Application of the Conjugate-Gradient Method for Analysing the Optimum Thrust Force for the Pico-Hydropower

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ABSTRACT

A water thrust from the nozzle is the main factor for the optimizing of the pico-hydropower system. The previous study indicated that the optimal water trust resulting from the balance between the forebay tank and penstock pipe. The study of the balances has been conducted by governing of Bernoulli equation that is related to the discharge and thrust of water. The Bernoulli equation is solved using numerical analysis namely conjugate gradient methods as an algorithm for optimizing the target function by determining the values of independent variables. The main target in the numerical calculation is to obtain the maximum values of water thrust from the nozzle for rotating the turbine. The independent variables considered in this computational process are the diameters of both forebay tank and penstock pipe, in conjunction with both elevations. The values obtained in this numerical analysis indicated that the dimension of both the forebay tank and penstock pipe has a significant effect in enhancing the water thrust from the nozzle. Besides, the elevations of both are indispensable in increasing the water thrust for rotating the turbine of the pico-hydropower system. Considering these results and literature study, a simple scheme of forebay tank and penstock pipe with nozzle is proposed, completed by size and dimension. The diameter and height of the forebay tank are about 2.5 and 3.0 m, respectively. An ideal diameter for the penstock pipe is 0.10 m while the nozzle is 0.05 m. The system is possible to produce a driving force approximately 11-12 N. The theoretical estimation with 50% of loss power indicates that the system is possible to generate 3.0-6.0 kW. The power may bigger than this estimation if the loss of power lower than 50% and an efficient system of more than 70%.

Keywords: *Conjugate-gradient, Forebay, Hydropower, Nozzle, Penstock.*

1. INTRODUCTION

The use of new and renewable energies available in nature is indispensable for supporting electric power especially in a remote area [1, 2, 3]. Development of technology and commercialization for clean, low cost and sustainable energies has been experienced rapid growth during recent decades. The types of electric power are hydropower, solar, biomass, wind, geothermal, wave, and tidal [4, 5, 6, 7]. Considering the simplicity, a pico-hydropower is a better option for supporting the availability of electric power in the remote area where the natural resource is the small water flow [8, 9].

The pico-hydropower generate electric under or equal to 5000 Watt with some typical advantages, i.e: easy to maintain, low water consumption, durability, and low construction cost [10, 11, 12]. The small scale hydropower systems do not require a large infrastructure with the least environmental impacts. Numerous inborn benefits of small hydropower generation projects are making them extensively acceptable worldwide [7], [9].

However, we seek a significant problem in regulating water discharge when some community implemented the system [13, 14]. In the condition of the rainy season, for example, the system experience excessive thrust due to the abundant water discharge. Conversely, in the dry season the system sometimes unable to operate due to the lack of water discharge to turn the turbine. Therefore, an optimization of water discharge at the forebay tank and penstock pipe for overcoming the problem is proposed in this study.

The components that greatly affect the pico-hydropower performance are forebay tank and penstock pipe [15, 16]. Accumulation of water in the forebay tank is flowed through the penstock pipe to obtain a high-speed fluid flow before being directed to push the turbine blade [14, 17, 18] To establish a high-speed water boost, it is necessary to analyze the compacted fluid flow in the penstock pipe starting from the head to the end of the penstock pipe which will push the turbine blade directly [15, 17]. The way of compacting fluid flow in the penstock pipes is to block the fluid flow. The most popular barrier methods are venturi, nozzles, and orifice [19].

The speed of fluid flow from the penstock pipe that is given an obstacle can be analyzed by using the Bernoulli principle [15, 19, 20]. Bernoulli's principle is a fluid flow formalism about speed, pressure, and elevation [21, 22]. Both physical parameters are inversely proportional to each other where an increase of velocity will cause a decrease of pressure on the fluid flow. This principle is a simplification of the Bernoulli equation which states that the amount of energy at a point in a closed flow is equal to the amount of energy at another point on the same flow path [19, 22, 23].

Considering the previous study about the optimization of water flow in penstock, the Bernoulli equation is applicable to regulate the speed and pressure of fluid flow in the penstock pipe depend on the barrier method. An effective blocking manner in adjusting of both speed and pressure of fluids is a nozzle method [19, 20]. In engineering, the nozzle is currently used as the name of the end of a penstock pipe where high-speed water is released to drive a turbine blade [14], [15, 23]. To achieve ideal conditions, a formulation is required by modifying the Bernoulli equation by considering the amount of thrust required by the turbine blade [20, 22]. The magnitude of potential energy produced from the water impulse of the nozzle corresponds to mechanical energy in the form of turbine angular speed [24].

