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Investigation on the Mechanism of American Put Option Pricing under Binomial Consideration

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ABSTRACT

American options are style of options which allows the holder to exercise their rights at any time before and including the expiration date. Different from European options, that only allows the holder to exercise their right at the expiration date, American options seems to bring the holders with more freedom. There are two common types of options: calls and puts. In this investigation, we will be focusing on American put options specifically. Puts give the buyer the right, but not the obligation for selling the underlying asset at the strike price in the contract. Moreover, options may have their own pricing theory, that is a model to estimate the value of options by assigning a price, which is known as premium. With the theory, American option holders could make the decision of holding the option or not. Therefore, the pricing model of an American option seems to be significant and important for the final decision.

Keywords: *American options, Binomial model, Put options, Python*

1. INTRODUCTION

The pricing of American-style options has been extensively studied over the past 20 years. However, due to the diversity of their sources of return, there is no efficient and accurate method for pricing them. The binomial model is a risk-neutral model that serves as the most flexible approach, simplifying option pricing calculation and adding intuition [1, 2, 3]. It calculates the strike price for different time periods by pre-specifying the number of steps in the tree using python and with the desired accuracy. Also, to ensure that the nodal prices of the different routes overlap, the upward and downward movements of the stock price should satisfy $u * d = 1$; where u and d are the upward and downward movements, respectively. The purpose of this article is to explore the pricing mechanism and early exercise fees for American options under different K (risk free rate) values.

This investigation will bring up two different approaches for investigating the pricing mechanism of American puts under two periods, the binomial pricing model and Python. In the first part of the investigation, the Binomial approach will be explained and calculated under different cases; in the second part of the investigation, the Python approach will be debuted and tested.

1.1 Methodology

As mentioned in the abstract, the investigation will be carried out within two types of methods: the binomial calculation approach, and the python approach.

The binomial option valuation is setting the option under two conditions in the future payoff consideration, whether it’s going up or going down with the given probability of each condition. Under the given probabilities, the value could then be computed [4]. To assure this investigation is fair and just, the cases should all ensure that there’s no opportunity for arbitrage. Under this big assumption, this investigation has been separated to 3 time periods: time 0, time 1, and time 2. Under each period, this report will discuss the cases and probabilities and reach a conclusion.

The second approach in the investigation will be adopted by Python. This report will discuss the Python mechanism devised and will use it to test several examples to demonstrate its effectiveness.

2. American option pricing (for two periods) with Binomial

2. CALCULATION APPROACH

The pricing of options should be placed under different times for consideration. The two-period binomial option pricing model will be adopted to stimulus real-world situations. In order to generalize our findings, this work will use different variables for modeling, then explain different cases with the variables [5].

2.1 Two periods binomial option pricing model

A popular pricing model which based on the idea that in the next period, the value of an asset can only be equal to one of the two values calculated, normally it's the case of the value increases and value decreases.

2.1.1 Variables and assumptions

There are a few variables that we will use in the following research. In the work will define $S0$ as the current value of the asset; u is the factor of increase, whereas d is the factor of decrease. K is the strike price of the option, which is the price that an option can be exercised; R is the risk-free rate. To ensure that no arbitrage opportunity existed, the constants d, u, and R should satisfy $d < R < u$.

2.1.2 The general pricing model and pricing

With the binomial model assumption and the variables, it is possible for this work to compute the value of the risky asset at different stages (times), *Figure 1* provides a visual depiction of a binomial tree model.

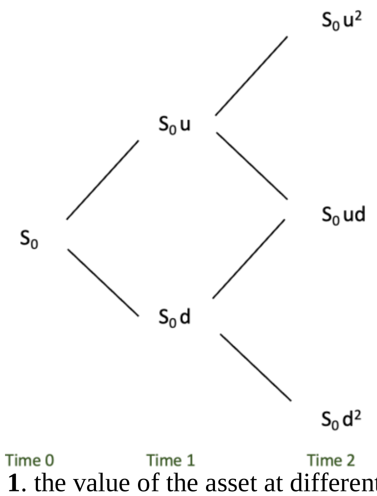

Figure 1. the value of the asset at different times

2.1.3 Computing the general formula for prices

The prices at time 2, obviously, could be computed as the difference between K and the value of the asset. Figure 2 gives a depiction of the situation.

