

Dark Matter Production from Inflation under Chiral Symmetry

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ABSTRACT

Many conceptual problems with the standard Big Bang model led us to the inflationary cosmology. The standard model is a broadly accepted theory in particle physics, in which chiral symmetry is a significantly useful property. Though a cosmological constant is just a constant, a time-varying cosmological constant is necessary for some modified theories. The cosmological constant can be coupled to the fermions through the chiral anomaly, and the time dependence of the cosmological constant is transferred to the rate of change of the number density of left-handed and right-handed fermions. During inflation, the cosmological constant was much larger than the present, so fermions were produced in large numbers, and after calculation, these fermions could just provide all the dark matter in the universe today.

Keywords: *Inflation, Cosmological Constant, Chiral Symmetry, Dark Matter*

1. INTRODUCTION

The origin of the universe and the determination of early models have long been questionable issues in cosmology. From the standard Big Bang model to inflation theory [1-4], “Matter Bounce” [5], and the toy model, “String Gas Cosmology” [6], there exist several potential candidates. They make great progress compared to the standard Big Bang model, each with its characteristics.

Evidence suggests [7] that after the Big Bang, the universe may have experienced a violent acceleration of expansion, but the model itself is born with some defects. We are supposed to set our eyes on the inflation theory to make things clearer.

Inflation is an early phase of the accelerated expansion process. It can avoid the above problems of the standard Big Bang theory, except for the singularity problem, and can also solve many problems that the standard Big Bang theory is not capable of, such as the origin of inhomogeneity.

The standard model is an excellent theory widely recognized in particle physics, though fails to answer questions like inflation, dark matter, matter-antimatter asymmetry, and the neutrino mass. There are many important properties in it, which we pay special attention to. For example, inflation will produce many fermions.

Fermions have an important feature, chiral symmetry, which is quite important in the standard model.

Chiral symmetry is an important and widely used property in particle physics. It is composed of flavor transformation and axis-flavor transformation and can be decoupled into a transformation group of left-handed and right-handed charges. Several studies have found a link between chiral symmetry and the cosmological constant.

Studies have shown that there is a close relationship between chiral symmetry and the cosmological constant. The cosmological constant is a long-standing, profound, and practical key cosmological question [8-11], and there are countless studies discussing the cosmological constant. For example, a recent study, through an analysis of the Planck experiment [12], showed that the dark energy component of the universe as $\Omega_\Lambda = 0.6847 \pm 0.0073$, differing from 0 at $>93\sigma$.

In this work, to follow the suggestion from a time-depending cosmological constant that there is a precious potential between inflation and dark matter production, we made an effort to explain where all the dark matter in the universe came from, and have found that the fermions produced during inflation can provide all the dark matter in our universe today as a possibility, which is rarely seen.

2. REVIEW OF INFLATION

Many observations prove the accelerated expansion of our universe, which is both homogeneous and isotropic. From a comparatively large perspective, at > 200 Mpc, the expansion significantly shows its homogeneity and isotropy, which is one of the broadest and most fundamental features of the universe, agreeing well with observations of the cosmic microwave background (CMB) [13] and the distribution of galaxies. But we don't yet know the early stage of this expansion process, in other words, the evolution of the early universe. There are many candidate models of the early universe. To introduce the following discussion of the cosmological constant problem and the production of dark matter, we will introduce the standard big bang model and the inflation theory, and point out why we choose the latter.

2.1. The Big Bang model and its defects

The standard Big Bang model has three necessary principles as a foundation. First comes the homogeneous and isotropic universe on large scales. The second one is the dynamics of space-time described by the Einstein field equations. The last one is the matter description which considers the matter as a superposition of a radiation fluid with the relativistic equation of state and matter of the properties of pressure-less or cold-less.