In the present study, the balance between the forebay tank and penstock pipe will be analyzed using a computational method to obtain a water flow with high pressure. The conversion of kinetic energy from accelerated flow to rotary motion for electrical power generation through the use of a turbine is also considered in the modelling. The theory on the subject reveals that close channel flows are governed by the Bernoulli equation [15,19]. The Bernoulli Equation is a different way of the conservation of energy principle, applied to fluid flow. It relates the pressure, the kinetic energy and the gravitational potential energy of a fluid in a close tube. The close tube means the penstock with a nozzle at the output part [22, 23]. The other form of Bernoulli for this subject is analyzed using numerical method namely conjugate gradient [25, 26, 27]. A method usually used to solve the problem of non-constrained optimization, specifically for non-linear functions is the conjugate gradient method [28, 29]. The method is also applicable for training the neural network system [30]. In the case of the viscous fluid velocity, the conjugate gradient algorithm is proven to produce a minimum error in calculating numerical stability [27].

2. METHOD

The balance of dimensions between the forebay tank and penstock pipe will be analyzed through the Bernoulli equation and the conjugate gradient method. The target of the study is to obtain the maximum water thrust at the output of the penstock pipe. Assuming that the water flows in the close tube, the Bernoulli equation is regulated into the driving force (F) as the dependent variable [17, 22]. The independent variables in this modeling are water flow speed (v) and pressure (P). Both acceleration of gravity (g) and water density (ρ). The physical parameters will be optimized are elevations (h) and cross-sections (A) of both forebay tank and penstock pipe.

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The Bernoulli equation is derived by reviewing the hydrostatics and hydrodynamics phenomena. The formulation of the Bernoulli equation is started by considering Figure 1 which shows as a close tube. In this case, the cylinder-shaped container is considered as a penstock pipe. The water flow as fluid elements is considered to move with speed (v) and acceleration (a). Furthermore, the length, cross-section, and density of the fluid elements are ds, dA, and ρ so that the weight of the unit element is $ds\, dA\, \rho\, g$ [19, 22].

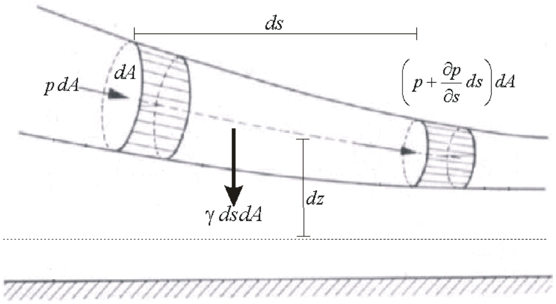

Figure 1 A scheme of fluid flow in close tube as representation of water flow in penstock pipe modified from references [17,22,23,27]

Assuming that the cross-section of the penstock pipe is slippery (frictionless), the forces are only from the weight and pressure at the end of the penstock pipe [15], [24]. From these assumptions, the Bernoulli equation in one-dimensional steady flow for the ideal fluid that can be used in this case is

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant} \tag{1}$$

where z is the elevation or height of the place, p/γ is the height of the pressure, $v^2/2g$ is the height of the speed. The integral constant, C is the total energy as a linear summation of elevation, pressure, and speed [21], [22]. In this subject, equation (1) will be developed through several assumptions related to fluid flow characteristic from the reservoir into the output of penstock pipe.

In the initial stage, the relationship between the forebay tank and penstock pipe is a system in which the scheme is shown in Fig 2. At the output of the penstock pipe is adjusted as a nozzle where the high pressure of water flow is produced for thrusting a blade of turbine [14, 15, 19].

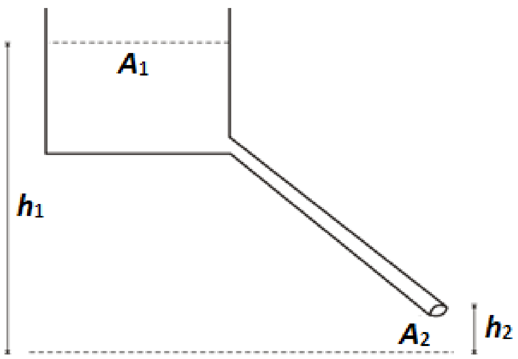

Figure 2 Scheme of the forebay-tank (cross-section A_1 and heigh h_1) and penstock-pipe (cross-section A_2 and heigh h_2).

