Exploring Secondary Mathematics Teachers' TPACK Development and Student-Centred Beliefs

Bohan Liu[1,*]

[1]*Institute of Education, University College London, 20 Bedford Way, London, WC1H 0AL, United Kingdom*
**Corresponding author. Email: bohan.liu.20@ucl.ac.uk*

ABSTRACT

While technology becomes progressively important in mathematics classrooms, teachers' technological pedagogical content knowledge (TPACK) attracts attention, and teachers' beliefs are regarded as a barrier to integrating technology in teaching. To remedy the shortage of investigations about the correlation between a wider range of the belief systems and TPACK in mathematics, based on the assumption that TPACK enhancement requires mathematics teachers' practical use of digital hardware and software, this essay focuses on student-centred beliefs and reveals that TPACK is an extension of the belief systems rather than an isolated area of knowledge, and student-centred beliefs tend to advantage TPACK development. The obstacles of strengthening TPACK include the unavailability of digital tools, scheduled teaching to prevent technology use and opportunities to experience the strengths of technology, and mathematics teachers' intrinsic resistance to change their practice to integrate more technology into teaching. In response to these, it is necessary to provide specific projects and ICT intervention schemes focusing on pedagogy, set standards for TPACK level before obtaining qualified teacher status, and construct a shared space that mathematics teachers, mathematics education researchers, policymakers, and digital technology designers can communicate and collaborate to promote TPACK learning and enhancement.

Keywords: *TPACK, student-centredness, digital technology, education, mathematics*

1. INTRODUCTION

A recent report The State of Digital Learning 2020 published by Schoology [1] highlighted the important role of digital teaching tools in education, such as increasing classroom efficiency, the demand for relevant and effective professional development, and the wide acceptance of digital learning as part of strategies for teaching and learning. As substantial research regarding technology-enhanced mathematics education drew on the potential of digital tools to boost learning experiences [2], Technological Pedagogical Content Knowledge (TPACK) has emerged as a powerful framework to analyse teachers' integration of technology [3]. In the past few decades, research massively explored the concept of TPACK theory, understanding of technological knowledge [4], and TPACK development in specific disciplines [5]. Although teachers' beliefs influence such incorporation of technology into classroom instructions [6], few studies looked into teachers' beliefs and TPACK [7] [8], especially in the context of mathematics [9].

This paper thus intends to first discuss a constructivist-oriented view—student-centred belief (SCB) in the complicated belief systems and TPACK in mathematics and how they associate with each other in secondary mathematics education. This is followed by investigating the difficulties of TPACK development and some suggestions on how to better support mathematics teachers in enhancing their TPACK.

2. DEFINITIONS

2.1 PCK and TPACK

Shulman [10] suggests a particular type of knowledge demanded for teaching—Pedagogical Content Knowledge (PCK)—an "integrated knowledge structure of the subject area, knowledge of students, pedagogical knowledge, and knowledge of the environmental context as teachers engaged in planning, teaching, and assessing activities" [11, p. 42]. Thus, the central idea of PCK is that learning to teach needs not only an understanding of subject-matter contents but also the acquisition of appropriate teaching strategies and skills tailored to the learner's characteristics. Specific to Mathematics, PCK requires teachers' competence in organising the representation of particular mathematics topics and problems, adapting them to meet students' different interests and abilities and presenting them in teaching, and knowing what would facilitate students' learning and what makes mathematics learning difficult, including the students' prior knowledge and possible misunderstandings. More importantly, mathematics teachers must value the interactions between mathematical content and pedagogy rather than oversimplifying them as two isolated segments.

Recognising the complicated and multifaceted feature of the knowledge required for teaching and capturing the key qualities needed for integrating technology in teaching, Shulman's PCK framework was extended to Technological Pedagogical Content Knowledge (TPACK) by incorporating a new category—technological knowledge (TK)—about standard equipment, such as blackboard and textbooks, and more advanced technologies such as digital tools and the Internet. Mishra and Koehler [12] first defined TPACK as a foundation of successful teaching with technology that includes, apart from what PCK covers, knowledge about understanding and using technologies to teach and how technologies contribute to solving difficulties that students encounter, constructing and consolidating the students' existing cognitive structure, and developing their new knowledge.

From these regards, locating at the intersection of content, pedagogy, and technology, TPACK implies that mathematics teaching is a highly complex activity that relies on multidimensional and comprehensive knowledge, a field in which all these aspects intertwine. What TPACK emphasises is that conducting effective teaching requires mathematics teachers' full comprehension and quality coordination of basic types of knowledge in technology, pedagogy, and subject content, and the overlapping and interactions between them—pedagogical content knowledge (PCK), technological content knowledge (TCK), and technological content knowledge (TPK).

