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Optimization Method and Application in Python and Aviation

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ABSTRACT

Optimization methods have been formed and developed in recent decades and play an important role in many fields. In the aviation sector, it plays a role in everything from the design of air vehicles to the planning of routes, ticketing, etc. Through the method of literature review, this paper introduces optimization methods and their applications in aviation and python. In the theoretical aspect, mathematical methods involved in optimization methods and some python applications of them are included in this paper. In terms of real-world applications, this paper discusses the application of optimization methods in the field of aviation. The paper finds that different optimisation approaches have certain pitfalls in use, such as the gradient descent method used by the authors of this paper, which in some cases may converge to a local minimum rather than a global minimum. Besides, it iterates at a reduced speed near the extremes.

Keywords: *Optimization Method, Application, Pyhton, Aviation*

1. INTRODUCTION

Optimization methods are the subject that has formed in recent decades. However, the history of it can be traced back to 500 B.C. A classic optimization problem, the golden ratio of 0.618, was found in Ancient Greece in this period. For a long time thereafter, scholars used mathematical methods to solve optimization problems, but no discipline was formed.

After the seventeenth century, the emergence of calculus led to the gradual formation of classical optimization methods as a discipline. The development and formation of modern optimization methods is accompanied by the formation of mathematical methods such as linear programming, nonlinear programming, dynamic programming, and the principle of extreme value.

In business, optimization methods are essentially mathematical methods to solve different extreme value problems. However, optimization problems in many areas can be reduced to mathematical problems and solved using optimization methods. This is why optimization methods are used and create value in different fields. Among other things, the application of optimisation methods covers almost the entire aviation sector. From aircraft manufacturing to airline ticket pricing and route planning, it is all at work.

As there is rare research focusing on this side, this paper aims to introduce the optimization methods and the application of them in Python and aviation. Optimization methods are the methods to solve optimization problems, which are the different solutions to achieve optimal purpose in mathematical meaning. The aim of them is to research optimization programs and scientific decisions for different systems that play a role in EI, electronics, transportation, aviation, and other fields. Besides, this research explores the implementation of optimization algorithms in Python and their application in the field of aviation, dissecting the development of optimization algorithms. It hopes to promote the further development of relevant research in this field and provide some meaningful insights.

2. GENERAL INSTRUCTIONS

2.1 Optimization Methods

Constrained optimization methods can be divided into unconstrained optimization methods and constraint optimization methods, and the former are the basis for many constrained optimization methods. Meanwhile, constraints can be divided into equation constraints and inequality constraints. The related methods are shown in table 1.

Table 1. Unconstrained Optimization methods and Constraint optimization methods

Unconstrained Optimization methods	Constraint optimization methods	
Descent method	Equation Constraints	Inequality constraint
Gradient descent	Optimality conditions-Karush-Kukn-Tucker	
Line Search, exact and backtracking	Line Search on residual of optimality conditions	
Newton's method	Newton's method for constrained problems	

2.2 Development

Optimization methods are the subject that has formed in recent decades. However, the history of it can be traced back to 500 B.C. A classic optimization problem, the golden ratio of 0.618, was found in Ancient Greece in this period. For a long time thereafter, scholars used mathematical methods to solve optimization problems, but no discipline was formed.

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2.3 Business Understanding

Optimization methods are essentially mathematical methods to solve different extreme value problems. However, optimization problems in many areas can be reduced to mathematical problems and solved using optimization methods. This is why optimization methods are used and create value in different fields. This article will introduce the application of optimization methods in the aviation field.

3. APPLICATION IN PYTHON

3.1 Application

The author has implemented the gradient descent method through python code. The code is shown in Figure 1. The function gradient descent had four parameters: start point, function, gradient, and epsilon, representing the starting point of gradient descent, function, gradient map, and the threshold of the iteration, respectively.

```
import numpy as np
import matplotlib.pyplot as plt


def gradient_descent(start_point, func, gradient, epsilon=0.01):
    """
    :param start_point: start point
    :param func: function
    :param gradient: gradient map
    :param epsilon: threshold to stop the iteration
    :return: converge point, # iterations
    """
    assert isinstance(start_point, np.ndarray)  # assert that input start point is ndarray
    global Q, b, x_0     # claim the global varience
    Q = np.array([[10, -9], [-9, 10]], dtype="float32")
    b = np.array([4, -15], dtype="float32").reshape([-1, 1])
    x_0 = np.array([5, 6]).reshape([-1, 1])
    x_k_1, iter_num, loss = start_point, 0, []
    xs = [x_k_1]

    while True:
        g_k = gradient(x_k_1).reshape([-1, 1])
        if np.sqrt(np.sum(g_k ** 2)) < epsilon:
            break
        alpha_k = np.dot(g_k.T, g_k).squeeze() / (np.dot(g_k.T, np.dot(Q, g_k))).squeeze()
        x_k_2 = x_k_1 - alpha_k * g_k
        iter_num += 1
        xs.append(x_k_2)
        loss.append(float(np.fabs(func(x_k_2) - func(x_0))))
        if np.fabs(func(x_k_2) - func(x_k_1)) < epsilon:
            break
        x_k_1 = x_k_2
    return xs, iter_num, loss
```

