Analysis on the Market Design of Teacher Assignment

Ruonan Gao[1,*]

[1]*McGill University, Montreal, Canada, H3A 0C8*
**Corresponding author. Email: ruonan.gao@mail.mcgill.ca*

ABSTRACT
Evidently, teachers are essential, but the unequal distribution of teachers' quality between our and affluent neighborhoods commonly exists across the world. This paper will discuss how to eliminate this inequality using market design. Previous evidence suggests a potential opportunity to improve teachers' quality by allocating teachers to schools. The paper examines two existing matching mechanisms and proposes a new allocation mechanism that is mathematically proven to be efficient, individually rational, and strategy-proof. The paper showed a new way of approaching the inequality of teachers' quality by better allocating teachers to schools.

Keywords: *Equality, Education, Market Design, Teacher Assignment*

1. INTRODUCTION

It is common sense that teachers are essential, and evidence today confirmed what we know, that teachers' quality strongly impacts students' achievements. As previous research showed, one standard deviation increases in teacher value-added in a single year increases students' earnings by $350 at age 28, which means a 1% increase in earnings from one year with a better teacher [1]. Thus, this effect compounds to be relatively large through the course of students' education.

So, how do we improve teaching quality overall? One proven policy about improving teaching quality simplifies the teaching profession by reducing class size. Krueger's paper in 1999 showed that students randomly assigned to smaller classes have better test scores [2], and these same students are more likely to take standardized tests going into college, suggesting they are more interested in higher education [3]. However, the class reduction policy has an unintended consequence of increasing the likelihood of poor and minority students being taught by underqualified or inexperienced teachers [4]. This phenomenon is caused by the unequal distribution of teachers' quality between poor and affluent neighborhoods [5].

The paper will focus on the potential to improve equality of teacher allocation with market design. The paper is organized as follows. Section I determines teacher preferences over schools and discusses fundamental problems associated with these determinants. In Section II, we introduce and compare the current teachers' allocation mechanism and discuss the pros and cons. Section III proposes a possible mechanism and examines the mechanism with strict priorities and under coarse conditions and discusses the expected outcome. Finally, section IV is the conclusion. This paper provides a method that could potentially improve the matching between teachers and schools and eventually approach the inequality in teachers' quality between neighborhoods.

2. LITERATURE REVIEW

Why is that such unequal distribution between poor and affluent neighborhoods? Research suggests that most public-school teachers are paid according to rigid salary schedules based on years of experience and education within the district [6]. So, teachers are essentially selected for their position not based on the position's wage but on other characteristics: geography and student characteristics [7]. As a result, teachers want to teach close to where they live and typically prefer to live in friendly neighborhoods; teachers also prefer more advantageous students because they have more at-home investments, which reduces teachers' workload. These two teachers' preferences increase the likelihood of inexperienced and less qualified teachers teaching students from poor and minority neighborhoods, creating education inequality.

A way approaches this inequality is to improve teacher school matching quality. Evidence shows that the quality of the match between teachers and schools explains 10%-40% of what we usually estimate as

teacher quality [8], so this suggests a potential opportunity to improve teacher quality by better allocating teachers to schools.

However, a matching problem arises with fixed salaries that schools and teachers would match based on the absolute advantages, as Table 1 shows. In this example, we assume a better school is better in all aspects than a worse school. Then with rigid salaries, teachers would prefer the better school unless they are altruistic that have incentives to teach at a more disadvantaged school. Therefore, better schools and more qualified teachers are matched up, and worse schools are stuck hiring fewer experienced teachers. To maximize overall students' achievements, we will have to allocate teachers based on comparative advantage. Hence, unrestricted wages in each match implement the first-best allocation by giving teachers incentives to seek positions that make the most of their comparative advantage. Evidence also suggests that flexible prices would produce achievement gains at a much lower cost [9].

Table 1. Assign by absolute advantage

	Better School	Worse School
Better Teacher	Y	X
Worse Teacher	X	Y

3. CURRENT TEACHERS ALLOCATION MECHANISMS

Looking at the teacher allocation problem from market design, people usually think of a teacher proposing a deferred acceptance algorithm [10], which we refer to as teacher DAA in the rest of the paper. For example, in France, they created a centralized teacher DAA, that teachers apply to a centralized application system using a standard application to their preferred schools [11]. The system assigns teachers to school based on their absolute advantage, and teachers have the option to apply to all schools they want to, and then the school picks their most preferred applicants. The advantage of this system is that it substantially cuts the application price of teachers, such as search costs time costs, allows teachers to apply to more schools, and expands the applicant pools for schools. However, the system still assigns teachers to schools based on absolute advantages, creating inequality in teachers' quality. Moreover, for schools, preferences are more aligned with student achievements than other characteristics that are less important to our social welfare, suggesting there might be gain from schools proposing deferred acceptance algorithms.

One mechanism similar to the school proposing DAA is the Teach DC program in Washington DC, the United States. In this system, applicants submit a detailed application to the district, including a paper application, teaching lesson, and several interviews. And then, the districts would send schools a list of the most likely successful applicants, which helps predict teacher quality by being a decision aid where experts examine the applicants based on what they know would correlate with future students' achievements. Then, they can recommend which teacher would be a good fit for a school. It is similar to the school proposing DAA because the school reaches out to applicants, and applicants get to decide whether they want to work at the school or not. The problem is that schools' preferences align with students' future achievement, which means schools would still try their best to hire teacher based on absolute advantage, and inequality still exist.

4. PROPOSED MECHANISM

Dealing with the inequality, we need to assign teachers to schools based on comparative advantage. Therefore, implementing pay for performance is crucial since it aligns teacher performance with comparative advantage, incentivizing teachers to consider disadvantaged schools during their job application process.