The formulation of the thrust at the output point of the penstock pipe is started from the basic assumption that the pressure is equal to the force. However, the pressure is inversely proportional to the cross-section of the penstock pipe [19, 24]. Based on the assumption, the basic equation of the thrust can be modified as

$$F_2 = P_2 A_2. \tag{2}$$

Here, the symbols of F_2, P_2, and A_2 are a representation of the thrust force, pressure, and cross-section of the penstock pipe, respectively. By assuming that the pressure on the penstock pipe similar to the forebay tank, the basic form of Bernoulli equation is obtained as:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2. \tag{3}$$

In this case, the subscripts 1 and 2 are a parameter for the forebay tank and penstock pipe, respectively [20], [24].

The equation of water thrust is formulated through procedure and assumption which is shown below. The first related pressure at forebay tank that is assumed as a natural force from the atmosphere and gravity,

$$P_1 = P_0 + \rho g h_2. \tag{4}$$

Here, P_0 is the atmospheric pressure standard (101,325 kPa). The next assumption is that the pressure in the penstock (P_2) is affected by the static pressure at the forebay tank (P_1). Consequently, equation (4) is substituted into equation (3) to obtain:

$$P_2 = P_0 + \frac{1}{2}\rho v_1^2 \left(1 - \frac{A_1}{A_2}\right) + \rho g \left(h_1 - h_2\right). \tag{5}$$

Due to the main target in this subject is to determine an ideal dimension for balancing between the forebay tank and the penstock pipe, the continuity principle employs the cross-section (A) and speed (v) as

$$v_1 = \frac{A_2}{A_1} v_2. \tag{6}$$

By substituting equation (6) into equation (5) is obtained

$$P_2 = P_0 + \frac{1}{2}\rho v_1^2 \left(1 - \frac{A_1}{A_2}\right) + \rho g \left(h_1 - h_2\right) \tag{7}$$

Equation (7) is a tool for determining the pressure at the penstock pipe. Finally, the formula for calculating the value of driving force at the output of the penstock pipe below, equation (7) is submitted into equation (2) to obtain

$$F_2 = P_0 A_2 + \frac{1}{2}\rho v_1^2 A_2 \left(1 - \frac{A_1}{A_2}\right) + \rho g A_2 \left(h_1 - h_2\right) \tag{8}$$

The step for obtaining equation (8) is developed from the references [15, 19, 20, 23, 24].

The parameter for obtaining the ideal pushing force at the output of the penstock pipe (F_2=F) is the extension of both balances between the forebay tank (A_1=f) and the penstock pipe (A_1=p). The others parameter in the computation process are constant so that the target function for this case is:

$$G_n^{f,p} = \left[\frac{\partial F}{\partial f} \quad \frac{\partial F}{\partial p}\right]. \qquad (9)$$

Both components of the target function are defined as:

$$\frac{\partial F}{\partial f} = \frac{1}{2}\rho v_p^2 f \qquad (10)$$

and

$$\frac{\partial F}{\partial p} = P_0 + \frac{1}{2}\rho v_p^2 + \rho g\left(h_f - h_p\right) \qquad (11)$$

Here, the target parameter of F is named as the function of f and p. In conjugate gradient methods, the optimization parameter is determined by using the procedure:

$$\max\left\{F\left(f, p\right)\right\} : f, p \in R^n \qquad (12)$$

The target functions in this procedure are only for the real values which are developed from the references [26, 29].