In standard cosmology, the homogeneity isotropy of the universe is based on the metric,

$$ds^2 = dt^2 - a(t)^2 dx^2. \quad (1)$$

The expansion rate, or Hubble parameter, is given by

$$H = \frac{\dot{a}}{a}. \quad (2)$$

In a homogeneous and isotropic universe, we have the Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \quad (3)$$

Omit the cosmological constant and space curvature, we have

$$H^2 = \frac{8\pi G}{3}\rho. \quad (4)$$

Combining (2)(4), we get

$$\dot{\rho} = -3H(p + \rho), \quad (5)$$

where p is the pressure density and ρ is the energy density. Both of them describe the matter.

However, the standard Big Bang model has a singularity problem, that is, when we look back, the energy density, temperature, and curvature of the universe will continue to increase, and eventually reach a singularity where all three are infinite. Inflationary

cosmology has the same problem, but the standard Big Bang model has some other serious problems that go against the actual nature of the universe.

First comes the size problem. Under standard cosmology, if the scale of the observation horizon is widened, and its temperature is equal to the Planck temperature. As the universe expands, the photon wavelength will gradually increase due to redshift, the wavelength at this time will be longer than $1\mu m$. And if the space of the universe is finite, to match the Planck time, we can only use the Planck length, which is much less than $1\mu m$.

The second one is the horizon problem. For two beams of photons to arrive from opposite angles in the sky, the source points of these photons will be separated by a longer distance than the horizon, which leads to the lack of causal relationship between the temperature of the two points, which goes against the homogeneity and isotropy of the CMB.

Third comes the flatness problem. In standard cosmology, the critical energy density of a spatially flat universe is a point of instability. According to the Friedmann equation, as the universe expands, the relative difference between the actual energy density and the critical energy density increases, leading to the fact that to achieve today's observations of how flat space is, the initial space curvature must be nearly unreasonably precise. It seems too harsh that the universe we live in will not be what it is now with just a few small changes [14].

These problems force us to look to a more promising candidate model, the inflation theory.

2.2. Inflation theory

In the early stages of the universe, compared to the universe today, the vacuum energy density was high, which caused the expansion of the universe to accelerate at a considerable acceleration, what we call inflation.

Inflationary cosmology "inserts" a new cycle of accelerated expansion with exponentially increasing scale factors in the life course of the early universe. Inflation led to a massive production of entropy, which constituted the non-adiabatic evolution of the universe. It was this that contributed to the final resolution of the flatness problem. On the other hand, if the inflation period lasts long enough and occurs on the energy scale of $10^{16} GeV$, the required inflation period is about $50H^{-1}$, which also solves the event horizon problem. Constant energy density provides exponential expansion of space, which solves the size problem [14].

The success of inflationary cosmology stems from the accelerated expansion of space during the inflationary phase. Even if it's not perfect, it still provides the theoretical background we need.

3. INTRODUCTION OF THE STANDARD MODEL AND CHIRAL SYMMETRY

The standard model is a widely recognized model in particle physics that unifies three of the four interactions except for gravity: electromagnetic, weak, and strong, and agrees well with numerous observations.

Symmetry and its breaking are natural properties that appear widely in quantum field theory. Various symmetries, such as flavor symmetry and chiral symmetry, are important subjects in QCD or particle physics research. Among the symmetries, chiral symmetry is a useful property in the standard model and QCD, and it is an important property of massless fermions in flat space. We will analyze chiral symmetry in this section to introduce the relationship between chiral symmetry and the cosmological constant.

3.1. Standard model

The theories and discoveries of thousands of physicists since the 1930s have resulted in a remarkable insight into the fundamental structure of matter: everything in the universe is found to be made from a few basic building blocks called fundamental particles, governed by four fundamental forces. Our best understanding of how these particles and three of the forces are related to each other is encapsulated in the standard model of particle physics. Developed in the early 1970s, it has successfully explained almost all experimental results and precisely predicted a wide variety of phenomena. Over time and through many experiments, the standard model has become established as a well-tested physics theory [15].

The 62 particles of the standard model were first divided into two types: fermions and bosons. Fermions possess semi-odd spins and follow the Pauli exclusion principle to form various substances. Bosons possess integer spins, do not obey the Pauli exclusion principle, and transfer various interactions.

