Optimal Oil-based Exotic Options Strategies Under the Background of War: An Empirical Study in the Context of the Russia-Ukraine Conflict

Zhuocheng Shen[1,*]

[1] *School of Economics, Yunnan University, Kunming, 650500, China*
[*] *Corresponding author. Email: shenzhuocheng@mail.ynu.edu.cn*

ABSTRACT
This paper discusses what kinds of oil-based exotic options will have the best performance under the background of wars, and suggests some exotic options combinations to improve overall performance. In order to draw a conclusion, this research takes the percentage return of each strategy as the key index to make a comparison. Therefore, the first thing in this research is to use Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model to obtain the volatility of underlying asset, and use Monte Carlo Simulation to value each option on the assumption that the crude oil futures prices follow Geometric Brownian Motion. Then, according to the actual price path, the percentage return of each strategy can be calculated to get the result. What is found in this research is that it is risky to invest only in a single option, except fixed lookback options, because the prices of crude oil futures fluctuate and do not always remain at a favourable level. However, if the constructed portfolio contains Asian options with average strike price in the opposite direction, the overall performance will be better. As a result, when some similar cases happen in the future, it is suggested that investors can choose fixed lookback options or portfolios consisting of some other exotic options with average strike price Asian options in different directions.

Keywords*: Exotic options, Strategies, War.*

1. INTRODUCTION

In recent decades, wars happen frequently all over the world, and it is known that crude oil prices have a close relation with wars. Not only can war have positive impacts on the prices of crude oil, but sometimes when wars break out, the prices of crude oil will drop sharply. Therefore, in these extreme backgrounds, it is meaningful to analyze which direction the crude oil prices will go and discuss whether there are some optimal investment strategies that can get favorable profits. With the outbreak of the Russia-Ukraine conflict, the prices of crude oil have soared and showed the characteristics of violent fluctuations. Although crude oil futures can also create many profit opportunities in these contexts, options can provide more flexibility because there are various kinds of options. By purchasing different types of options or combining different options, investors can construct portfolios suitable for specific situations which are most in line with their own expectations and tastes. However, compared with crude oil futures, the trading of options is still relatively inactive, and what kinds of options and portfolios can perform well remains to be explored.

Phoenix Satellite TV reported that according to a well-known scholar, for the reasons why crude oil prices soared after the Russia-Ukraine conflict broke out, the conflict was only one of the factors. There were also three other factors related to supply that led to rising prices [1]. Wang and Sun studied the influence of some factors on crude oil prices by structural equation model. Then it came to a conclusion that if there were wars between some big oil-producing countries and their neighbors, the prices of crude oil would soar in an indirect way, that was, the ability of the war to reduce oil supplies [2]. Engel proposed that it was unreasonable to assume that the forecast variance of a period was constant as in traditional econometrics. Therefore, a new autoregressive conditional heteroscedasticity (ARCH) process was proposed, in which the error of fitting equation was regarded as a function of lag terms rather than a function of explaining variables, thus solving the heteroscedasticity problem. Then, it was used to estimate the means and variances of inflation in the UK, and it was

found that in that case the ARCH effect was significant, and the variance could be estimated well [3].

Bollerslev continued to study the autoregressive conditional heteroscedasticity model (ARCH) put forward by Engle, and improved it in 1986, introducing a more general model called Generalized Autoregressive Conditional Heteroskedasticity (GARCH). Compared with ARCH model, it was difficult to choose the optimistic lag order, and if the sample size was not large enough, the degree of freedom may be seriously lost. The GARCH model was more concise, and could better predict the future conditional variance. [4]. Agnolucci tried to compare the volatility of crude oil futures obtained from GARCH with the implied volatility, and concluded that the volatility of GARCH models might be better than that calculated by inverting the Black-Scholes equation in terms of prediction accuracy. In addition, assuming that the errors follow GED distribution was the best among all GARCH-type models in that study. However, in practice, it was suggested to combine the two methods, because implied volatility contained some information that was not available in time series [5]. Black and Scholes studied the way to price options and corporate liabilities and proposed a method that could value options and corporate liabilities under some assumptions. For examples, the prices of underlying assets followed Geometric Brownian Motion, investors could borrow money at risk-free rate and so on [6].