Furthermore, the balance of extensive between the forebay tank (f) and penstock pipe (p) as shown in Figure 2 will be analyzed using the conjugate gradient method. By regulating equation (8) based on conjugate gradient method in references [25, 27, 30, 31] an algorithm for the subject are:

(0) Choosing initial values (f_0, p_0, α_0^f, α_0^p, δ_0^f, δ_0^p, $G_0 \equiv G_0^{f,p}$, d_0^f, d_0^p), determining constant parameters (P_0, v_p, ρ, g, h_f, h_p), estimating F_0, then calculating:

(1) $f_n = f_{n-1} + \alpha_{n-1}^p d_{n-1}^p$ and $p_n = p_{n-1} + \alpha_{n-1}^f d_{n-1}^f$

(2) $G_n^{f,p} = \left[\frac{\partial F}{\partial f} \quad \frac{\partial F}{\partial p}\right]$

(3) $\beta_n^{f,p} = \frac{G_n^T G_n}{G_{n-1}^T G_{n-1}}$ (T: transpose)

(4) $\delta_n^f = f_n - f_{n-1}$ and $\delta_n^p = p_n - p_{n-1}$

(5) $d_n^f = -G_n^f + \beta_n \delta_n^f$ and $d_n^p = -G_n^p + \beta_n \delta_n^p$

(6) $\alpha_n^f = \alpha_{n-1}^f \frac{\left\| d_{n-1}^f \right\|}{\left\| d_n^f \right\|}$ and $\alpha_n^p = \alpha_{n-1}^p \frac{\left\| d_{n-1}^p \right\|}{\left\| d_n^p \right\|}$

(7) $F_n = P_0\, p_n + \frac{1}{2}\rho v_p^2\, p_n\left(1 - \frac{f_n}{p_n}\right) + \rho g\, p_n\left(h_f - h_p\right)$

(8) $\varepsilon_n = \left|\frac{F_n - F_{n-1}}{F_{n-1}}\right| \times 100\%$.

The target of the estimation in the conjugate gradient algorithm is to obtain the values of pushing force (Fn) at the output of penstock pipe.

Overall, the Bernoulli formalism and conjugate gradient method regulated in this algorithm are developed from the previous study with the slow viscous fluid flow. In this present procedure, the algorithm is to analyze the fast non-viscous fluid flow in the close tube [20, 24, 25, 27]. For obtaining an ideal driving force at the nozzle, A computational procedure is employed for estimating several values that have an impact on the pushing force (F_2). In this computation procedure, some parameters are used i.e: elevation of both forebay tank and penstock pipe, speed of lowering the water in the forebay tank, and elevation of forebay tank, and diameter of the penstock pipe. The estimation procedure is compared with the previous numerical and experimental study for evaluating the balance between the forebay tank and penstock pipe dimensions.

3. RESULT AND DISCUSSION

Figure 3 shows the result of the computational process using the conjugate gradient algorithm. The water thrust is obtained by employing the constant parameter, atmosphere pressure, gravity acceleration, and water density at approximately 1000 kg/m^{1}s^2, 10 m/s, and 1 kg/m^3, respectively. The lowering speed of the water level at the forebay tank is assumed as a constant number. The others are elevations of both the forebay tank and nozzle. The target function for optimizing is the diameter of the forebay tank and penstock pipe which are derived by inputting the diameter of both. The result confirms the Bernoulli formulation in references [22, 23], that the original dimension of both the forebay tank and penstock pipe is a bigger and smaller tubular, respectively. The conjugate gradient method implemented for this case corresponds to the problem in the references [25, 27].

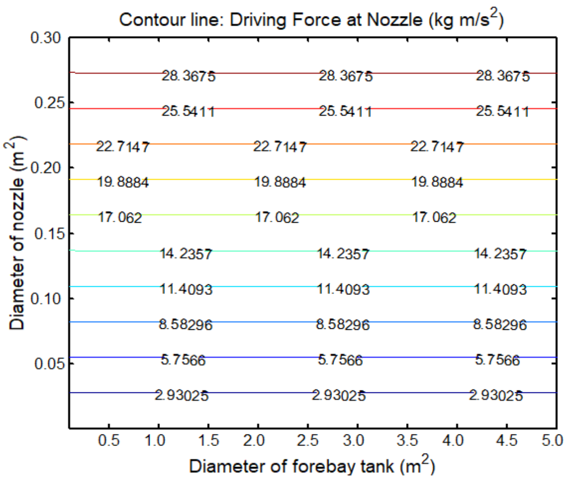

Figure 3 A scheme of fluid flow in close tube as representation of water flow in penstock pipe modified from references [17, 22, 23, 27] A contour of driving force at nozzle part in Newton (kg m /s^2) resulted from the computation process using the conjugate gradient method. The result is obtained by inputting the constant parameter the lowering speed of the water at forebay tank, elevation forebay tank, and elevation of the nozzle at 0.05 m/s, 5 m, and 1 m, respectively.