Judging from properties such as the type of color and whether it has anti-particles, there are seven classes of particles, namely quark, lepton, gluon, W boson, Z boson, photon, and the Higgs boson. Among them, there are only two kinds of W bosons. But for Z bosons, photons, and Higgs bosons, there is only one kind of each of them.

3.2. Chiral symmetry

Symmetry is a ubiquitous phenomenon in nature, especially important and worthy of study, revealing many secrets of the objective world, among which chiral symmetry is an indispensable member. To introduce chiral symmetry, we first introduce two kinds of pre-symmetry: flavor symmetry and axial flavor symmetry.

There are six different flavors of quarks. $SU(N_f)$ flavor symmetry is achieved by assigning several quarks with different flavors to the same mass in different scenarios. For an N_f quarks field where all quarks have the same mass, the interaction term is invariant under group transformation, so renormalization does not change the property of equal mass.

Integrating with space the conserved current j_0^α obtained by Noether's theorem, we get the charge,

$$Q^\alpha = \int d^3x j_0^\alpha(x). \quad (6)$$

For each quark from 1 to $N_f^2 - 1$, they satisfy the Hamiltonian commutation and group formulation,

$$[H, Q^\alpha] = 0, \quad (7)$$

$$[Q^\alpha, Q^\beta] = if_{\alpha\beta\gamma} Q^\gamma. \quad (8)$$

In the above algebra, $f_{\alpha\beta\gamma}$ are the structure constants. (8) expresses the group transformation in the N_f group in detail.

Similar to the flavor, the charge of axis flavor is obtained by the following formula,

$$Q_5^\alpha = \int d^3x j_{50}^\alpha(x). \quad (9)$$

The axial flavor transformation, in group formulation,

$$[Q_5^\alpha, Q_5^\beta] = if_{\alpha\beta\gamma} Q_5^\gamma. \quad (10)$$

Considering the group representation of flavor transformation and axis-flavor transformation, we can get

$$[Q^\alpha, Q_5^\beta] = if_{\alpha\beta\gamma} Q_5^\gamma. \quad (11)$$

(8)(10)(11) constitute the algebra satisfied by the charge in the chiral transformation.

The chiral transformation set is a combination of flavor transformations and axial flavor transformations, and on this basis can be spontaneously decoupled into the left-handed and right-handed components. If a transformation group can be regarded as the direct product of the left-handed and right-handed transformation groups, then the transformation group is a chiral transformation group, which indicates that the quark flavor has chiral symmetry.

We introduce Q_L^α and Q_R^β as the charges of left-handedness and right-handedness, written as

$$Q_L^\alpha \equiv \frac{1}{2}(Q^\alpha - Q_5^\alpha), \quad (12)$$

$$Q_R^\alpha \equiv \frac{1}{2}(Q^\alpha + Q_5^\alpha). \quad (13)$$

Substituting (8)(10)(11) into (12)(13), we can get

$$[Q_L^\alpha, Q_R^\beta] = 0, \quad (14)$$

$$[Q_L^\alpha, Q_L^\beta] = if_{\alpha\beta\gamma} Q_L^\gamma, \quad (15)$$

$$[Q_R^\alpha, Q_R^\beta] = if_{\alpha\beta\gamma} Q_R^\gamma. \quad (16)$$

In this way, through the above formula, we decompose the chiral transformation group into each independent $SU(N_f)_L$ and $SU(N_f)_R$ group. The chiral transformation group,

$$SU(N_f)_L \otimes SU(N_f)_R = SU(N_f)_{chiral} \quad (17)$$

is the direct product of these two groups.

Being able to decouple and recouple is an important sign of chiral symmetry and convenient property.

3.3. Chiral symmetry breaking.

Chiral symmetry breaking is a kind of spontaneous symmetry breaking under the function of QCD gauge theory. Perfect chiral symmetry is only established when the mass of the quark is zero. Once this condition is not met, we say that the chiral symmetry is broken. Because quarks have tiny masses relative to the particles they are made of, actually we can still consider chiral symmetry to exist within a certain range. In cosmology, it is believed that the breaking of chiral symmetry originates from the chiral phase transition that occurs in the physical system due to the drop in temperature 10^{-6} second after the Big Bang, which breaks the original symmetry of the universe. This is relevant to what we will cover in the next section.

