A Multi-product Newsvendor Problem Model Based on Demand Transfer and Mental Accounting in the Context of E-Commerce

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Abstract. With the development of digital economy which multi-product co-management is an essential feature, e-commerce becomes harder to make decisions in the process of management. This paper studies the Multi-Product Newsvendor Problem with demand transfer, budget constraints and mental accounting under demand uncertainty. We build a model to depict the multi-product demand and profit gain from e-commerce, and give the optimal order strategy for e-commerce by heuristic algorithm analysis. Then, a sensitivity analysis is conducted to study the effects of product transfer rate, budget constraints and mental accounting on the order strategy. The conclusion demonstrates that e-commerce should enhance the product demand transfer rate by various means and overcome the impacts of mental accounting and budget constraints.

Keywords: Multi-Product Newsvendor Problem · demand transfer · mental accounting · genetic algorithm · budget constraints · e-commerce

1 Introduction

E-commerce has developed rapidly in recent years. According to the E-COMMERCE IN CHINA [16] released by the Ministry of Commerce, e-commerce has become the most innovative component of the digital economy with the largest scale of development at the quickest speed. It plays an indispensable role in fighting epidemics, resuming work and production, preserving supply and recovering consumption. With the increase in China's economic income and product varieties, there are more alternative choices in similar products and the demand of e-consumer is also diversified with more uncertainty. All these aspects bring more indeterminacy and management difficulties to e-commerce enterprises.

In this paper, the Multi-Product Newsvendor Problem (MPNP) is used to study e-commerce operations and ordering decisions, taking into account the diversity and substitution of e-commerce products and the bounded rationality and budget constraints of e-commerce decisions. As the development of e-commerce expands the range of product choices and the number of alternatives available to the customers, there is a large number of demand transfer in the actual operation of e-commerce. Demand transfer

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means that when consumers purchase a product that is out of stock, they will choose a similar product as a substitute. According to the study by Smith's team, they found that only 12% to 18% of consumers will give up their purchase if the product is out of stock [18]. In order to be closer to reality, this paper introduces capital budget constraints and mental accounting. Mental accounting, a theory of cognitive psychology research, means that in reality, enterprises and consumers' decisions are irrational under the influence on their own benefits from a single decision among a combination of decisions.

In this paper, we introduce the Newsvendor Problem with demand transfer, mental accounting and budget constraints in e-commerce and its operation to facilitate e-commerce merchants to correctly evaluate the operation and choose the right operation strategies. We comprehensively study the influence of demand transfer and mental accounting on e-commerce operation, utilize genetic algorithm to analyze two different grades of goods in the same category so as to enhance the applicability of the study, and finally explore the law of e-commerce product demand and order through sensitivity analysis. Our goal is to support e-commerce enterprises effectively to make ordering decision.

2 Literature Review

Newsvendor Problem (NP), a classical inventory model in terms of monocyclic and stochastic demand, is initially invented then implemented by the ecologist Edgeworth in 1888. This paper studies Multi-Product Newsvendor Problem which was first put forward by Hadley and Whitin in 1963 [8]. It studies how the decision-makers determine the order quantity of multiple products in order to maximize total profits under the certain demand distribution and constraints amid products. This paper aims to provide an overview of Multi-Product Newsvendor Problem based on two perspectives: model formulation and solution approaches.

2.1 Relative Literature of Multi-product Newsvendor Problem

Many scholars have extended the relative research of Newsvendor Problem. For instance, Chen introduced multivariate Markovian demand to the original demand [3]; In Selim's study, he utilized mixed-integer programming to seek optimal decision of local and cross boundary procurement with risk constraints [1]. Feng discussed the substitution of products in Multi-Product Newsvendor Problem [5]. Zhou analyzed the MPNP model with budget and loss constraints by linear approximation [26]. Huang conceived strategic behavior of customers and introduced rational expectation equilibrium to his construction of the relative model [9]. Meanwhile he provided the solution of rational expectation equilibrium with both strategic customers in the static game. Whitin, Mills, Karlin and Carr enriched the newsvendor decision of MPNP model and maximized the profits by inventory control and price theory. Zhen focused on a vague demand model and constructed three decision models, which takes maximum expected revenue, chance-constrained programming, chance programming as three criterions respectively [27].

In generalization, there are three major perspectives in studying MPNP model: (1) Introduce more practical constraints to the model, such as budget constraints and risk

constraints. (2) Depict more details in the characters of products, including the demand fluctuation, the relationship between products, the contracts (quantity discount, repurchase) of product, etc. (3) Enrich the decision of possible newsvendor, like price decision, order quantity decision, or contract decision, etc.