Macbeth and Mervill tested the Black-Scholes Model and concluded that for in-the-money options, their average values would be less than market prices, and vice versa. Those options that were in-the-money(out-of-the-money), the degree of market overpricing(underpricing) got higher as the extent of in-the-money(out-of-the-money) increased, and got lower as the time to expiration decreased, except those out-of-the-money with less than 90 days to expiration, which was proved had no significant relationship between the two variables [7]. Agustini et al. tried to verify the usefulness and accuracy of geometric Brownian motion in stock prices prediction by analyzing the differences and relation between predicted prices and actual prices, and found that Geometric Brownian Motion was an ideal prediction method. In addition, in the short-term forecast of the study, the predicted MAPE value was less than or equal to 20% [8].

A stochastic process is defined as Brownian Motion when the increments of each period are independent of each other and obey the normal distribution. If the logarithm of random variable follows Brownian Motion, then it follows Geometric Brownian Motion, which is the simplest continuous stochastic process and the basic model for describing the randomness of securities prices.

Postali and Picchetti tried to analyze oil price's path and proved that the movement of oil prices could be regarded as Geometric Brownian Motion. The conclusion was that as long as the speed of mean reverting of oil prices were low, Geometric Brownian Motion could represent the movement of oil price well [9]. Suresh summarized main options combination strategies, including spread, straddle, strips, straps and strangles, and stated the situations where each portfolio should be used. By simulating spot prices and calculating the return of the portfolios, it was concluded that as long as investors correctly understood the market conditions and chose the appropriate option portfolio strategies, it was easier to make a profit in the option market [10].

This paper is going to explore whether some exotic options and portfolios fit the background of wars and will make more profits. The structure is as follows. Section 2 talks about the relation between wars and oil prices, especially in the case of the Russia-Ukraine conflict. Section 3 introduces some potential optimum exotic options. In section 4, the method and results of comparing each strategy with the help of GARCH (1, 1), Monte Carlo Simulation and Geometric Brownian Motion are presented. The last part is the conclusion obtained by comparing the results.

2. RELATION BETWEEN WARS AND OIL PRICES

2.1. Wars can have significant influence on oil prices

Generally speaking, wars can influence oil prices from two aspects, mean and volatility. According to oil price path during many wars in the past decades, it can be found that wars can have large impacts on oil prices, but it is not necessary. In most wars, the volatility of oil prices sees obvious rises, and many wars can also have influences on the average prices of oil. While some of them only bring short-term shocks, some others generate persistent impacts on the prices.

2.2. Practical reasons for rising and volatile oil prices during the Russia-Ukraine conflict

By observing the oil price path during the Russia and Ukraine conflict, it is hard to claim that there is no relation between the volatile oil prices and the conflict since the oil prices dramatically rise just after the time that the war breaks out. In general, many factors make the oil prices present such a phenomenon instead of only the war itself. On the one hand, because of the need for fuel and market sentiment, the demand for oil has begun to increase. On the other hand, due to the economic sanctions imposed by some countries on Russia, the Iranian nuclear deal, the COVID-19 and the low oil prices in the recent past, crude oil is in short supply either because buyers do not want to purchase or suppliers do not produce [1]. Therefore, these factors mutually contribute to insufficient supply elasticity, which means

that the supply capacity is maintained at a low level corresponding to the previous demand, and cannot be adjusted rapidly. Once there is an event where the demand increases or is expected to increase, the prices will go up because of the shortage of supply.

However, recent research concluded that if wars happened among some large oil producing countries, the oil prices would always be affected since in market, there would be a concern that oil supply might be disrupted even though that actually had not yet happened [2]. Although Russia is undoubtedly a large oil producer, in this case the main battlefield is in Ukraine, which largely protects Russia's oil production from the threat of war. Therefore, this factor might have little or no impact on crude oil prices.

3. SOME POTENCIAL OPTIMAL EXOTIC OPTIONS

3.1. Lookback Options

Lookback options are a kind of special European options whose payoffs are determined by the prices that are most favorable to the holder. Therefore, this kind of option can capture optimal prices and maximize payoffs. However, the prices of these options are very high, and it remains to be proved whether these options can create more profit.

3.2. Asian Options

Asian options are a kind of options that are relatively more conservative, and the payoffs of this kind are determined by the average price of underlying asset within the investment term. Some Asian options have many excellent properties that can fit the conflict background well.