As previously mentioned, once a group is chiral symmetric, it can be decomposed into the left-handed and right-handed components. That is exactly what Weyl spinors do. For example, we can construct

$$\psi \equiv \begin{Bmatrix} \psi_L \\ \psi_R \end{Bmatrix}, \quad (18)$$

where ψ_L and ψ_R are fittingly named left-handed and right-handed Weyl spinors.

The Dirac equation is

$$(i\gamma^\mu \partial_\mu - m)\psi = 0. \quad (19)$$

With the help of Weyl spinors and a no mass fermion field, (19) is separated into two equations,

$$i(\partial_0 - \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\psi_L = 0, \quad (20)$$

$$i(\partial_0 + \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\psi_R = 0. \quad (21)$$

Just as the chiral symmetric transformations above, the equation of ψ_L and ψ_R decouple. (20) and (21) are called Weyl equations.

The fermion fields ψ_L and ψ_R decoupled from the chiral symmetric fermion field are called a pair of Weyl spinners. Couple them to get the Dirac spinor describing the original chiral symmetric fermion field.

There are two kinds of breaking of chiral symmetry.

The first one is explicit chiral symmetry breaking. We know that the six quarks can be divided into two groups, one lighter and one heavier. They each form a left-handed or right-handed transformation group of three quarks $SU(3)_L$ and $SU(3)_R$. According to (17), the direct product of these two transformation groups is a chiral transformation group. But this group of chiral transformations is approximate because the actual quarks do not satisfy the zero-mass condition, in other words, the chiral limit, although they are very close. A small violation of the chirality limit leads to explicit chiral symmetry breaking.

The second one is spontaneous chiral symmetry breaking. Spontaneous chiral symmetry breaking means that the particle energy state has two stable energy minimum points, but the particle state can only be perturbed around one point. When the point around which the particle is perturbing is identified, a mass effect occurs and the symmetry is broken.

4. CHIRAL SYMMETRY AND DARK MATTER PRODUCTION FROM INFLATION

Based on the above summary, in this section, we will explore the time dependence of the cosmological constant and its role in the production of dark matter through the chiral anomaly.

4.1. Time-varying cosmological constant

Recent studies [16] have shown that new requirements for the cosmological constant are put forward under the coupling of BF theory with fermion fields. The cosmological constant will be no longer a constant, but a time-dependent variable. In this section, we introduce why and pave the way for the generation of dark matter in the next section.

In the fermion-free case, we define a physical cosmological constant Λ . Then, we can establish

$$\Lambda(t) + \bar{\Lambda} = \lambda_0 f(t), \quad (22)$$

where the function $f(t)$ embodies the time-dependent nature, λ_0 is an integration constant and $\bar{\Lambda}$ is the radiation correction term.

The time dependence of $f(t)$ is derived from the time dependence of dark energy. This formula gives the time dependence of the cosmological constant through $f(t)$.

4.2. Dark matter provided by chiral symmetry

We can represent the chiral anomaly as

$$\partial_t(n_{tot}) + 3Hn_{tot} = \frac{2g^2}{\pi^2} \Lambda(t)^2, \quad (23)$$

where $n_{tot} \equiv n + \bar{n} = n_L - n_R$, which represents the sum of left-handed and right-handed fermion number densities, or the sum of the number densities of positive and negative particles. g represents the order, and the whole of $\frac{2g^2}{\pi^2}$ can be recorded as a constant. (23) pass the time dependence of $\Lambda(t)$ to the derivative of n_{tot} concerning time, which results in the time dependence of the rate of the particle density changing.

For n_{tot} , we have

$$\frac{n_{tot}}{s} = 5 \times 10^{-84} g^2, \quad (24)$$

where $s \simeq 2.3 \times 10^{-11} eV^3$ now, represents the entropy.

As can be seen from (24), n_{tot} is extremely small in today's universe. That is, the chiral asymmetry of dark fermions is extremely small. This would explain why the cosmological constant we observe is very small, and why we cannot observe the vacuum particles caused by the cosmological constant.

