRODUCTION

The development of the era and the increasing number of people in the world resulted in more and more problems faced related to water pollution, especially in urban areas. Humans cannot consume water that has been polluted because it is dangerous. The characteristics of urban wastewater can be seen from its high organic matter content, namely by looking at the amount of Biological Oxygen Demand (BOD) in it. Urban wastewater is commonly referred as domestic wastewater, which usually comes from the disposal of dirty water from bathrooms, latrines and kitchens [1]. Currently, there have been many scientific developments that study ways to treat wastewater both biologically, chemically and biochemically so that polluted water can be reused by humans. One of the wastewater treatment methods that is widely used in Asian countries is using wastewater

stabilization ponds, this is because the processing and maintenance costs are quite cheap.

Wastewater stabilization ponds are very large ponds where wastewater is stored for a long time. So that in the stabilization pond there is a biological process of organic matter in the sludge produced from primary deposition and a stable biological treatment of wastewater occurs. Stability depends on both anaerobic and aerobic conditions [2]. Wastewater treatment with stabilization ponds is used to reduce organic matter and remove pathogens to the desired level according to water quality standards. Wastewater treatment relies on natural processes by utilizing the presence of bacteria, algae and zooplankton [1].

Research has been carried out on the formation of dynamic models and identification of parameters of the effluent stabilization pond based on mass balance involving biochemical reactions with microorganisms

[3]. Research has also been carried out on modeling of domestic wastewater treatment in facultative stabilization ponds which produces 13 differential equations consisting of concentration equations for organic and inorganic materials in the pond. In this modeling the author uses biological, chemical and physical phenomena as well as interactions between variables that affect the wastewater treatment system. In addition, the authors consider the influence of the inflow and outflow of the pond [3,4,5].

Over time, research on wastewater treatment modeling has also become more diverse, such as research conducted [6,7] that study about wastewater treatment using Thermophilic bacteria. In 2017 Sunarsih et al [8] conducted a study on the dynamic modeling of the facultative stabilization pond wastewater treatment system involving 4 main components, namely algae, bacteria, dissolved oxygen and Organic Matter, which is calculated using the BOD value with, the assumption that there is no inflow and outflow of wastewater during the process.

A wastewater treatment model for substrate reduction and nitrogen removal has also been developed by modifying the ASM1 (Active Sludge Model) model for bacterial growth coupled with substrate diffusion into flocs and biofilms [9]. The activated sludge model (ASM) recommended by the International Water Association (IWA) is a widely accepted model, in which ASM1 can successfully simulate the process of removing organic matter and ammonia-nitrogen (SNH) in WWTPs [10]. The Complexity of Activated Sludge Model No. 1 (ASM1) is one of the main obstacles that slows down its widespread use, especially among professionals in wastewater treatment plants (WWTP). A simplification procedure based on steady-state mass balance has been proposed for a conventional activated sludge process

(ASP) configuration, consisting of a fully aeration and settling bioreactor (without any particular compound at the outlet) [11].

Evaluation of wastewater quality control in facultative stabilization ponds needs to be done using dynamic system mathematical modeling. The
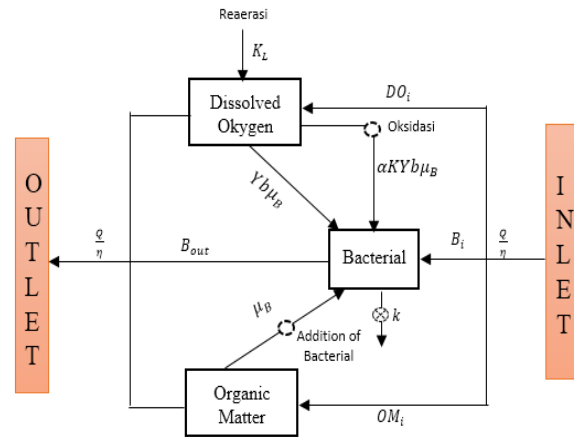

Figure 1. Flowchart of the Biological Wastewater Stabilization Ponds

mathematical model used is based on [3,4,8] which is then re-constructed into a system of non-linear differential equations of 3 (three) concentrations (variables), namely bacteria, organic matter (OM) which in the calculation uses the BOD and Dissolve Oxygen (DO) using the monod model as a microbial growth model and considering the inflow and outflow of the wastewater during the process.