3.3. Barrier Options

Barrier option are a kind of options that the payoffs are determined by whether the prices of underlying assets within the investment term reach a certain level. Because there are certain restrictions on the exercise of options, the prices of this kind of options are lower than similar common options, which can create more profit opportunities.

3.4. Gap Options

Gap option are a kind of European options. The characteristic of gap options is that there will be a trigger price which is different from the strike price. Only when the prices of underlying asset on settlement day breaks through their trigger prices, the option can be excised. As with barrier option, gap options have some restrictions, so the prices of this options will be lower, creating more profit opportunities.

4. OPTIMAL OPTION INVESTMENT STRATEGIES IN THE BACKGROUND OF RUSSIA-UKRAINE CONFLICT

This research considers percentage return as the standard to evaluate which strategies have better performance during the Russia-Ukraine conflict. However, because the price data of some exotic options is inaccessible since these kinds of options are traded over-the-counter, and there will be some discrepancies between market prices and values generated from models, it is unfair to take actual prices as the cost of some strategies but take theoretical prices as that of others. Therefore, this research regards theoretical prices as all strategies' cost, and each strategy's profit is calculated under three different strike prices (for those who need to set a strike price) using actual underlying asset prices for better comparison. Moreover, due to the high risk of derivatives, it is assumed that investors would enter strategies on the first trading day (Feb 28,2022) when the market begins to respond to the war to reduce uncertainty, and all the contracts would be terminated on Apr 14,2022, because this is the last trading date and settlement date of the May crude oil options traded in CME (Chicago Mercantile Exchange Holdings Inc.).

4.1. Volatility of underlying asset

In this research, crude oil futures due in May, 2022 is chosen as the underlying asset, and 10 years' daily trading prices of that futures (a total of 2516 observations) are collected, from Feb 28, 2012 to Feb 28, 2022 but the price data on Apr 20, 2022 is excluded because that data can be considered as an outlier since the crude oil futures price on that day is -37.63 dollar per barrel. All the price data is obtained from Yahoo Finance, and a time series diagram of daily return is shown below.

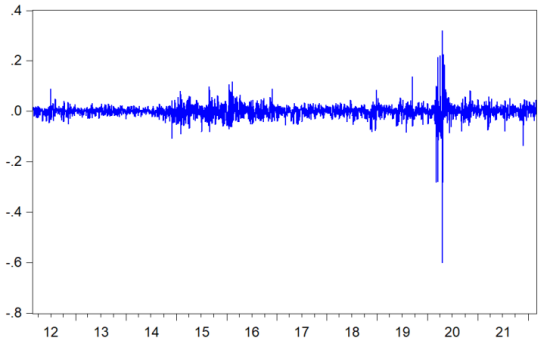

Figure 1 Time series of crude oil futures' daily return

As volatility is one of the basic elements of option pricing, it is an important issue in this research whether it can be accurately predicted.

By observing Figure 1, it is obvious that clustering exists, which means that within some periods, the volatility of return is high, but within some other periods the volatility is low. Therefore, it cannot simply calculate standard deviation of daily return and let the standard deviation act as the volatility of underlying asset because the volatility is various in different periods instead of being constant.

Two approaches can be used to address this problem and achieve more appropriate volatility. First, in 1982 Engle proposed autoregressive conditional heteroscedasticity (ARCH), which regarded the error of fitting equation as a function of its lagging terms, making volatility of lagging data can have influence on predicted data to work out the issue of heteroscedasticity, and this process was improved in 1986 by Bollerslev who put forward Generalized Autoregressive Conditional Heteroskedasticity (GARCH) [3, 4]. The second way is to calculate implied volatility which contains expectations of the whole market and can represent the future to some extent. However, recent research showed that GARCH was more likely to perform better as compared to implied volatility when these two approaches were used to predict the volatility of WTI futures [5]. Thus, in order to obtain more accurate results, this research uses GARCH to forecast future volatility because underlying asset's volatility will suddenly go up after the outbreak of conflict. It is better to use GARCH to catch the rising volatility.