Figure 1 Python Code

Hence, the user needs to set the appropriate parameters to call the function. The data in Figure 2 is the author’s parameters. Meanwhile, the author used Python's built-in plt package to draw the corresponding result graph.

```python
if __name__ == '__main__':
    # function and its gradient defined in the question
    func = lambda x: 0.5 * np.dot(x.T, np.dot(Q, x)).squeeze() + np.dot(b.T, x).squeeze()
    gradient = lambda x: np.dot(Q, x) + b

    x0 = np.array([4, 4], dtype="float32").reshape([-1, 1])
    xs, iter_num, loss = gradient_descent(start_point=x0,
                                          func=func,
                                          gradient=gradient,
                                          epsilon=1e-6)
    print(xs[-1])  # last point of the sequence
    plt.style.use("seaborn")
    plt.figure(figsize=[12, 6])
    plt.plot(loss)
    plt.xlabel("# iteration", fontsize=12)
    plt.ylabel("Loss: $|f(x_k) - f(x^*)|$", fontsize=12)
    plt.yscale("log")
    plt.show()
```

Figure 2 Parameters

The final result is shown in Figure 3.

```
[[4.999424]
 [5.998852]]

Process finished with exit code 0
```

Figure 3 Result

According to the author's parameters, the results of the iteration are finally infinitely close to five and six.

The drawing result is in Figure 4.

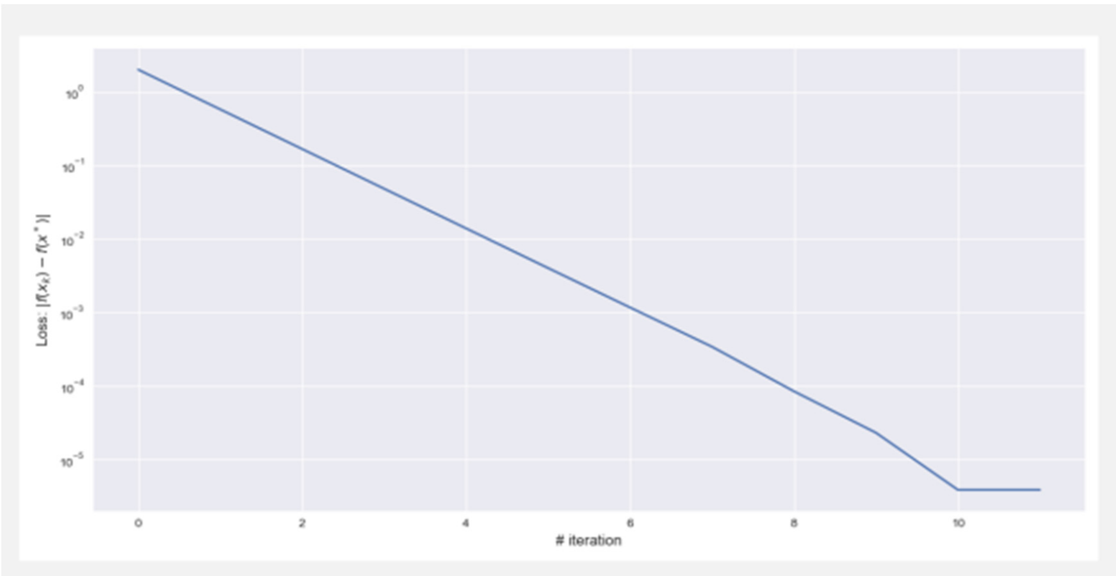

Figure 4 Drawing result

After several iterations, the algorithm gradually approaches the final result according to the step size set by the author.

3.2 Evaluation

Gradient descent can be used to solve least-squares problems, whether linear or nonlinear. The application in

Python by the author is for the problem of least squares. In addition, it usually plays a role in machine learning and artificial intelligence disciplines and is an effective method to solve problems for minimal values.