2.2 Teachers' student-centred beliefs

Broadly speaking, researchers categorise teachers' practice and their beliefs into two contrasting types—traditional (teacher-centred) views and constructivist (student-centred) views [9] [13]. Teachers with teacher-centred perspectives regard obtaining relevant content knowledge as the primary learning objective [14], and they view mathematics as an unchanged, certain, absolute, and applicable collection of facts, procedures, and skills [15]. As a result, mathematics teachers of this type usually act to transfer rigid knowledge to students, and their instructions often include explaining how students should follow fixed and universal rules and processes [8]. In comparison, the constructive view, mainly developed from Jean Piaget's work, supports that knowledge is gained through reflections on the learners' experiences and active construction in their minds [16]. From a constructive standpoint, mathematics learning is that, rather than having knowledge transferred or imbued by others, the learners actively engage in constructing their own knowledge, keep incorporating new mathematical knowledge into their previously established understanding, and constantly update mathematical knowledge through active interaction with the outside world.

As a belief originated from the constructivist learning theory, student-centredness involves "a high degree of discovering knowledge" by students themselves to improve "learning and knowledge retention, with less dependency on the teacher as the owner of knowledge", and teachers "cannot offload information into the students' brains and expect them to process and apply it" [17, p. 320] , but can act as facilitators and someone who provide resources of learning [18]. Hence, student-centredness attaches importance to the students' irreplaceably principal positions as primary agents of learning and recognises their greater subjectivity and responsibility throughout discovering and building mathematical knowledge structures and the students' initiative, different abilities, and needs are prioritised. Compared to teacher-centredness that values procedure, mathematics teachers who favour student-centredness see mathematics as a tool for sense-making and a dynamic field to be continually expanded, thus tend to encourage inquiry-based activities to enhance students' mathematics learning experience [19]. It can be therefore seen that teachers' established perceptions towards the nature of mathematics in practice will influence their beliefs about mathematics teaching and learning, which in turn impact their preference for practical classroom design and instructions.

3. RELATIONSHIP BETWEEN MATHEMATICS TEACHERS' TPACK AND STUDENT-CENTRED BELIEFS

Unlike subject knowledge, pedagogical knowledge shared between teachers in different disciplines, or technology experts' knowledge [12], TPACK is an emerging region of knowledge that transcends all three single-dimensional elements. There is a correspondence between low TPACK and teachers' limited experiences working with technology in mathematics teaching [9][20] and teachers' preference for computers. That is, TPACK cannot be transferred to teachers' minds, learning to teach is also an iterative process where teachers construct their knowledge through their implementations and practice of technology use in person within their own classrooms to expand their technology-related knowledge. This highlights the importance of an advantaged environment, such as technology-enhanced mathematics classrooms, where there are rich opportunities to use digital tools in practical teaching and reflect on the teachers' experience for developing mathematics teachers' TPACK. Thus, the basic assumption of this piece of work is that mathematics teachers' practical use of digital technologies in classrooms can underpin their TPACK enhancement.

Technology-enhanced mathematics teaching encompasses structured approaches to activities altered by digital technology usage, promoting more inquiry, exploration, and collaboration, and teachers usually serve as facilitators of student learning in such context [2]. With this said, teacher-centred beliefs may face more serious difficulties adapting to a technology-enhanced environment since accommodating technology into mathematics classrooms appears to demote teachers away from being the controllers and weaken their "central" position in lecture-based classrooms. Reversely, with the aid of technological tools, more abstract notions and advanced mathematics beyond manual calculation and algorithms, such as mathematical modelling and visualisation [21], could be covered, and students are better encouraged and supported to understand mathematical concepts [8]. Additionally, mathematics teachers with strong TPACK used more student-centered strategies, including discovery learning and collaborative group activities [11].

Therefore, the use of technology facilitates the student-centred views of learning by conducting active discovery, arousing students' interest, and enhancing the opportunities of understanding mathematical concepts. More importantly, mathematics teachers' role as a facilitator in technology-enhanced classrooms appears to be aligned with that in the student-centred beliefs (SCB). Meanwhile, the greater exposure under a rich resource of technologies tackles the difficulties in TPACK development due to the insufficient use of technology and allows teachers' attempts at digital tools. Consequently, mathematics teachers can develop understanding concerning better applying technologies to provide greater autonomy to the students in the way that enables them to manage their learning progress through materials.