Table 1. Parameter Value on Model.

Parameter	Name	Value	Units	Source
α	Oxygen consumption by bacteria for metabolism	1.2	mg/mg	[1]
K	Kinetic Constant at T	0.0005	day^{-1}	[1]
Y_b	The rate of bacterial product	0.09		[1]
K_{OM}	Half saturation of substrate bacteria	50	mg/l	[2]
K_{DO}	Half saturation DO bacteria	0.1	mg/l	[1]
μ_B	Specific growth rate bacteria	0.01	day^{-1}	[2]
K_L	Dissolved oxygen's inter transfer coefficient	0.8566	$mday^{-1}$	[1]
DO^{sat}	DO saturation	5	mg/l	[1]
Q	Flow rate entering the pond	10697.53	m^3/time	[13]
η	Pond Volume	8085	m^3	[13]
DO_i	Dissolved Oxygen in Inlet	0.9	mg/l	[1]
OM_i	OM in inlet	250	mg/l	[1]
B_i	Bacteria in inlet	490	mg/l	[1]
k	bacterial death coefficient	0.06	day^{-1}	[1]

2. MATHEMATICS MODEL

The mathematical model of a non-linear system differential equation is developed based on the biological processes that occur in the facultative stabilization pond. Wastewater entering through the inlet carries sludge containing organic matter with certain concentration. Organic matter and dissolved oxygen are consumed in the growth of bacteria. Metabolic rates are all assumed to depend nonlinearly on the substrate (organic matter) and oxygen concentration [6]. It is assumed that the pond bottom is not active. The illustration of a biological wastewater treatment system in a facultative stabilization pond can be seen in Figure 1.

2.1. Dynamical Model

The mass balance equation is formed on 3 (three) non-linear differential equations developed from the Monod equation against time correction as maximum growth [1,3] and the flow diagram of the biological process between components can be seen in Fig 1. Taking into account the existing assumptions, a mathematical model of a first-order non-linear differential equation system was developed that describes the rate of change in concentration: Bacteria (B), Dissolve Oxygen (DO) and Organic Matter (OM). With developing the model by [7], considering the model by [1] and taking into account the inflows and outflows during the process as in the model by [4], we get the following model that is the biological treatment process in the facultative stabilization pond by considering the inflow and outflow of wastewater during the process.

$$
\begin{cases}
\dfrac{dDO}{dt} = -aKY_b\mu_B \dfrac{OM(t)}{K_{OM}+OM(t)}\dfrac{DO(t)}{K_{DO}+DO(t)}B(t) \\
\qquad + K_L(DO^{sat}-DO(t)) + \dfrac{Q}{\eta}(DO_i - DO(t)) \\
\dfrac{dOM}{dt} = -\mu_B \dfrac{OM(t)}{K_{OM}+OM(t)}\dfrac{DO(t)}{K_{DO}+DO(t)}B(t) \\
\qquad + \dfrac{Q}{\eta}(OM_i - OM(t)) \\
\dfrac{dB}{dt} = Y_b\mu_B \dfrac{OM(t)}{K_{OM}+OM(t)}\dfrac{DO(t)}{K_{DO}+DO(t)}B(t) \\
\qquad - kB(t) + \dfrac{Q}{\eta}(B_i - B(t))
\end{cases}
\tag{1}
$$

with $\mu_B, K, K_{OM}, K_{DO}, K_L > 0$ and $\alpha, Y_b, DO^{sat}, Q, \eta,$ $DO_i, OM_i, k, B_i > 0$. In the general μ_B, K, K_{OM}, K_{DO} and K_L is the kinetic parameters.

2.1. Fourth Order Runge Kutta Method and Program Model

The solution of three-variable non-linear differential equation system in equation (1) can be solved using the fourth-order Runge Kutta method. This method is an approach method to find a solution of a system equations that has high accuracy and is easy to calculate. The Runge Kutta method is used as an integration technique to get the concentration of each component against the simulation time that calculated using the help of Matlab program (R2015a). The procedure for completing the fourth-order Runge Kutta method is

$$
m_1 = g(k_n, l_n),\ m_2 = g\left(k_n + \frac{h}{2}, l_n + \frac{h}{2}m_1\right),
$$

$$
m_3 = g\left(k_n + \frac{h}{2}, l_n + \frac{h}{2}m_2\right),\ m_4 = g\left(k_n + h, l_n + hm_3\right)
$$

and the solution is $l_{n+1} = l_n + \dfrac{h}{6}[m_1 + 2m_2 + 2m_3 + m_4]$ [12].