The result in Figure 3 indicates that there is no impact of the dimension from the forebay tank to the water thrust from the nozzle. On the other hand, the wider the diameter of the nozzle, the higher the water thrust. Some parameter which is employed as a constant number shows a significant impact. The other treatment to the forebay tank is an adjustment of forebay tank elevation. In Figure 4, a correlation between the diameter and thrust of water from the nozzles is shown by considering four elevations as the sample. From the previous numerical studies about hydropower [32] and sea-wave [33] power, the small design may rotate the energy converter which are constructed from several whells. The wheels are connected with the same axis and chain.

At the same diameter of the nozzle but different elevation, there is an elevation impact is approximately 0.1-0.2 N for every 2.5 m. For the elevation of forebay tank at 2.5 m, the water thrust is about 5.2 N. When the elevation is increased to 5.0, 7.5, and 10.0 m at the same diameter of the nozzle (0.05 m), the thrust escalates to 5.3, 5.4, and 5.6 N, respectively. The condition indicates that the impact of forebay tank elevation will increase two times bigger at the higher elevation. The result confirms the previous study about low head pico-hydropower [15, 16]. Besides, the diameter of the nozzle also shows a significant effect with a reasonable diameter for the pico hydropower type is a maximum of 10 cm (0.10 m) [23, 26]. Considering some correlation in this estimation, we analyze the effect of the speed lowering of water level at forebay tank.

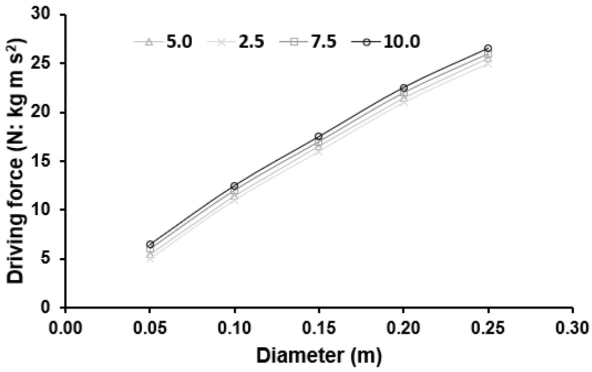

Figure 4 Correlation between driving force and diameter of nozzles with a different elevation of forebay tank (2.5, 5.0, 7.5, and 10.0 m).

Table 1 lists the effect of the speed of lowering the water level to the thrust from the nozzle. The estimation procedure is carried out in the condition of the invariable of both elevation and diameter of the forebay tank. Both 5 m and 2.5 m of elevation and diameter, respectively are used in this calculation. The results indicate that the lowering speed of the water at the forebay tank also contributes to magnify the water thrust from the nozzle. The procedure to manage the speed of lowering the water level corresponds to the fluid debit [21, 22]. The parameter can be controlled by adjusting the cover of the forebay tank [19]. The reasonable of the speed lowering of water which corresponds to the water debit and supply is 5 cm (0.05 m) [14, 16, 19]. This means for the forebay tank designed as a big tube with a diameter 2.5 m is optimum to generate the force around 11.4093 N with the speed of lowering water at 0.05 m.

Table 1 Correlation between driving force and diameter of nozzles with a different elevation of forebay tank (2.5, 5.0, 7.5, and 10.0 m).

v_f (m/s)	p (kg m/s^2)
0.01	11.4031
0.05	11.4093
0.10	11.4287
0.15	11.4610
0.20	11.5062
0.25	11.5644
0.30	11.6355

Table 2 Invariable of the lowering water level (0.05 m s^{-1}) and diameter (2.5 m).

h_f (m/s)	**p (kg m/s^2)**
4.5	11.3556
5.0	11.4093
5.5	11.4631
6.0	11.5169
6.5	11.5706
7.0	11.6244
7.5	11.6782

Table 2 shows the effect of the elevation of the forebay tank to the water thrust from the nozzle. The estimation procedure is carried out in the condition of invariable of both the speed of lowering the water level and diameter of the forebay tank. In this calculation, we used 0.05 m/s and 2.5 m of the speed of lowering water and diameter, respectively. The result confirms the previous estimation related to the diameter of the nozzle in Figure 4. The pushing force in Table 2 bigger than in Figure 4 due to the estimation process in the latest table is conducted by improving the diameter of the nozzle from 0.05 to 0.10 m. The diameter of the nozzle used for resulting the water thrust in Table 2 is recommended for the low head of pico-hydropower [15, 23, 24]. The recommended elevation for pico hydropower for reducing the cost according to the previous study is 5.0 m from the baseline [6, 15, 16]. The result in Table 2 confirms the recommendation about the elevation of the forebay tank for the small scale of hydropower system.