There is a possibility that the origin of the dark matter lies in inflationary physics [17-23], where taking particle generation caused by a chiral anomaly as an entry point. There is a surprising coincidence that, according to our analysis, through the time-dependent cosmological constant, we get the time-dependent relationship of the rate of particle production. This coincides with the change in the acceleration of the expansion of the universe over time in the inflation theory [24].

We can consider $\Lambda \simeq H^2$, where H_{inf} is the Hubble scale during inflation. With this in mind, depending on the chiral anomaly, n_{tot} can be written as

$$n_{tot}(t) = \frac{2g^2}{3\pi^2 a(t)^3} H_{inf}^3, \quad (25)$$

where $a(t)$ is the scale factor in cosmology. (25) comes directly from (22), with Λ approaches 0 in the post-inflationary universe.

We can find something similar between the cosmic evolution of primordial baryons and the "superheavy dark matter" scenario [23,25-30]. During the radiation-dominant period, the number of particles per Hubble volume increased with the scale factor through a redshift phenomenon that could provide the full amount of dark matter in the universe today. In this case, the superheavy dark matter is an equal mixing of dark baryons and dark antibaryons.

We can write the energy density of the dark baryons as

$$\rho_{darkmatter} \simeq \frac{1}{3} \Lambda_{darkmatter} n_{tot}(t), \quad (26)$$

where n_{tot} is given by (25). We can get such a relationship between $\Lambda_{darkmatter}$ and H by calculation [23], which is

$$\Lambda_{darkmatter} = \left(\frac{5.92 \times 10^{11}}{H} \right)^{\frac{3}{2}} GeV^{\frac{5}{2}}. \quad (27)$$

When (27) holds, the abundance of dark matter we observe is sufficient to result from the chiral anomaly in the inflation process. That is to say, if the cosmological constant is not a constant, but decreases with time, then through chiral anomaly during inflation, the fermion number density n_{tot} in the universe will increase at a gradually slowing rate. Therefore, the large cosmological constant during the inflation period will lead to the production of a large number of fermions which can provide all the dark matter in the universe today.

5. CONCLUSION

In this paper, firstly we introduced several candidate models of the early universe, analyzed their advantages and disadvantages, and made our choice, the inflation theory for the coming work. After that, we introduced the standard model and chiral symmetry for the final part of our work. In the next part, based on the possibility [16] that the cosmological constant is time-dependent and the relation (23) we have found, we built a bridge from the cosmological constant to the changing rate of the number density of fermions, which carry the time-dependency to the latter. We have found that the cosmological constant is decreasing with time, so (23) means there may be a period during which a significant number of fermions were produced. On this basis, we choose the inflation theory, which believes that there was an early period during which the universe accelerated at a considerable acceleration. Following the calculation in [23], we finally found that the fermions produced during inflation can provide the dark matter observed today. That opens up new possibilities for studying the creation of dark matter. But there are still some important possibilities to be figured out. In this paper, we just study the impact of the chiral symmetry on the fermion production, and eventually on the dark matter production, but what the role chiral symmetry breaking plays in the dark matter production remains unclear. A recent paper [31] shows the dark matter candidate after inflation, produced by spontaneous chiral symmetric breaking in the QCD regime, hence it is possible to consider the cosmological constant relating to chiral symmetry breaking, which may generate a new project for later work.

REFERENCES

- [1] Guth AH (1981). The Inflationary Universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* 23 347
- [2] R. Brout, F. Englert and E. Gunzig (1978). The creation of the universe as a quantum phenomenon *Annals Phys.* 115 78