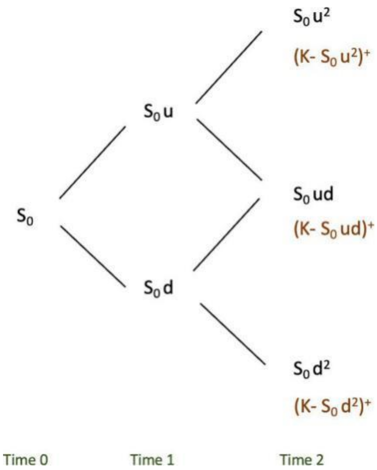

Figure 2. the prices of the asset at time 2

For computing the prices at time 1 and time 0, this work should adopt risk-neutral pricing.

Ration, q. The risk-neutral pricing ratio is computed by the following formula:

$$q = \frac{r-d}{u-d} \tag{1}$$

With the formula, the prices can be computed at time 1 (N^S). N^S is the representation of the change in option price over the change in stock price; it could be shown with the mathematical formula:

$$N^s = \frac{F(S_0 u) - F(S_0 d)}{S_0(u-d)} \tag{2}$$

Hence, the prices of both time 1 and time 2 could be represented as (see Figure 3):

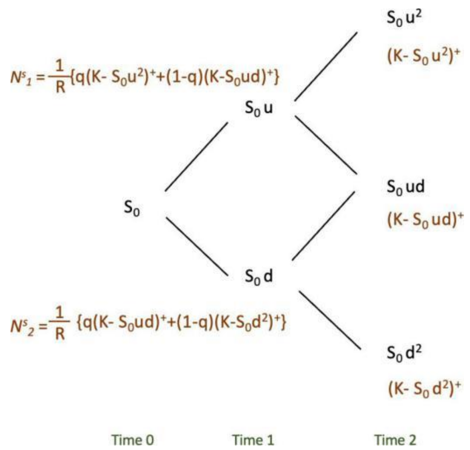

Figure 3. prices (N^s) at both time 1 and time 2 with value S0

The pricing mechanism at time 0 is the difference between the value of the asset at time 1 ($S_0 u$ and $S_0 d$, separately) and the prices at time 1 (N^s). But the value that could be generated at time 0

depends largely on the cases adopted at time 1. In other words, the decision at time 1 has a direct correlation to the pricing mechanism at time 0. Therefore, it's significant to investigate the possible decisions at time 1 first.

2.2 Early exercise condition in American options

As mentioned, American options give the holder a right [5]. Under the binomial approach, an early exercise condition in American options happens when N_1^s or N_2^s is smaller than $K - S_0 u$ or

$K - S_0 d$. The theory behind this condition is that holders always tend to make a profit from optioning the puts. When the prices of the puts (*N*s*1* or *N*s*2*) they hold are larger than the strike price in the market (*K- S0 u* or *K- S0 d*), it will not exercise early as the put is profitable at this point. When the prices (N_1^s or N_2^s) are actually lower than the market ($K - S_0 u$ or $K - S_0 d$), early exercising occurs [6].

2.2.1 The possible decisions at time 1

Decisions that could be made at any time before or on the expiration date is the specialty of an American option. This work separated the decisions into two categories: the cases that the value of the asset goes up and the value of the asset goes down (referring to $S_0 u$ and $S_0 d$).

2.2.2 Discussion over the cases

The comparison should be placed under different lenses: though the values of the variables are constant, but this work still don't know the relationship between the variables. Hence, this work could divide them up into 6 cases under this category (shown as following):

Case 1 $S_0 u^2 < K$

Case 2 $S_0 u < K < S_0 u^2$

Case 3 $S_0 < K < S_0 u$

Case 4 $S_0 d < K < S_0$ Case 5 $S_0 d^2 < K < S_0 d$

Case 6 $K < S_0 d^2$

With the cases, the decisions are able to be evaluate separately. But there's still one big assumption, aside from the non-arbitrage opportunity ($d < R < u$), that is the discussion of $u * d$. This investigation assumes that $u * d = 1$, which makes the cases simpler to interpret. This work are going to interpret the cases individually.