Figure 5 is the scheme of the forebay tank and penstock pipe, completed by the legend for every component. The scheme is developed based on the literature study from references [10, 11, 15, 16]. and our analysis using the conjugate gradient method. From the estimation which has been done, some specific size and dimension for keeping the balance between the forebay tank and penstock pipe are proposed, together with the nozzle. The main target of the proposed scheme is to obtain an optimum the water thrust from the nozzle for rotating the turbine and energy converter system for the small scale of hydropower.

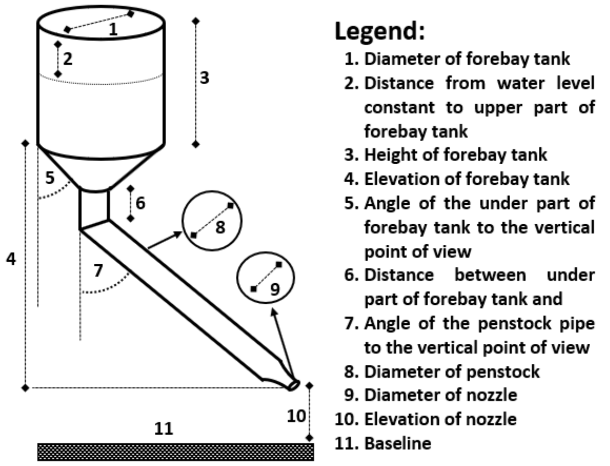

Figure 5 A scheme of the balance between forebay tank and penstock pipe

Both effectivity and efficiency of the system are considered for developing the size and dimension of the component in Figure 5. Therefore, the dimension of the forebay tank as a big tube is recommend with the diameter and height about 2.5 and 3 m, respectively. The recommended elevation of the forebay tank from the baseline is approximately 5 m. In the under part of the forebay tank, it is designed in a cone with the tilt around 45^o from the vertical point of view. The cone is connected to the penstock and nozzle with the diameter at approximately 0.10 and 0.05 m, respectively. The water thrust that is possible to produce the system is around 11-12 N.

The electric power which is possible to generate by the system in Figure 5 can be estimated by developing formula from references [17, 18]. The pushing force of water flow from the nozzle can generate a torsion of the turbine for rotating the electric generator. In this sample case, the small scale of the pico hydropower is usually operated with a rotation speed of approximately 300-500 rad s^{-1} and diameter of the turbine about 1.0-1.5 m [17]. The torsion that may generate is obtained from the diameter of the turbine and the water thrust of the nozzle. By employing the pushing force in Tables 1 and 2 (11-12 N), the torsion of the system may vary around 11-18 N m (kg m^2 s^{-2}). As a result, the electric power that is possible to generate by the system is approximately 3-6 kW (kg m s^{-3}). The relationship of mechanical energy to the values of electric power is calculated using the method in references [32, 33]. This estimation is developed from references [16, 17, 18] with the loss of power around 50%. Therefore, in implementing the system, the performance can be improved by reducing the loss of power and increasing the water thrust. The results correspond to the pico hydropower which is generated electric power under or equal to 5000 Watt (5kW) [10, 11, 12].

4. CONCLUSION

The result shows that several parameters are indispensable to consider before implementing the system. Based on the conjugate gradient analysis, there is no significant impact of forebay tank diameter. Therefore, the dimension which is applied in this modeling is choice from some previous study about the low head of the Pico hydropower. The parameters that are indispensable to improve for the forebay tank side are the speed of lowering the water level and the elevation. The diameters of penstock and nozzle are also having a significant effect to magnify the pushing force.

From the numerical analysis in this study and previous literature, a simple scheme of forebay tank and penstock pipe is proposed with the size and dimension. Both effectivity and efficiency are considered for developing the size and dimension. All components are assumed as a tube. The diameter and height of the forebay tank are about 2.5 and 3.0 m, respectively. An ideal diameter for the penstock pipe is 0.10 m while the nozzle is 0.05 m. The system is possible to produce a pushing force of approximately 11-12 N. According to the theoretical estimation with 50% of lost power, the system is possible to generate 3-6 kW. The results correspond to the Pico hydropower which is generated electric power under or equal to 5000 Watt (5kW). The system is possible to implement by reducing the loss of power lower than 50% and increasing efficient system of more than 70%.