This research uses Eviews to estimate GARCH (1, 1) model because GARCH (1,1) is the most popular and frequently used one among all GARCH-type models currently. The model in this case can be represented as:

$$R_t = U_t + \varepsilon_t \quad (1)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (2)$$

where t is time point in days, R_t is daily return, U_t is mean of daily return, ε_t is error of mean equation, σ^2_t is daily variance, ω, α and β are parameters to be estimated.

With the help of Eviews, the parameters of GARCH (1, 1) model is estimated and the output is shown below.

Table 1. Estimation output of GARCH (1, 1) Model

Variable	Value	Std.Error	Z-Statisitc	Prob.
		Mean Equation		
U_t	0.000869	0.00031	2.792395	0.0052
		Variance Equation		
ω	9.54E-06	2.43E-06	3.931739	0.0001
α	0.116651	0.014255	8.183170	0.0000
β	0.871648	0.013615	64.02328	0.0000

According to Table 1, all the parameters in both mean equation and variance equation are significantly non-zero at the significance level of 5%. Therefore, the GARCH (1, 1) model can be represented as:

$$R_t = 0.000869 + \varepsilon_t \quad (3)$$

$$\sigma_t^2 = 0.00000954 + 0.116651\varepsilon_{t-1}^2 + 0.871648\sigma_{t-1}^2 \quad (4)$$

Thus, the variance of crude oil futures' return on Mar 1, 2022 can be forecast based on previous day's data, and the value of the forecast daily variance on Mar 1,2022 is 0.000522. The variance is converted to daily standard deviation, and its value is 0.022847319. Assuming the daily income is independent and identically distributed, the annual standard deviation can be calculated by the following equation:

$$\sigma_{year} = \sigma_{day} \times \sqrt{252} \quad (5)$$

where σ_{year} is annual standard deviation, σ_{day} is daily standard deviation and 252 is the number of trading days in a year.

Thus, by calculation the forecast annual volatility is 0.36268995.

4.2. Options valuation and profits

In this research, the first thing that needs to be done is valuation. In 1973, Black and Scholes developed a model to price common European options under certain assumptions [6]. Then in 1979, MacBeth and Merville conducted an empirical examination of the Black-Scholes Model, and found that it was better to use the BS Model valuing short term options which were near-the-money [7]. However, the biggest limit of this method to value options here is that it can only price common European options. Therefore, based on the idea of this model, this research uses Monte Carlo Simulation to price exotic options under the assumption that crude oil futures prices follow Geometric Brownian Motion. Recent research showed that Geometric Brownian Motion could predict stock prices well especially in short term [8]. Also, the study done by Postali and Picchetti concluded that Geometric Brownian Motion could be an excellent proxy for oil prices as long as the pace of mean reverting was slow [9]. Thus, this research assumes that the prices of crude oil follow Geometric Brownian Motion, so the theoretical prices can be obtained by

calculating the present values of the expected payoffs. Then, actual payoffs of strategy are calculated using actual price path in investment term, and subtracting the costs from actual payoffs to obtain profits.

According to GBM (Geometric Brownian Motion), the asset price at time T can be presented as:

$$S_T = S_t e^{\left(\alpha-\frac{1}{2}\sigma^2\right)(T-t)+z\sigma\sqrt{t}} \quad (6)$$

where T and t are time points in years, S_T and S_t are asset's prices at corresponding time point, α is expected return, σ is standard deviation and z is a random value of inverse function of cumulative probability distribution function of normal distribution.

Under the assumption of risk-neutral, α is equal to risk-free rate. According to the data on Trading Economics, on Feb 28,2022, the YTM (Yield to Maturity) of one month treasury bills is 0.061%, and that of two months treasury bills is 0.193%. In this research the investment term is one month and 14 days. By linear interpolation, treasury bills' YTM of corresponding term is approximately 0.1226%, which can be regarded as risk-free rate.

To get random number z, in this research two functions of Excel, rand() and normsinv() are used. Rand() is a function that can generate a random number ranging from 0 to 1 and normsinv() is the inverse function of cumulative probability distribution function of normal distribution. By combining these two functions as normsinv(rand()), z can be obtained.

In addition, because z is a random value, the results of each process can be different. Therefore, in this study, every prediction process is made 10 million times and the mean is calculated to reduce standard error.

4.2.1. Lookback Options

Having analysed the factors that will influence the crude oil futures prices, this research selected two kinds of lookback options, fixed call and floating call, to calculate profit as the alternative optimal strategies.