Notably, there exist complicated relationships between the strength of SCB and teachers' TPACK level—Various levels and biases of these beliefs exert different degrees of impact on TPACK, and pedagogical beliefs alone may not well predict TPACK. Concretely, as Lai and Lin [8] found, the presence of technology value, an additional variable, influences the association between SCB and TPACK: Mathematics teachers with stronger SCB and higher technology values tend to perform better TPACK, but stronger SCB does not necessarily indicate higher technology values. However, mathematics teachers with higher SCB who view technology as an instructional strategy rather than an essential teaching strategy have lower TPACK in practice. The situation of pre-service secondary mathematics teachers appears to be more complicated. A case study [9] focused more specific to SCB towards different elements in mathematics teaching and revealed a more intricate association—Despite more student-centered or constructivist-oriented beliefs to technology use and strong TCK, teachers' TPACK might still be low because of the traditional beliefs about mathematics itself and learning mathematics, and teachers' low mathematical knowledge (CK) and PCK.

Under the progressively more considerable impact of TPACK and its constituents on a variety of K-12 disciplines [22], several TPACK-focused teacher professional development programmes attracted some attention, and TPACK catering or teachers have been developed to promote the use of student-centred information technology. Koh [23] invented three kinds of TPACK design scaffolds—a meaningful learning rubric, lesson design heuristics, and TPACK Activity Types—to support the change of teachers' concept from teacher-centredness to student-centred and help the teachers better match technology, pedagogy, content, and their practice and context, thus further developing teachers' faith in TPACK. Jimoyiannis's [5] TPACK model for integrating ICT into teachers' teaching as part of a broader programme aimed at preparing teacher educators, including secondary school mathematics teachers.

Thus, on the one hand, TPACK cannot be regarded as a separate region of knowledge to beliefs, which is a complex system—In reality, beliefs are not as oversimplified as binary options between teacher-centredness and student-centredness. Although substantial theories and models were proposed to improve pre-service and in-service teachers' knowledge and technology integration skills, such as designing curricula and professional training workshops based on TPACK frameworks, the influence of contextual factors in the TPACK framework is recognised [24]. This embodies the need for special attention to the context when assessing the relationship between TPACK beliefs—Probing different constituent elements in teaching separately may give different results, other factors such as teachers' technology values and experiences or knowledge with technology will jointly influence the connections between SCB and TPACK.

On the other hand, TPACK can be deemed as an internal concept existing in teachers' cognition based on the facts, and it is a dynamic set of knowledge that can be developed through external training and learning and actual teaching in classrooms. Mathematics teachers' beliefs are fluids that can flow between teacher-centredness and student-centredness, and teachers need time to cultivate and develop student-centred beliefs, which is formed and extended from their past experience, understanding of different types of knowledge, including

TCK, TPK, and TPACK, and information evaluation of information over their long-term career, followed by these perceptions and skills impacting on the teachers' use and attitudes of technologies. Although stronger SCB does not necessarily correspond to stronger TPACK, with what student-centred beliefs advocate, teachers who possess more student-centred views tend to more easily realise that students need different knowledge delivery and progress at different paces. Thus, it is reasonable to speculate that in these teachers' lessons, various learning opportunities offered by different software and digital hardware are more welcomed, while mathematics teachers can get access to use these digital tools and then enhance technological skills as a part of TPACK.

Nevertheless, even with student-centred perspectives, mathematics teachers' tasks are not as simple as expected, and learning, applying, and disseminating TPACK are not only crucial for incorporating mathematical content and technology in mathematics but may also change the way teachers are trained and how technology is used in student-centred situations. Therefore, prior to investigating how to better support mathematics teachers to develop TPACK while having SCB, there is a need to recognise the impact of contexts in TPACK, particularly the difficulties of using technology and applying TPACK concerning the belief and implementation of student-centredness.

4. SUPPORTING MATHEMATICS TEACHERS' TPACK DEVELOPMENT

4.1 External difficulties

Mathematics teachers with student-centred beliefs may not know how to use technology effectively to teach mathematics, or they are unfamiliar with student-centred ICT practices despite their strong general knowledge and positive attitude towards technology use in mathematics classrooms. This is because they may be unable to use digital tools [25], or their knowledge of technology is restricted by socio-cultural factors, including the shortage of technological or administrative support, the lack of access to computers, and insufficient time for lesson planning with digital tools. Sunderland also highlighted that the shortage of student-led mathematical modeling, problem-solving, and computer programming that "makes use of the powerful mathematical digital technologies that are widely used in society and the workplace" are inadequate [26, p. 24]. In response to such external constraint, many countries introduced professional TPACK development programmes [27]-[30] to address the lack of technical resources and financial support for in-service and pre-service teachers. These efforts created facilities for simplifying the procedures for scheduled access to digital tools and formed a basis for timely and maximal usage of technology, translating the latest technology into classroom practice, and enhancing teachers' TPACK during using these facilities.