AUTHORS' CONTRIBUTIONS

Concept, S.S, and J.A, WAS, AF, UNP, method, J.A and WAS, analysis, S.S, JA, AF, editing UNP, ZA. All authors provided feedback, discussed result and contributed to the final manuscript.

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REFERENCES

[1] F. Tumiwa, H. Imelda H, The Energy-poor: the facts in civil society, Institute for Essential Services Reform (IESR), 2011, pp 70-85.

[2] M. Felix, S.H. Gheewala, Environmental Assessment of Electricity Production in Tanzania, Energy for Sustainable Development, vol. 16, no. 4, 2019, pp 439-447.

[3] Z. Arthouros, Global status report Paris: REN21 Secretariat 336, 2019.

[4] W. Beabpimai, T. Chitsomboon, Numerical Study of Effect of Blade Twist Modifications on the Aerodynamic Performance of Wind Turbine, Int. J. of Renewable Energy Development, vol. 8, no. 3, 2019, pp 285-292.

[5] A. Santoni, G. Cavazzini, G. Pavesi, G. Ardizzon, A. Rossetti, Techno-Economical Method for the Capacity Sizing of a Small Hydropower Plant, Energy Conversion and Management, vol. 52, no. 7, 2011, pp 2533-2541.

[6] S.K. Singal, R.P. Saini, C.S. Raghuvanshi, Analysis for cost estimation of low head run-of-river small hydropower schemes, Energy for Sustainable Development, vol. 14, no. 2, 2010, pp 117–126.

[7] M. Berkun, Hydroelectric Potential and Environmental Effects of Multi-dam Hydropower Projects in Turkey, Energy for Sustainable Development, vol. 14, no. 4, 2010, pp 320-329.

[8] S.P. Adhau, R.M. Moharil, P.G. Adhau, Mini-hydro Power Generation on Existing Irrigation Projects: *Case study of Indian sites*, Renewable and Sustainable Energy Reviews, vol. 16, no. 7, 2012, pp 4785-4795.

[9] R. Bakis, Electricity Production Opportunities from Multipurpose Dams (case study), Renewable Energy, vol. 32, no. 10, 2007, pp 1723–1738.

[10] H. Zulkarnain, Soekarno, A. Berlian, Pico Hydropower System for Rural Area, Journal of P3TEK, vol. 1, no. 1, 2002, pp 20-26.

[11] H. Soekarno, T. Anggono, A. Heriansyah, A Basic Component in Designing of Micro Hydropower, Journal of P3TEK, vol. 1, no. 1, 2002, pp 8-12.

[12] Darmawi, R. Sipahutar, S.M. Bernas, S.M. Imanuddin, Pico Hydropower Application on Tidal Irrigation Canal Supporting the Indonesian Agricultural Activities Case Study: Telang II – Banyuasin, Int. J. on Adv. Sci. Engineering & Information Tech. vol. 3, no. 1, 2013, pp: 89-92.

[13] J.C. Blanco, Y. Secretan, A.L.A. Mesquita, Decision Support System for Micro-hydro Power Plants in the Amazon Region under a Sustainable Development Perspective, Energy for Sustainable Development, vol. 12, no. 3, 2008, pp: 25-33.

[14] J.S. Anagnostopoulus, D.E. Papantonis, Optimal Sizing of a Run-of River Small Hydropower Plant, Energy Conversion and Management, vol. 48, no. 10, 2007, pp 2663-2670.

[15] K.V. Alexander, E.P. Giddens, Optimum Penstocks for Low Head Micro Hydro Schemes, Renewable Energy, vol. 33, no. 3, 2008, pp 507-519.

[16] B. Ho-Yan, W.D. Lubitz, Performance Evaluation of Cross-Flow Turbine for Low Head Application, World Renewable Energy Congress: Lingköping-Sweden Sweden, 8-13 May 2011, pp 1394-1399.

[17] G.A. Caxaria, D.M. Sousa, H.M. Ramos, Small Scale Hydropower: generator analysis and optimization for water supply systems, World Renewable Energy Congress: Lingköping-Sweden, 8-13 May 2011, pp 1386-1393.