2.1.1 Multi-product Newsvendor Model with Demand Transfer

In multi-product sales model dominated by E-commerce, there is always high substitution among the main categories. When the products for customers are in short supply, the unfulfilled purchasing demand will be transferred into the other products that have the same effect, function and feature. Then the demand transfer appears. Chinese scholars have made achievements in demand transfer research. For hybrid-channel supply, Guo made discussion about inventory control strategy in demand transfer, constructed and optimized buy back contract which promote the coordination of supply chain [6]. Wang studied the impact of customers' demand transfer on order strategies and solved the optimal order quantity in the context of canceling order behavior of e-commerce [22]. For multi-channel demand transfer, Wang approached the optimal decision and maximum benefits under different decision-making mode and facilitate the coordination by the incentive compatible contract [23]. Recently, there are numerous studies which combined e-commerce with demand transfer. However, most of them focus on gaming optimization between online and offline channels, and heterogeneity between product return and customer's demand, research on demand transfer and multi-product newsvendor model are still inadequate.

2.1.2 Mental Accounting

Classic studies on newsvendor model are to maximize the expected profit by decision on the basis of the hypothesis that the decision-maker is completely rational. Kahnman found out that the decision-maker largely relies on their psychological expectations rather than objective facts, that is, the decision-maker isn't absolutely rational [11]. A typical irrational behavior is the mental accounting founded by Kahnman in 1981 [20]. From the concert psychological experiment, he found that the decision-maker would form different mental accounting for different decision tasks. Though the cash values of these decisions are the same, there are distinctive values in each mental accounting. In multi-product newsvendor model, the rational decision refers that the decision-maker merely pursues the maximum of expected profit of all products without considering every product income in detail. But in reality, the decision-maker is always influenced by own mental accounting before making decisions and every product are endowed specific targets based on risks by the decision-maker. Gu applied the prospect theory to the newsvendor model from the mental accounting perspective and found that the ordering decisions of retailers were determined by the combination effect of preferences [7]. Shi introduced the mental accounting revenue illusion theory under the channel integration strategy of retailers to evaluate E-coupon [19]. Mental accounting is a vital direction of current newsvendor model research and explains the convergence phenomenon, but there are fewer studies on multi-product newsvendor models and mental accounting.

2.2 Review of Solution Approaches for MPNP

In terms of solution approaches for MPNP, for small-scale solutions, planning algorithms are mainly used at present. For example, when Hadley and Whitin first proposed MPNP under constraints, they gave a solution using Lagrange Multipliers, Leibniz Rule and Dynamic Programming. Some scholars also proposed a quadratic programming model to solve a multi-constraint MPNP model [2]. And the planning algorithms methods have been proved to have high accuracy in solving the model with simple constraint requirements. While facing more complex factors, Zhang utilized the gradient iteration; khanra used heuristic algorithms [12, 25]; Moon used both dynamic planning and heuristic algorithms to solve the MPNP problem with restocking cost constraints [14]; Niederhoff solved the MPNP by replacing the Lagrangian relaxation with the approximate planning algorithm [17]. In generalization, the two main methods are planning algorithm with specific solutions and large-scale heuristic algorithm, and both have their applicability.

In sum, this paper introduces the unique features of e-commerce operations such as demand transfer, mental accounting and budget constraints in the model in order to better simulate the current e-commerce operations for optimal decision of e-commerce enterprises. We also take into account that there are more constraints but less correlated with each other in the model. As e-commerce enterprises need to analyze and react quickly according to the changes in their own operations and demand, it is obvious that the exact algorithm cannot meet e-commerce's need in solution speed and multi-constraints. Therefore, this paper designs the genetic algorithm for the solution and conducts case studies and sensitivity analysis to confirm the feasibility of the model and provide better suggestions for e-commerce operations.

3 Model Formulation and Solution Approaches

3.1 Model Description and Hypothesis

Suppose that the e-commerce manages two products in similar efficacy with demand transfer between each other, in other words, the unmet demand of Product 1 will be transferred to Product 2 in the proportion of λ_1. But the demand transfer does not necessarily occur and will not be completely transferred. Therefore, $0 < \lambda_1 < 1$. The same goes to Product 2. For each product, its cost is c, price is p, and the order quantity is q at the beginning of the period. The unsold part can be recovered at price s at the end and satisfy the condition of $p > c > s$.