The calculationi is done by adding the initial data input or the initial value of bacteria, namely $B(0) = 490\ mg/l$, organic matter (BOD) $OM(0) = 250\ mg/l$ and dissolved oxygen $DO(0) = 0.9$ mg/l [1]. The initial value of the concentration is based on the measurement results at the inlet of the stabilization pond.

3. RESULTS AND DISCUSSION

The system of differential equations (1) describes how the biological processes that occur in the system during wastewater treatment can reduce organic matter (BOD). The WWTP is a system that illustrates the process of interaction between the concentration variable elements, causing a reduction in organic matter. This condition is called a dynamic state, where the system changes with time which is indicated by the state of the system from an unsteady state to a steady state [8]. Changes from an unsteady state to a steady state occur when each variable or component in the system reaches the concentration points where at that point all components no longer can affect each other, so it can be said that there is no longer change in the concentration of these components.

3.1. Determining the Equilibrium Point

To perform a stability analysis in the model (1), what needs to be done is to find the value of the equilibrium point of the system. Finding the equilibrium point of a dynamic system means finding the value of the variables when the system is in a steady state (no changes with the system). The steady state can occur if it meets $\dfrac{dDO}{dt} = 0, \dfrac{dOM}{dt} = 0$ and $\dfrac{dB}{dt} = 0$. So the system (1) become
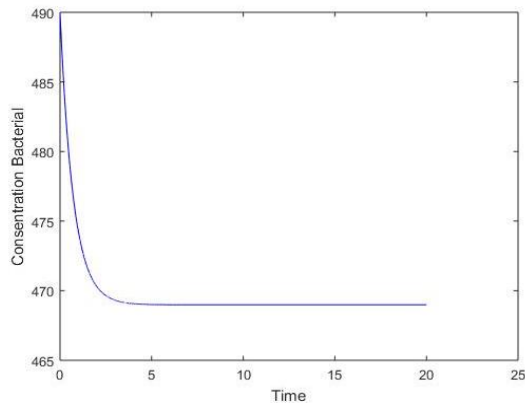
$$-aKY_b\mu_B \frac{OM(t)}{K_{OM}+OM(t)} \frac{DO(t)}{K_{DO}+DO(t)} B(t) + K_L$$

$$(DO^{sat}-DO(t)) + \frac{Q}{\eta}(DO_i - DO(t)) = 0$$

$$-\mu_B \frac{OM(t)}{K_{OM}+OM(t)} \frac{DO(t)}{K_{DO}+DO(t)} B(t)$$

$$+\frac{Q}{\eta}(OM_i - OM(t)) = 0$$

$$Y_b\mu_B \frac{OM(t)}{K_{OM}+OM(t)} \frac{DO(t)}{K_{DO}+DO(t)} B(t)$$

$$-kB(t) + \frac{Q}{\eta}(B_i - B(t)) = 0$$

$$(2)$$

With the help of Maple 18 and some calculation, obtained the equilibrium point of the system, there is

$$E(DO^*, OM^*, B^*) = \left(\frac{K(OM)QY_b\alpha(OM - OM_i)}{K_L\eta + Q} + \right.$$

$$\left. \frac{DO^{sat}K_L\eta + DO_iQ}{K_L\eta + Q}, OM^*, B^* \right).$$

3.2. Model Stability Analysis

Stability analysis around the equilibrium point is done by first finding the Jacobian matrix of system. Jacobian matrix of the system (1) is

$$J = \begin{bmatrix} -\dfrac{\alpha KY_b\mu_B OMB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{DO}{K_{DO}+DO}\right) - K_L - \dfrac{Q}{\eta} \\ -\dfrac{\mu_B OMB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{DO}{K_{DO}+DO}\right) \\ \dfrac{Y_b\mu_B OMB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{DO}{K_{DO}+DO}\right) \end{bmatrix}$$

$$-\dfrac{\alpha KY_b\mu_B DOB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{OM}{K_{OM}+OM}\right)$$