Nevertheless, having rich availability of technological resources does not necessarily improve practical use and integration of technology. An overemphasis on student academic performance by schools and parents can hinder the perception of the perceived opportunities of ICT in education, as this pressure and competition can force the curriculum to involve more process-driven and exam-oriented questions [31]. Moreover, it takes time for teachers to acquire new skills, knowledge, and confidence in the usage of software and hardware [32]. Niess's interviews found that although sometimes teachers have a strong technological and mathematical background and are very skillful in lesson planning, TPACK development is still impeded [33]—It takes time to construct the teachers' ideas regarding technology, prescribed timetables and scheduled teaching force their narrations to dominate most of the lesson times, resulting in students having to focus on following rigid instructions, and thus pushing the lesson towards the teacher-centred terminal and resisting SCB.

By the aforementioned features, this works against SCB and technology-enhanced mathematics classrooms in a way that creative activities using digital tools are constrained, and students are more rarely given chances for exploration and active discovery and less freedom to manage their study. As a result, schools that concentrate on ICT facilities in classrooms or labs may find that teachers are left limited time to effectively develop TPACK by using technologies and exploring relative skills due to the conflicts. Hence, along with the provision of digital software and hardware, schools need an ethos that welcomes teacher innovation in the classrooms, allowing the teachers to spend time experiencing the use of technologies and devise new approaches to fit technology into their own practices while conforming to their SCB.

4.2 Researchers' and teacher educators' contributions

While researchers keep investigating different exemplars and digital tools in teaching practices, teachers must explore technological devices alone and attempt to learn technological knowledge from their current classroom environment without the assistance of the research teams, resulting in preventing many potential digital innovations from entering mainstream education [34]. This reflects the gap between research and practice and the need to keep focusing on TPACK in a more practical and operational way to guide mathematics teachers in the design of ICT interventions. Regarding the classroom implements, mathematics teachers must be given chances to share their experience of using TPACK in more student-centric lessons and allow them to observe each others' classrooms where TPACK is well demonstrated and applied.

Teachers' practical challenges in these TPACK-focused ICT professional development projects were manifested in ambiguous outcomes. Despite some TPACK professional development programmes that effectively strengthened mathematics teachers' technological proficiency and the manipulation of specific digital tools, not all teachers involved felt confident when incorporating digital tools to facilitate student-centered learning after the participation [29]. Thus, a more powerful contribution is to researching and developing specific pedagogy-focused programmes for supporting teachers articulate student-centred ICT curriculum design in the context of technology [11]. Typically, these projects need to help in-service and pre-service mathematics teachers understand the complexities of the environment, including how to use digital tools and adapt instructions to take students' different needs into account while considering what types of digital tools are suitable to engage students and what approaches are most effective for learning.

From the perspective of TPACK per se, teachers' TPACK cannot stand in isolation from the components linked to technology. Apart from being the interconnections and constraints of technology, TPACK is also the interaction between technology-supported mathematics curriculum resources and the students' needs and abilities as it requires teachers to effectively employ technology-related knowledge. In light of this, robust knowledge within TCK, TPK, and PCK in mathematics classrooms are crucial since these mediating variables bridge the single dimensions of knowledge and effective cultivation of knowledge in more complex contexts. To some extent, PCK advancement can well help prepare for knowledge of teaching and learning using digital technology [11]—analysing "how to teach" requires mathematics teachers' deep understanding of the students. Teacher educators, therefore, must provide necessary and adequate opportunities to consolidate pre-service and in-service mathematics teachers' PCK and prepare them for extending PCK to knowledge that integrates and employs a deeper understanding of technology in teaching and learning.

4.3 Internal difficulties

Some internal factors that stop teachers from enhancing TPACK under student-centred views have also been pointed out. For example, Chai, Koh, and Tsai [35] revealed that teachers' perspective in teaching is one of the key causes that influence their knowledge and practice, and teachers' attitudes towards digital tools, including their preference, confidence, and anxiety of using computers, which will influence the beliefs of teachers who have constructivist views [8]. This is consistent with what was drawn on that teachers' technology values, as another component in teachers' internal belief systems other than SCB, will interfere with the use of technology and the opportunity to develop TPACK. It follows that teachers' knowledge, past experiences, and beliefs about teaching are evolved over long teaching careers and will impact teachers' perceptions of pedagogical change, and their internal refusal or reluctance to adapt to the original concepts creates greater resistance than external barriers—Any effort to improve mathematics teachers' TPACK should first detect the deep-rooted beliefs and then design programmes and teaching skills that are catering to the corresponding beliefs.