- [3] A. A. Starobinsky (1980). A new type of isotropic cosmological models without singularity *Phys. Lett. B* 91 99
- [4] K. Sato (1981). First order phase transition of a vacuum and expansion of the universe *Mon. Not. Roy. Astron. Soc.* 195 467
- [5] R. H. Brandenberger (2009). Alternatives to cosmological inflation *arXiv* 0902 4731 [hep-th]
- [6] R. H. Brandenberger (2008). String gas cosmology *arXiv* 0808 0746 [hep-th]
- [7] Feuerbacher, B. B., & Scranton, R.. Evidence for the Big Bang. <https://www.uwa.edu.au/study/-/media/Faculties/Science/Docs/Evidence-for-the-Big-Bang.pdf>
- [8] Y. B. Zeldovich (1967). Cosmological constant, and elementary particles *JETP Lett.* 6 316
- [9] S. Weinberg (1989). The cosmological constant problem *Rev. Mod. Phys.* 61 1
- [10] A. Padilla (2015). Lectures on the cosmological constant problem *arXiv* 1502 05296
- [11] R. Bousso (2008). The cosmological constant *Gen. Relativ. Gravit.* 40 607
- [12] N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, and M. Ballardini et al. (2020). Planck 2018 results. vi. Cosmological parameters *Astron. Astrophys.* 641 A6
- [13] Palo Bernardis and Silvia Masi (2012). The Cosmic Microwave Background: observing directly the early universe *arXiv* 1208 0298 [astro-ph.IM]
- [14] R. H. Brandenberger (2011). Introduction to early universe cosmology *arXiv* 1103 2271v1 [astro-ph.CO]
- [15] CERN. The standard package. <https://publicarchive.web.cern.ch/en/Science/StandardModel-en.html>
- [16] Stephon Alexander, Gabriel Herczeg, Jinglong Liu, and Evan McDonoughBrown (2020) Chiral symmetry and the cosmological constant *Phys. Rev. D* 102 083526
- [17] S. Alexander, E. McDonough, and D. N. Spergel (2018) Chiral gravitational waves and baryon superfluid dark matter *J. Cosmol. Astropart. Phys.* 05 003
- [18] T. Tenkanen (2019) Dark matter from scalar field fluctuations *Phys. Rev. Lett.* 123 061302
- [19] N. Herring, D. Boyanovsky, and A. R. Zentner (2020) Nonadiabatic cosmological production of ultra-light dark matter *Phys. Rev. D* 101 083516
- [20] Y. Ema, K. Nakayama, and Y. Tang (2019) Production of purely gravitational dark matter: The case of fermion and vector boson *J. High Energy Phys.* 07 060
- [21] D. J. H. Chung, L. L. Everett, H. Yoo, and P. Zhou (2012) Gravitational fermion production in inflationary cosmology *Phys. Lett. B* 712 147
- [22] P. W. Graham, J. Mardon, and S. Rajendran (2016) Vector dark matter from inflationary fluctuations *Phys. Rev. D* 93 103520
- [23] L. Li, S. Lu, Y. Wang, and S. Zhou (2020) Cosmological signatures of superheavy dark matter *J. High Energy Phys.* 07 231
- [24] M. P. Hertzberg and M. Sandora (2019) Dark matter and naturalness *J. High Energy Phys.* 12 037
- [25] E. W. Kolb, D. J. H. Chung, and A. Riotto (1999) WIMPzillas! *AIP Conf. Proc.* 484 91
- [26] D. J. H. Chung, E. W. Kolb, and A. Riotto (1998) Superheavy dark matter *Phys. Rev. D* 59 023501
- [27] V. Berezhinsky, M. Kachelriess, and A. Vilenkin (1997) Ultrahigh-energy cosmic rays without GZK cutoff *Phys. Rev. Lett.* 79 4302
- [28] V. A. Kuzmin and V. A. Rubakov (1998) Ultrahigh-energy cosmic rays: A window to postinflationary reheating epoch of the universe? *Phys. At. Nucl.* 61 1028
- [29] M. Birkel and S. Sarkar (1998) Extremely high-energy cosmic rays from relic particle decays *Astropart. Phys.* 9 297
- [30] P. Bhattacharjee and G. Sigl (2000) Origin and propagation of extremely high-energy cosmic rays *Phys. Rep.* 327 109
- [31] Mayumi Aoki, Jisuke Kubo, and Jinbo Yang (2022) Inflation and dark matter after spontaneous Planck scale generation by hidden chiral symmetry breaking *arXiv* 2109 04814v2 [hep-ph]