2.3 Investigation under each case

2.3.1 Case 1 $\boldsymbol{S_0 u^2 < K}$

In order to make a decision of early exercising if the put asset's value goes up, we shall compare the if $N^s \leq (K - S_0 u)^+$. Therefore, we shall prove if the following inequality holds true or not:

$$N^s = \frac{1}{R}\{q(K - S_0 u^2)^+ + (1-q)(K - S_0 ud)^+\} \leq (K - S_0 u)^+ \quad (3)$$

With $S_0 u^2 < K$, the value of $(K - S_0 u^2)^+$ will actually have a positive value, giving out $(K - S_0 u^2)$. As $u * d = 1$, the equation could be rewritten as:

$$N^s = \frac{1}{R}\{q(K - S_0 u^2) + (1-q)(K - S_0)^+\} \quad (4)$$

$$N^s = \frac{1}{R}\{q(K - S_0 u^2) + (1-q)(K - S_0)\} \quad (5)$$

Then computing the inequality into

$$\frac{S_0(u-R)(ud-1)}{u-d} \leq K(R-1) \quad (6)$$

As $u * d = 1$, and $R > 1$, the inequality holds true under the condition of the value goes up to $S_0 u$

with $S_0 u^2 < K$. Therefore, this work shall early exercise the option at that time.

When the value of the put goes down to S0d, under this condition, this work shall prove if the inequality below holds true or not:

$$N^s = \frac{1}{R}\{q(K - S_0 ud)^+ + (1-q)(K - S_0 d^2)^+\} \leq (K - S_0 d)^+ \quad (7)$$

However, with the information given, it is hard to prove this inequality.

2.3.2 Case 2 $\boldsymbol{S_0 u < K < S_0 u^2}$

Let's start the investigation under this case when the value goes up with *S0u*. As the value

$S_0 u < S_0 u^2$, and $u * d = 1$ as this big assumption, it can conclude that *u* is positive and larger than

1. In order to consider the case when the price goes up, this work shall prove if the equation below exists or not.

$$N^s = \frac{1}{R}\{q(K - S_0 u^2)^+ + (1-q)(K - S_0 ud)^+\} \leq (K - S_0 u)^+ \quad (8)$$

With $K > S_0 u$, the value of $(K - S_0 u)^+$ is positive and will result $(K - S_0 u)$. But, the condition states that $K < S_0 u^2$, which means that the value for $(K - S_0 u^2)^+$ will eventually become 0. As $u * d = 1$, the equation of $(K - S_0 ud)$ could be rewritten as $(K - S_0)$, which $K > S_0$. Hence, $(K - S_0 ud)^+$ will give $(K - S_0)$. Since $S_0 u > S_0$, we are able to see that the right-hand side, $(K - S_0 u)^+$ must be larger than the left-hand side, N^s. Therefore, the condition is true, hence, this work shall early exercise the put when the price goes up and the condition of $S_0 u < K < S_0 u^2$ exists.

If the value of the option goes down, to $S_0 d$, with the condition $S_0 u < K < S_0 u^2$ the decision shall be made after the computation. This work shall again, prove if the equation holds under the condition and our assumption, that is $u * d = 1$.

$$N^s = \frac{1}{R}\{q(K - S_0 ud)^+ + (1-q)(K - S_0 d^2)^+\} \leq (K - S_0 d)^+ \quad (9)$$

As the value of d is positive but smaller than S_0, we are able to deduce that $K > S_0 d$ and $K > S_0 d^2$. This makes the right-hand side of the inequality, $(K - S_0 d)^+$, $(K - S_0 d)$. Moreover, the value of $(K - S_0 d^2)^+$ will also have a numerical value, $(K - S_0 d^2)$. With $u * d = 1$; and our possible deduction of $K > S_0$, the value of $(K - S_0 ud)^+$ will then become $(K - S_0)$. But, since the left-hand side has a reciprocal, $\frac{1}{R}$, this makes the left-hand side smaller than the right-hand side. As a result, the condition is true under the proof. Thus, the decision of early exercising the option should be made if the value of the put goes up and the condition of $S_0 u < K < S_0 u^2$ exists.