[18] Budiarso, Warjito, D. Adanta, N.S. Puta, H. Vohra, Cut-out Types Analysis on Pico Hydro Pelton Turbine, Int. J. on Adv. Sci. Engineering and Information Tech. vol. 8, no. 5, 2018, pp 24-30.

[19] K. Nirmaladevi, M. Mythilee, M. Navein, K. Manikandaprabhu, A Study on Bernoulli's Equation and Its Application in Fluid Mechanics, Int. J. for Sci. Research & Development, vol. 6, no. 12, 2019.

[20] J.P. Thavamani, Bernoulli Equation in Fluid Flow, Int. J. of Current Research, vol. 8, no. 10, 2016, pp 459-461.

[21] H.R. Malonek, G. Tomaz, Bernoulli Polynomials and Pascal Matrices in the Context of Clifford Analysis, Discrete Applied Mathematics, vol. 157, no. 4, 2009, pp 838-847.

[22] D. Azevedo, M.C. Valentino, Generalization of the Bernoulli ODE, Int. J. of Math. Education in Sci. and Tech, vol. 48, 2016, pp 256-260.

[23] M.M. Alnakhlani, Mukhtar, D.A. Himawanto, A. Alkurtehi, D. Danardono, Effect of the Bucket and Nozzle Dimension on the Performance of a Pelton Water Turbine, Modern Applied Science, vol. 9, no. 1, 2015, pp: 25-3.3.

[24] S. McKee S, A.D. Sneyd, M.F. Tome, Time-dependent Flow in Penstock during Head-gate Closure, Appl. Math. Modelling, vol. 20, 1996, pp: 614-623.

[25] S. Sanmatías, E.A. Vercher, Generalized Conjugate Gradient Algorithm, J. of Optimization Theory and Appl., vol. 98, 1998, pp: 489–502.

[26] Z. Wei, G. Li, L. Qi, New Nonlinear Conjugate Gradient Formulas for Large-scale Unconstrained Optimization Problems, Appl. Math. and Computation, vol. 179, no. 2, 2006, pp: 407-430.

[27] T. Johansson, D. Lesnic, A Variational Conjugate Gradient Method for Determining the Fluid Velocity of a Slow Viscous Flow, Applicable Analysis, vol. 85, no. 11, 2006, pp: 1327-1341.

[28] W.W. Hager, H. Zhang, A Survey of Nonlinear Conjugate Gradient Method, Pacific Journal of Optimization, vol. 2, no. 1, 2006, pp: 35-58.

[29] I. Abdullahi, R. Ahmad, Global Convergence Analysis of a New Hybrid Conjugate Gradient Method for Unconstrained Optimization Problems, Malaysian J. of Fundamental and Appl. Sci., vol. 2, no. 13, 2017, pp: 40-48.

[30] N.M. Nawi, N.A. Hamid, N.A. Samsudin, M.A.M. Yunus, M.F.A. Aziz, Second Order Learning Algorithm for Back Propagation Neural Networks, Int. J. on Adv. Sci., Engineering and Information Tech., vol. 7, no. 4, 2017, pp: 1162-1171.

[31] J. Aminuddin, Wihantoro, Bilalodin, Sunardi, N. Rauf, Designing of Muffler Part for Car Exhaust System with Low Emission and Noise using Conjugate Gradient Method, J. Phys.: Conf. Ser. **1494** 012041, 2020. https://doi.org/10.1088/1742-6596/1494/1/ 012041

[32] J. Aminuddin, Nurhayati, A. Widiyani, P. Razi, Wihantoro, A.N. Aziz, R.F. Abdullatif, A. Arifin, Estimation of Ideal Configuration and Dimension of Pico Hydropower using Euler-Lagrange Equation and Runge-Kutta Method, J. Phys.: Conf. Ser. **1494** 012039, 2020. http://doi.org/10.1088 /1742-6596/ 1494/1/ 012039

[33] J. Aminuddin, M. Effendi, Nurhayati, A. Widiyani, P. Razi, Wihantoro, A.N. Aziz, R.F. Abdullatif, Sunardi, Bilalodin, A. Arifin, Numerical Analysis of Energy Converter for Wave Energy Power Generation-Pendulum System, Int. Journal of Renewable Energy Development, vol. 9, no. 2, 2020, pp: 255-261, doi:10.14710/ ijred.9.2.255-261