Internal factors also come from teachers' resistance to change their practices as pedagogical concepts are gradually constructed through years of teaching [23] unless they can clearly see the benefit from using technologies and assessments genuinely value the relevant skills strengthened through technology use [36]. Moreover, considering that the changes in pedagogy and instruction often reflect educational visions or curriculum objectives [37], the policies seem more decisive to tackle the lack of secondary mathematics teachers' motivation and willingness to utilise digital technologies. More specifically, mathematics curricula need to push teachers to learn technology by, for instance, specifying expectations of ICT operation and manipulation for both teachers and students, setting standards for technology use in teacher training schemes before the approval of qualified teacher status, and requiring trainee teachers' school mentors to have stronger TCK, TPK, and TPACK to guide the trainee teachers when needed. However, it is not always possible to force all in-service or pre-service teachers to have good knowledge of technologies due to the lack of digital devices in some rural or lower-income areas.

Combining the above suggestions, contributions from individual sectors seem far insufficient. Practical teaching requires an overall climate in mathematics education with an ethos welcoming the teachers to innovate in their classrooms, try out new technologies wherever possible, and apply TPACK to support students to explore problems based on a good understanding of the students' knowledge and potential. Additionally, a shared space between researchers, teachers, policymakers, and digital technology tool designers is needed. First, the space enables the researchers to publish their investigation results about the general patterns of effective ways of using and developing TPACK in mathematics classrooms, based on the teachers' implementation. Second, the space should encourage communications between mathematics teachers and allow the teachers to inform other parts of the space of their feedback of implementing different pedagogy and applying new digital tools. Where SCB is available, teachers, as facilitators, must carefully consider how they can use their TPACK to design lesson activities and enable them to interact creatively with other elements of the classrooms. Third, designers can promote their products and take teachers' views and feedback on board to modify the functions of the tools. Fourth, policymakers and teacher educators should integrate views from different sectors, push the construction and intensification of teachers' technological values, provide seminars or learning communities to boost mathematics teachers' understanding of digital tools and technology use.

5. CONCLUSION

Intending to suggest better help mathematics teachers integrate technology and use TPACK into their practice favouring student-centred views of learning, this piece of work penetrated from the interpretation of the notions of TPACK and a student-centred view of teaching in mathematics and then proposed some potential measures.

TPACK is a dynamic set of knowledge existing in internal concepts. Like SCB, TPACK is constructed through mathematics teachers' information evaluation and knowledge expansion over their teaching careers and self-reflection and is enhanced through external training. However, the correlation between TPACK and SCB requires a more comprehensive context. Despite that SCB embraces technology-enhanced classrooms and technology use, which underpins TPACK development, other elements in the belief systems also matter—Even with low SCB, high technology value corresponds to stronger TPACK. Thus, future research can concentrate on other aspects of the belief system, such as self-efficacy, to obtain a bigger picture of how beliefs relate to TPACK.

Some external obstacles, including the shortage of digital tools and scheduled teaching, result in mathematics classrooms favouring teacher-led and preventing teachers from trying out new technology. Gaps between research and implementation about technology use often leave mathematics teachers to explore technology use without support. Meanwhile, internal restrictions such as teachers' negative perceptions and unwillingness to use digital tools more significantly hinder TPACK development. Therefore, apart from adequate digital tools, specific models and ICT intervention schemes focusing on pedagogy are more essential to articulate student-centredness and understand technology-enhanced classroom environments. Professional training must provide instructions to strengthen technology-related skills and, for less experienced or pre-service mathematics teachers, set requirements to ensure a satisfying level of TPACK for approving qualified teacher status. More essentially, researchers, practitioners, technology designers, and policymakers must be aware of the importance of shared communities across all sectors involved in mathematics education to make research, classroom TPACK, digital tools, curriculum, and policies a closer integration consisting of frequent and effective communication and collaboration.

Considering that teachers may not implement constructivist pedagogy although they favour them [8], follow-up empirical-based investigations would be more convincing to reveal how secondary mathematics teachers with SCB regard and overcome the challenges of using technologies and developing TPACK under practical classroom environments. A multi-case study approach is appropriate by the means of allowing descriptions and identification of individual teachers' beliefs, TPACK, and the underlying associations between the two through cross-case analysis [38].

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