2.3.3 Case 3 $\boldsymbol{S_0 < K < S_0 u}$

Under the consideration of the value going up, with *S0u*, in the work can decide whether the value for u is larger than 1 or not. As $u * d = 1$, and the case gives $S_0 < S_0 u$, this work can figure out that u is positive and must be larger than 1. As a result, d must be smaller than 1 but not negative. Then, this work can compare the numerical value of two sides for the equation:

$$N^s = \frac{1}{R}\{q(K - S_0 u^2)^+ + (1-q)(K - S_0 ud)^+\} \leq (K - S_0 u)^+ \tag{10}$$

Let's start the discussion on the right-hand side. Given that $K < S_0 u$, which indicates that the value for $K - S_0 u$ will be negative. Therefore, $(K - S_0 u)^+$ will become 0. While the left-hand side of this inequality could be evaluated as well. With u being positive and larger than 1, the result of $S_0 u^2$ will be much larger than *K*, hence $(K - S_0 u)^+$ is eventually 0. With $u * d = 1$, $(K - S0ud)$ could be rewritten as $(K - S0)$. With the case $S0 < K$, the value of $(K - S0)^+$ will be the value of $K - S0$. After the computation, it can be concluded that the left-hand side has a non-zero value, while the right-hand side is *0*. Thus, the inequality doesn't hold; under condition three $(S0 < K < S0\,u)$ with the value of the asset going up, this work shall not early exercise the option.

The second condition for the case is when the value of the asset decreases, $(S0d)$. in this case, we should prove if the equation is true under the condition of

$$N^s = \frac{1}{R}\{q(K - S_0 ud)^+ + (1-q)(K - S_0 d^2)^+\} \leq (K - S_0 d)^+ \tag{11}$$

That this work has *d* is smaller than one and $u * d = 1$, hence $(K - S0ud)$ could be computed as $(K - S0)$. Given that $K > S0$, hence the value of $(K - S0ud)^+$ gives the value of $(K - S0ud)$. As d is smaller than *1*, the value of $S0d^2$ must be smaller than the value of K. Therefore, it can deduct that $(K - S0d^2)^+$ gives the value of $(K - S0d^2)$. On the other side of the inequality, the value for $(K - S0d)^+$ will give the value of $(K - S0d)$. Since both sides are positive having the value, a more quantitative approach should be adopted for comparing the two sides. That, a reciprocal appears on the left-hand side of the inequality, so the value might be smaller than the right-hand side. As a result, the inequality is true. Hence, this work can make the conclusion that should early exercise the option when the value goes down and the case $(S0 < K < S0\,u)$ holds true.

2.3.4 Case 4 $\boldsymbol{S_0 d < K < S_0}$

Under Case 4, this work still, as usual, separate the discussion under the value goes up and the value goes down. If the value goes up, with $S0\,u$, then computing the value of $(K - S0\,u)^+$. With $u * d = 1$, and the condition gives $S0\,d < S0$, it can be found out that the value of $d < 1$. Therefore, $u > 1$; with which $K < S0$, $K - S0\,u$ will be negative. As a result, $(K - S0\,u)^+$ will be 0. Hence, this work can reach the conclusion that we should not early exercise when $S0\,d < K < S0$ at the state which the value goes up.

When the value goes down, $S0\,d$, an early exercise happens when

$$N^s = \frac{1}{R}\{q(K - S_0 ud)^+ + (1-q)(K - S_0 d^2)^+\} \leq K - S_0 d \tag{12}$$

Which the equation could be computed as

$$K\left\{\frac{1}{R}\frac{u-R}{u-d} - 1 \leq S_0 d\,\frac{u(d-R)}{R(u-d)}\right\} \tag{13}$$

With the given condition states that $S0\,d < K$, which gives the $(K - S0\,d)$ will be positive.

$(K - S0\,d)^+$ will give *K-S0 d* as a result. With $u * d = 1$ and $K < S0$, this work only need to prove if

$$u - d \leq R(u - d) \tag{14}$$

With the assumption that $R > 1$, the equation is always true. Therefore, this work should always exercise early when $S0\,d < K < S0$ if the value of the asset goes down $(S0d)$.

2.3.5 Case 5 $\boldsymbol{S_0 d^2 < K < S_0 d}$

Under the condition of the value going up $(S0u)$, and with $S0d2 < S0d$, $u * d = 1$, this work can interpret that $d < 1$ while $u > 1$. This, however, makes the section of $(K - S0\,u)$ negative, which $(K - S0\,u)^+$ will eventually be *0*. In other words, this work should compare the value of N^s with the value of 0.