$$-\dfrac{\mu_B DOB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{OM}{K_{OM}+OM}\right) - \dfrac{Q}{\eta}$$

$$\dfrac{Y_b\mu_B DOB}{(K_{OM}+OM)(K_{DO}+DO)}\left(1 - \dfrac{OM}{K_{OM}+OM}\right)$$

$$-\dfrac{\alpha KY_b\mu_B OMDO}{(K_{OM}+OM)(K_{DO}+DO)}$$

$$-\dfrac{\mu_B OMDO}{(K_{OM}+OM)(K_{DO}+DO)}$$

$$\dfrac{Y_b\mu_B OMDO}{(K_{OM}+OM)(K_{DO}+DO)} - k - \dfrac{Q}{\eta} \Bigg].$$

After get the Jacobian matrix next, substitute the equilibrium point to the Jacobian Matrix and fine the eigenvalues. But, because it is very difficult to do exact

calculations, then do numerical simulations and drawing phase portraits using Matlab (R2015a).

4. NUMERICAL SIMULATION

Numerical simulations were carried out to find the stability of the system (1) using the help of maple 18 and to find the solution of the system numerically using the Runge Kutta method of Order four with calculations using the help of Matlab (R2015a). The values of the parameters for the simulation are listed in the Table 1.

The phase portrait of the system (1) can be seen in figure 2 with DO(OM)-plane with the initial value of trajectory 1 is $DO(0) = 0.9, OM(0) = 250$ and $B(0) = 490$, the initial value of trajectory 2 is $DO(0) = 1.5$, $OM(0) = 240$, $B(0) = 450$, the initial value of trajectory 3 is $DO(0) = 0.8, OM(0) = 265$ and $B(0) = 480$, the initial value of trajectory 4 is $DO(0) = $

Figure 2. Phase Potrait of System 1 in DO(OM)-plane

$1.32, OM(0) = 255$ and $B(0) = 460$. From that figure can concluded that the system is **asymtotically stable** because all of the Trayektory is lead to the same point.

Figure 3. The Dynamical Changes of Dissolve Oxygen (DO) Concentration with The Time

Figure 4. The Dynamical Changes of Bacterial (B) Concentration with The Time
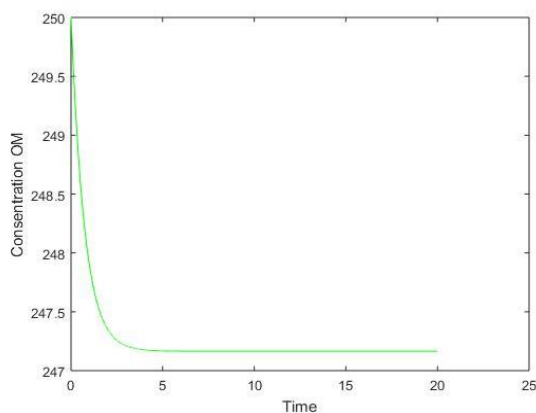

Figure 5. The Dynamical Changes of Organic Matter (OM) Concentration with The Time

The dynamical changes of each concentrations can be seen in the following diagram solution of the system differential non linear equation (1) figure 3 to 5.

The diagram solution in figure 3 to 5 obtained from calculations using matlab with the initial value $B(0) = 490 \, mg/l$, $OM(0) = 250 \, mg/l$ and $DO(0) = 0.9 \, mg/l$ [1]. It shows that the concentrations of dissolve oxygen will increase to a point and eventually stabilize in the other side the concentrations of organic matter and bacteria will decrease to a point and eventually stabilize.

5. CONCLUSION

The biological wastewater treatment process can be formed in the mathematical model of a non-linear ordinary differential equation system by considering the biological processes between bacteria, organic matter and dissolved oxygen. Taking into account the inflow and outflow of the pond, it can be concluded that the model is asymptotically stable.

Numerical analysis was carried out using the fourth-order Runge Kutta method and the help of Matlab aims to predict pond performance based on the

movement of DO (Dissolve Oxygen), OM (Organic Matter) and Bacterial concentrations.

AUTHORS' CONTRIBUTIONS

MNI and SS have contributions about CONCEPT, METHOD, and ANALYSIS. The EDITING has contributed by MNI and RHT. All autors (MNI, S, and RHT) provide feedback, discussed results, and contributed to the final manuscript.

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