The proposed mechanism is a centralized teacher DAA application system with decision aid and performance pay to allocate teachers based on comparative advantages. The matching system consists of 2 parts, school, and teachers. Teachers' preferences are related to geography, student characteristics, which refers to schools' location, reputation, student composition, past exam score, and salary after adding performance pay. School preferences are aligned with students' achievement, referring to teachers' incentives, educational background, academic achievements.

The process is that first applicants send detailed applications, including general application sheets and mock teaching lessons, and interviews that test their non-cognitive skills, to the centralized application platform and checkboxes to their preferred schools. Then, experts from the platform would examine all applicants and rank them according to the teaching quality and send lists of the top applicants to schools so that each school would receive a ranking of applicants that applied to their school. Next, the school would reach out to the most preferred applicants, and teachers can collect their offers and decide which school they would like to work for.

5. MATHEMATICAL MODEL

T: a finite set of teachers

S: a finite set of schools. School s has q_S seats

Teacher t has a ranking $\succeq_t$ over S ∪ {t} School s has a ranking $\succeq_S$ over T ∪ {s}

- A matching assigns each teacher to at most one school.
- A school cannot be matched with more teacher than the number of seats available at the school.

For a matching to be stable, we need:

- if a teacher and a school are matched, it must be that they find each other acceptable
- there are no blocking pairs, that is, if a teacher t prefers school s to her matched school, then it must be that either s does not find t acceptable or that s prefers all its matched teacher to t.

The outcome is teacher-optimal stable matching and teachers report their preferences truthfully since there is no profitable opportunity for any teachers to game the system to his or her advantage.

With strict priorities:

X: a finite set of schools q_X: the capacity of school x

N: a finite set of teachers

A matching specifies for each teacher her/his school allocation.

μ(i) = i means teacher i is not assigned to any school in X.

Teacher i has strict preferences $\succeq_i$ over X ∪ {i }. At each school x there is a priority ranking $\succeq_X$, possibly with ties.

$i \succ_X j$ means i has higher priority than j for school x

Respecting priorities means: "no waste":

$$\text{if } x \succ_i \mu(i), \text{ then } |\mu(x)| = q_X \quad (1)$$

"no justified envy": if μ(j) = x and $x \succ_i \mu(i)$, then $j \succeq_X i$ (2)

If priority orders of the schools are all strict, then we know that the teacher-proposing DA yields the unique teacher-optimal stable matching. That is the matching which respects priorities and is the teachers' most preferred one among all matchings which respect priorities. Moreover, we know that if priorities are strict, the teacher-proposing DA is strategy-proof for teachers. Hence, we can implement in dominant strategies the desired outcome with a direct revelation mechanism. The outcome is going to respect priorities, so no teacher can complain that he or she is missing out on a preferred school at the expense of another teacher who has lower priority, and it is impossible to match any teacher with a more preferred school without violating priorities.

With coarse priorities:

Given a priority structure $\succeq_X$, suppose we break the ties arbitrarily to obtain a strict priority structure $\succ'_X$, so

$$i \succ'_X j \Rightarrow i \succeq_X j \quad (3)$$

In words, if the strict priority order for school x ranks i above j it is either because the original priority order ranks i above j, or because i and j were originally in a tie, but the tie-breaking favoured i over j.

So, to find a matching which respects the original priority structure $\succeq_X$, we can simply break the ties to obtain a strict priority structure $\succ'_X$, and then run the algorithm.

The outcome will be stable with respect to $\succ'_X$, and therefore will also be stable with respect to the original priorities $\succeq_X$.

(Set of matchings which respect $\succ'_X$) ⊆ (Set of matchings which respect $\succeq_X$)

The resulting matching depends on the tie-breaking. Outcome may well be Pareto dominated by another stable matching which is ruled out in the process because the tie-breaking (but not the actual coarse priority structure) renders unstable.

6. CONCLUSION

The paper discussed the determinants of teacher preferences over schools with rigid salaries: geography, and student characteristics and pointed out the inequality distribution of teachers' quality because of the assignment of absolute advantage caused by fixed wages. Since previous evidence suggests that matching between teachers and schools explains 10-40% of what we usually estimate as teacher quality, the paper focuses on the potential opportunity to improve teacher quality by better allocating teachers to schools.

The current policy for teacher assignments is centralizing teacher proposing DAA used in France and Teach DC Program, which is similar to the school proposing DAA. They have the advantages of reducing opportunity costs for teachers and expanding school application pools. Teach DC program also offers a decision aid that helps schools better predict teachers' quality. However, both mechanisms allocate teachers to schools based on absolute advantages, creating inequality in teacher quality between rich and poor neighborhoods under district school program, which assign students to schools base on their geographic location.

The proposed mechanism combines the two instruments and adds performance pay to incentivize teachers to allocate based on their comparative advantages. The centralized application would still save teachers' search costs and expand the applicant pool for schools. We examined that the mechanism under strict priorities is desirable with the mathematical model. The outcome respects priorities, and among all matching that respects priorities, the outcome is the teacher-optimal one. It is also strategy-proof, thus making it safe for teachers to be truthful and removing the burden on strategies, making it safer for the policymaker to rely on the revealed preferences to carry out welfare analysis. Furthermore, under coarse conditions, the result matching depends on the tie-breaking, but it is still feasible.

The paper examines two existing matching mechanisms between teachers and schools. Then, it proposes a new allocation mechanism, which theoretically improves the allocation between teachers and schools, potentially leading to a more equalized distribution of teachers' quality between rich and poor neighborhoods.

AUTHORS' CONTRIBUTIONS

This paper is independently completed by Ruonan Gao.

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