The value of N^S gives the equation:

$$N^s = \frac{1}{R}\{q(K - S_0 u^2)^+ + (1-q)(K - S_0 ud)^+\} \tag{15}$$

With $u > 1$, $(K - S0u^2)$ will be negative, which means that $(K - S0u^2)^+$ will be *0*. With $u * d = 1$, $(K - S0ud)$ will turn into $(K - S0)$. As $K < S0d$, $d < 1$, then $K < S0$. Therefore, $(K - S0ud)^+$ will eventually give *0* as well.

$$N^s = \frac{1}{R}\{q(K - S_0 u^2)^+ + (1-q)(K - S_0 ud)^+\} \leq K - S_0 u \tag{16}$$

After the comparison, this work find out that the equation is true under this case. Therefore, this work should exercise the put option early the value goes up ($S0u$) and follows the condition, $S0\,d^2 < K < S0\,d$. However, there're instances of the values of the asset going down ($S0d$). First, calculating the section of $(K - S0d)^+$. As the prerequisite of the case, $K < S0d$, which gives $(K - S0d)$ negative. Thus, $(K - S0d)^+$ will eventually become *0*. Recalling back to the other side,

$$N^s = \frac{1}{R}\{q(K - S_0ud)^+ + (1-q)(K - S_0d^2)^+\} \quad (17)$$

This work should evaluate the size of N^s, if it's > 0, or < 0.

As mentioned, with $S0d^2 < K < S0d$, $u * d = 1$, gives us $K < S0$; $(K - S0ud)^+$ becomes *0*. Since $d < 1$, $(K - S0d^2)^+$ is positive, gives us $(K - S0d^2)$. Hence, the side of N^s is larger than *0*, larger than $(K - S0d)^+$. Therefore, the condition of early exercise,

$$N^s = \frac{1}{R}\{q(K - S_0u^2)^+ + (1-q)(K - S_0ud)^+\} \le K - S_0u \quad (18)$$

doesn't hold. as a result, this work should not exercise early when the value decreases (*S0d*) under the case $S0\,d^2 < K < S0\,d$.

2.3.6 Case 6 $K < S_0d^2$

As the condition only gives the relationship between K and $S0\,d^2$, which this work is unsure about the value of d, this work cannot deduct the value of u. Therefore, the given information might not be sufficient to compare the prices and make the final decision.

However, if the value goes down, with $u * d = 1$ and $K < S0\,d^2$, this work shall prove if the inequality below exists:

$$N^s = \frac{1}{R}\{q(K - S_0ud)^+ + (1-q)(K - S_0d^2)^+\} \le (K - S_0d)^+ \quad (19)$$

According to the information given, $(K - S0d^2)^+$ could give out the value of *0*. As $S0d^2 > K$, then whenever $d > 1$ or $0 < d < 1$, both of them make $S0d > K$. Hence the value on the right, $(K - S0d)^+$ will be *0*. $(K - S0ud)^+$ could be rewritten as $(K - S0)$. This makes the left-hand side has a positive value while the right-hand side is *0*. Thus, the inequality doesn't hold. Therefore, this work shall not early exercise our option when the value of the put goes down and the condition $K < S0\,d^2$ is true.

2.3.7 Decision table

This work can put our conclusions in the table 1. Key:

Table 1: Decision on whether to exercise earlier

	When the value goes up (S0u)	When the value goes down (S0d)
Case 1 $S0\,u^2 < K$	✓	*NR*
Case 2 $S0\,u < K < S0\,u^2$	✓	✗
Case 3 $S0 < K < S0\,u$	✗	✓
Case 4 $S0\,d < K < S0$	✗	✓
Case 5 $S0\,d^2 < K < S0\,d$	✓	✗
Case 6 $K < S0\,d^2$	*NR*	✗

✓ *early exercising the option;* ✗ *not early exercising the option; NR not able to make the decision*

3. PYTHON APPROACH (FOR TWO PERIODS) PUT OPTION PRICING

With the binomial model and the discussion over the different conditions of the two periods put option pricing, this work have first thought about how it shall make the decision. The next step for us is to use the information this work has and test those decisions under the examples generated by Python.