Unlike the least squares method, the step size setting is important when using gradient descent. Setting the gradient too small, the gradient descent will be extremely slow, and conversely, there may be a missed extreme value. Simultaneously, the defects of gradient descent are also obvious. Firstly, it may converge to a local minimum rather than a global minimum in certain cases. Secondly, the iteration rate will be reduced near the extremes. Hence, it needs a threshold to limit the iterations. This means the result from gradient descent is an approximation of the extreme value.

4. APPLICATION IN AVIATION

In the airline sector, the ultimate goal of the application of optimization methods is to increase the revenue of airline interests. This is mainly reflected in the three aspects of ticket pricing, ticket sales, and flight planning.

4.1 Ticket pricing

In terms of airline ticket pricing, the optimization method requires a rational model to find the optimal airline ticket prices and to ensure the number of tickets sold for different time periods. Ultimately, the aim of this is to obtain the highest airline ticket revenue. This is the problem of "dynamically pricing such inventories when demand is price sensitive and stochastic, and the firm's objective is to maximize expected revenues." [1]

In the case of China's high-speed rail sector, an optimisation model with the objective of maximising total expected revenue was constructed, with seat allocation limits and dynamic fares for each ticketing period as decision variables, taking into account both upper and lower price limits and train transport capacity constraints. A solution method based on a swarm algorithm was designed. This method resulted in a 4.28% increase in expected revenue.[2]

4.2 Ticket sale

Ticket pricing and ticket sales are different problems. Even if airfares are fixed, increasing the number of tickets sold to boost revenue is a problem.

Airline ticket overselling is a legal sales practice in the airline industry. On general voyages, there are customers who miss flights for various reasons, which obviously raises the vacancy rate of aircraft. Optimization methods help to find the best over-sell rate and keep all customers on board, which is a way for companies to increase revenue.

Airfare search engines offering the lowest airfare options will attract more customers. Hence, the revenue will be increased. In 2007, according to an independent analysis conducted by Topaz International, Sabre searched for the lowest airfares more frequently than other GDSs. This is made possible by Sabre's use of complex optimization algorithmic models.[4]At the same time, Talluri and Ryzin (2004) think customer choice behavior plays an important role in revenue management contexts. Learning from customers' browsing history and providing them with airfare options based on their buying behavior is also a key point for the optimization method to work.[3]

4.3 Flight planning

In the field of aviation, the area where optimization methods are most useful is undoubtedly the flight planning problem. It involves a number of factors, including weather interference, high labor costs, airspace and airport congestion, security, and so on.

Hence, sophisticated optimization tools have been used to help its planners make scientific decisions and increase airline profits. [5]: These optimization tools play a role in route, time, and personnel planning.

One of the methods used in the aircraft's flight management system is the optimisation of the aircraft flight plan by means of an optimisation method, whereby the actual cost index of performing the mission is limited to the optimal cost index CIopt, pre-determined by the aircraft operator, by acting on the target values of ETAopt and EFOBopt and the EFOB values to be achieved.[6]

4.4 Aircraft manufacturing

Aerodynamic design issues are at the heart of an aircraft development project. The advancement of this problem relies on traditional computational fluid dynamics analysis capabilities and aerodynamic optimization methods.[7]

During the design of the wing, Mohebbi and Sellier provided a very effective sensitivity analysis solution to calculate the sensitivity of the pressure distribution to changes in the position of the grid nodes. In the process, both the conjugate gradient method (CGM) and a version of the quasi-Newton method (i.e., BFGS) are used as optimization algorithms to minimize the difference between the computed pressure distribution on the airfoil surface and the desired one. [8]

5. CONCLUSION

This paper presents a python application for gradient descent and least squares methods. And it mainly compares the advantages and disadvantages of the two optimization methods. At the same time, the author also presents a detailed introduction to the role of optimization methods in the field of aviation, showing the breadth of application and development prospects of optimization methods.

Optimisation methods play a major role in many aspects of aviation, reducing losses, costs, etc. at all levels. A major contribution to the advancement of the aviation sector

Although the optimization method is only mathematical, its application in combination with mainstream programming languages such as Python allows it to play a crucial role in different disciplines and fields.

As for the limitations, the paper lacks a mathematical model for problems related to the field of aviation, resulting in a python application of optimization methods that only demonstrates the relevant methods and analyses their advantages and disadvantages, without correlating them to the field of aviation.

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