Since lookback options are a kind of path-dependent options, in this research, it is needed to forecast daily prices of crude oil futures. Therefore, this study predicts the daily crude oil futures prices according to the previous day's prices, which means that in Equation (6), T is equal to t+1, so it can complete a price path.

The results are shown as:

Table 2. Process results of lookback options

Type	Strike	Value	Std.err	Profit
Fixed call	95	9.80	0.0027	18.90
	100	5.84	0.0024	17.86
	105	3.25	0.0019	15.45
Floating call	-	8.42	0.0025	4.24

4.2.2. Asian options

Because of the low price and low risk, the two types of call Asian options are initially selected as the best alternative strategies.

This research uses the same method as lookback options to value Asian options. After obtaining the price path, the average price is calculated and used to calculate simulated profits.

The results are shown as:

Table 3. Process results of Asian options

Type	Strike	Value	Std.err	Profit
Average price call	95	3.33	0.0015	7.46
	100	1.37	0.0010	4.42
	105	0.46	0.0006	0.33
Average strike price call	-	2.84	0.0014	-1.68

4.2.3. Barrier Options

According to the expected price fluctuation direction, up-and-in call barrier option and down-and-out call barrier options may be optimal strategies because these kinds of options are cheaper.

The method to value barrier options is the same, and the barrier prices for up-and-in are set to 100, 105 and 110, for down-and-out are set to 80, 85 and 90.

The results are shown as:

Table 4. Process results of Barrier options

Type	Strike	Barrier	Value	Std.err	Profit
Up-and-in call	95	100	5.31	0.0026	6.64
		105	4.82	0.0027	7.13
		110	2.83	0.0026	9.12
	100	105	3.18	0.0021	3.77
		110	2.76	0.0021	4.19
	105	110	1.79	0.0016	0.16
Down-and-out call	95	80	5.36	0.0026	6.59
		85	5.25	0.0026	6.70
		90	4.48	0.0026	7.47
	100	80	3.27	0.0021	3.68
		85	3.23	0.0021	3.72
		90	2.72	0.0020	4.23
	105	80	1.87	0.0016	0.08
		85	1.86	0.0016	0.09
		90	1.70	0.0015	0.25

4.2.4. Gap Options

Call gap options may also be optimal strategies because of their low cost.

According to Equation (6), by setting the time interval (T-t) to 0.131 (33/252) and S_t to the price of February 28, the price of April 14 can be predicted. The trigger prices are set to 100 105 and 110.

The results are as follows:

Table 5. Process results of call gap options

Strike	Trigger	Value	Std.err	Profit
95	100	5.00	0.0026	6.95
	105	4.08	0.0026	7.87
	110	2.97	0.0026	-2.97
100	105	2.98	0.0021	3.97
	110	2.32	0.0020	-2.32
105	110	1.66	0.0016	-1.66

4.2.5. Common Option

Common options act as benchmarks for better comparison. This study uses the same method to estimate the prices of common options.

The results are shown as:

Table 6. Process results of common options

Strike	Value	Std.err	Profit
95	5.37	0.0026	6.58
100	3.28	0.0021	3.67
105	1.87	0.0016	0.08

4.3. Comparison of different kind of options

Having obtained the profit of each kind of options that may have better performance under the background of Russia-Ukraine conflict, this research collates all the data to make a comparison. Since different options have different prices, the percentage return should be calculated as the key index to determine which strategies are better. Thus, a table that contains all the information is shown as (the rank is based on percentage return):

Table 7. Comparison of different options

Type	Strike	Barrier/Trigger	value	Profit	% Return	Rank
Fixed call lookback	95	-	9.80	18.9	192.86%	7
	100	-	5.84	17.86	305.82%	4
	105	-	3.25	15.45	475.38%	1
Floating call lookback	-	-	8.42	4.24	50.36%	23
Average price call Asian	95	-	3.33	7.46	224.02%	5
	100	-	1.37	4.42	322.63%	2
	105	-	0.46	0.33	71.74%	22
Average strike price call Asian	-	-	2.84	-1.68	-59.15%	28
Up-and-in call barrier	95	100	5.31	6.64	125.05%	15
		105	4.82	7.13	147.93%	11
		110	2.83	9.12	322.26%	3
	100	105	3.18	3.77	118.55%	18
		110	2.76	4.19	151.81%	10
	105	110	1.79	0.16	8.94%	25
Down-and-out call barrier	95	80	5.36	6.59	122.95%	16
		85	5.25	6.70	127.62%	14
		90	4.48	7.47	166.74%	8
	100	80	3.27	3.68	112.54%	20
		85	3.23	3.72	115.17%	19
		90	2.72	4.23	155.51%	9
	105	80	1.87	0.08	4.28%	27
		85	1.86	0.09	4.84%	26
		90	1.70	0.25	14.71%	24
Call gap	95	100	5.00	6.95	139.00%	12