3.1 Mechanism and variables of the Python approach

The basic idea behind this approach is to test the "stock tree" with a series of variables [7]. It first needs to define the variables: in this case, this work brought up with $T = 2$ for indicating a two- period model; the other variables are *u*, *d*, *R*, *S0*, and *K*, which follows the same definition from the start of this investigation. But in the work gives them numerical values in order to compute the final price. Figure 4 shows the step of defining the variables on the Python notebook.

```
In [1]: T = 2

In [2]: u = 2
        d = 1/2
        R = 1
        S0 = 1000
        K = 800
```

Figure 4. step of defining the variables on the Python notebook

After defining those terms, a stock tree is depicted using the mathematical idea of matrices. By importing a library to the Python notebook, this work can portray the stock tree by using the mathematical values for expressing matrices. Figure 5 shows the condition.

```
In [3]: import numpy as np

In [4]: stock_tree = np.zeros([T+1, T+1])

In [5]: stock_tree

Out[5]: array([[0., 0., 0.],
               [0., 0., 0.],
               [0., 0., 0.]])
```

Figure 5. the second step for pricing the stock tree

Next, this work input the $S0$ value into the stock tree model and model the other terms by the $S0$. We will get other values of $S0u$, $S0d$, $S0ud$, $S0u^2$, $S0d^2$, which are the values of the asset we could get during the two periods. Therefore, figure 6 show the following stock tree:

```
In [6]: for t in range(T + 1):
            for i in range(t + 1):
        #         print('t = ', t, ', i = ', i)
                stock_tree[i, t] = S0 * (u ** (t - i)) * (d ** i)

In [7]: with np.printoptions(precision=2, suppress=True):
            print(stock_tree)

        [[1000. 2000. 4000.]
         [   0.  500. 1000.]
         [   0.    0.  250.]]
```

Figure 6. stock tree after inputting the values

Then, this work should define the payoff function of the model; the value of q could be calculated with through the given variables. While having all the variables, this work could compute the payoff function. The formula for N^s is the same to the ones in the binomial approach, shown in the following:

$$N^s = \frac{1}{R}\{q(K - S_0ud)^+ + (1-q)(K - S_0d^2)^+\}$$

$$N^s = \frac{1}{R}\{q(K - S_0u^2)^+ + (1-q)(K - S_0ud)^+\}$$

By calculating the variables for N^s, this work finally get the option tree. These procedures were shown in Figure 7.

```
In [8]: def option_pricer(payoff):
            q = (R-d) / (u-d)
            option_tree = np.zeros([T + 1, T + 1])
            option_tree[:, T] = payoff
            for t in range(T - 1, -1, -1):
                for i in range(0, t + 1):
                    option_tree[i, t] = 1/R * (q * option_tree[i, t+1] + (1-q) * option_tree[i+1, t+1])
                    option_tree[i, t] = max(option_tree[i,t], K - stock_tree[i,t])
            return option_tree
```

Figure 7. generating the option tree

the put option will only be early exercised if the price of the put in time 2 is larger than that in time 1. If not, we will finally get the price 0. With that condition, it can calculate the profit by early exercising. Hence, the put option price tree with payoff would finally look like the following (see Figure 8):

```
In [9]: put_payoff = np.maximum(K - stock_tree[:, T], 0)

In [10]: with np.printoptions(precision=2, suppress=True):
             print('The put price tree:')
             print(option_pricer(put_payoff))

         The put price tree:
         [[244.44   0.     0.  ]
          [  0.   366.67   0.  ]
          [  0.     0.   550.  ]]
```

Figure 8. put option tree profit

3.2 Usage of the approach

The model this work has generated are using the ideal values for u, d, $S0$, K. In other words, the values might not be the same in real-world conditions. But this approach offers us a style of thinking in pricing the put option, that it is able to change the values of inputs and run the Python to get the results.

4. Conclusion

Overall, this work has cleared up a route for the decision of the American put option problem; both methods did help to evaluate the problem and reach out a final decision. However, these efforts are not enough: the topic of American options and early decisions is very wide. Though we have a brief finding on the put option pricing and decisions, they're still calls; it could even extend this topic on a trinomial model [8]. Simultaneously, this work shall not stop by the decisions we have, more investigations should be extended on this topic.

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