		105	4.08	7.87	192.89%	6
		110	2.97	-2.97	-100.00%	29
	100	105	2.98	3.97	133.22%	13
		110	2.32	-2.32	-100.00%	29
	105	110	1.66	-1.66	-100.00%	29
Common call	95	-	5.37	6.58	122.53%	17
	100	-	3.28	3.67	111.89%	21
	105	-	1.87	0.08	4.28%	27

4.4. Option portfolio strategies

Considering that after the conflict breaks out, the prices of crude oil futures will be influenced by speculators' operation and market's expectation of war (such as the progress of peace talks), the prices will show a high volatility and will not always stay at a high level. As a result, it is possible that when the option contracts terminate, the price of underlying asset is at a relatively lower level, which will cause the option to miss the profit opportunity or even incur loss. What's more, options are leveraged financial products, and there are also high risks hidden under high returns, so the loss can be severe.

According to recent research, it will be easier to make profits if proper option combination strategies are used in the option market [10]. Therefore, some put options can combine the above exotic options to form portfolios. If the price of crude oil futures unfortunately drops on the settlement day, put options can make profits to hedge the loss from the other position.

Among all available put options, average strike price put Asian options can tackle this problem well because of its two advantages. First, investors do not need to determine an appropriate strike price for a put option because the strike price of this option is the average price within the investment term. For some other put options, too high strike price will largely increase the cost while too low strike price will not bring enough hedging effect. Second, the price of this kind of option is much lower than many other similar options.

For a more intuitive comparison, this research assumes that portfolios terminate on Mar 8, Apr 11 and Apr 14 because the prices of crude oil options on these days are the highest, lowest and moderate, respectively, which can provide a better comparison. Then prices and profits of average strike price put Asian option are calculated. The risk-free rate of each investment term is also obtained by linear interpolation.

4.4.1. Valuation for Average Strike Price Put Asian Options

The method of valuing Asian options has been mentioned above. The value and standard error of each average strike price put Asian option are shown as:

Table 8. Process results of average strike price Asian options

Terminate Date	Value	Stderr	S_T	Profit
Mar 8, 2022	1.29	0.0006	123.7	-1.29
Apr 11, 2022	2.69	0.0012	94.29	9.00
Apr 14, 2022	2.83	0.0012	106.95	-2.83

According to Table 8, if the price of underlying asset is very low at settlement day, this Asian option can earn a large profit to hedge the loss from the other position. If the price of underlying asset is moderate or high on settlement day, this option will only cause a relatively small loss and take its price as the upper bound.

5. CONCLUSION

This study discusses some exotic options and their portfolios that can get better profits under the background of the Russia-Ukraine conflict. According to research results, fixed lookback options have the best performance, because they can capture the best price and will not be affected by the price drop. If the market is not efficient enough, momentum effect might exist, which will enable the lookback option to capture these market anomalies and make profits. As for other exotic options mentioned above, those options with relatively low strike prices tend to perform better, because those options with higher strike prices are vulnerable to sudden price drops. In addition, the Asian options with average strike price are found to have great properties to hedge the downward risk.

Overall, if there is a similar case in the future, this paper suggests that investors should first analyse the whole background through demand and supply. If events only bring a temporary shock and make the prices of the underlying assets fluctuate violently, which means that the prices will not always remain at a favourable level and will lead to greater uncertainty of position return, two option investment strategies are recommended. First, it will be ideal to buy fixed lookback options. In addition, investment portfolios that are composed of other exotic options with relatively conservative strike prices and average strike price Asian options are also advisable.

However, the direction of Asian options needs to be opposite since they act as hedge positions to reduce risks.

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