At present, there are two main views on the study of market demand relationship in MPNP model: one believes that the demand of the whole commodities is fluctuating while the demand proportion of different products in total is fixed, and the demand fluctuates with the change of total demand. For instance, Wang studied the optimal decision MPNP model with demand transfer in this situation [21]. The other believes that the demand of the whole commodities in a period of time can be viewed as relatively constant, and the demand relationships between different products are variable and subject to the changes of seasonal, market environmental, consumer preferential, and other factors. In this paper, we argue that the second view depicts more reasonably about the

Table 1. Meanings of the parameters

parameters	Explanation
d_1, d_2	Market demand of Product 1, 2
p_1, p_2	Market price of Product 1, 2
c_1, c_2	Wholesale price of Product 1, 2
s_1, s_2	Recovered salvage value of Product 1, 2 at the end of the period
λ_1	Percentage of market demand transfer from product 1 to Product 2
λ_2	Percentage of market demand transfer from product 2 to Product 1
K_1, K_2	minimum profit requirements of Products 1, 2 for e-commerce
C	Budget constraints of e-commerce
q_1, q_2	Decision variable, order quantity of Product 1, 2
π_i^j	Profit of product i in the jth case

product demand of e-commerce when making multi-product ordering decisions in Phase I. Therefore, we adopt the second hypothesis: set total demand relatively constant as 1, the demands of different products denote respectively as $\mathrm{d}_1 = x$, $d_2 = 1 - x$. $f(x)$ and $F(x)$ are probability density function (PDF) and cumulative distribution function(CDF), π is the product benefit. Model parameters are as in Table 1.

3.2 Model Formulation

As e-commerce pursues profit maximization, the total amount of order is possible to exceed product demand, formulated as $q_1 + q_2 \geq 1$. For all possible inventory conditions, the expected profits under different situations are as follows:

There is no demand transfer, i.e. the demand for each product is less than the order quantity, as $x \leqslant q_1$, $1-\mathrm{x} \geqslant q_2$. The profit for each product can be formulated as:

$$\pi_1^1 = \int_{1-q_2}^{q_1} (p_1 x - s_1(q_1 - x) - c_1 q_1) f(x) dx \tag{1}$$

$$\pi_2^1 = \int_{1-q_2}^{q_1} (p_2(1 - x) - s_2(q_2 - 1 + x) - c_2 q_2) f(x) dx \tag{2}$$

The demand of Product 1 exceeds its order quantity and it can be completely transferred to Product 2. The profit for each product can be formulated as:

$$\pi_1^2 = \int_{q_1}^{1} ((p_1 - c_1) q_1) f(x) dx \tag{3}$$

$$\pi_2^2 = \int_{q_1}^{1} \begin{bmatrix} p_2(1 - x + \lambda_1(x - q_1)) - \\ s_2(q_2 - (1 - x) - \lambda_1(x - q_1)) - c_2 q_2 \end{bmatrix} f(x) dx \tag{4}$$

The demand of Product 2 exceeds its order quantity and it can be completely transferred to Product 1. The profit for each product can be formulated as:

$$\pi_1^3 = \int_0^{1-q_2} \begin{bmatrix} p_1(x + \lambda_2(1 - x - q_1)) - \\ s_1(q_1 - x - \lambda_2(1 - x - q_2)) - c_1 q_1 \end{bmatrix} f(x)dx \tag{5}$$

$$\pi_2^3 = \int_0^{1-q_2} ((p_2 - c_2)q_2) f(x) f(x)dx \tag{6}$$

It is worth noting that in reality there is no transfer demand between products and no situation that the demand cannot be fully transferred due to the lack of inventory. The reason is that the total order quantity of e-commerce is greater than the total demand and the demand transfer between products does not necessarily occur, so the order quantity must be greater than the demand quantity. In the model, taking Product 1 as an example, if $x > q_1$, $(1 - x) + \lambda_1(x - q_1) > q_2$, that is, $q_1 < x < \frac{1-\lambda_1 q_1 - q_2}{1-\lambda_1}$ but the equation: $q_1 - \frac{1-\lambda_1 q_1 - q_2}{1-\lambda_1} = \frac{q_1 + q_2 - 1}{1-\lambda_1} > 0$ is impossible, there will be no alike situation in this model.

E-commerce have irrational behaviour due to mental accounting. They have the minimum expected value for every sort of product and are confronted with budget constraints when ordering goods. We can formulate as follows:

$$\pi_1^1 + \pi_1^2 + \pi_1^3 > K_1 \tag{7}$$

$$\pi_2^1 + \pi_2^2 + \pi_2^3 > K_2 \tag{8}$$

$$c_1 q_1 + c_2 q_2 < C \tag{9}$$

3.3 Algorithm Design Paradigm and Solution Approaches

In this paper, we simulate that two products operated by e-commerce are two commodities of different grades under the same major category. They have similar functions but differ in grades, so each can be substitute in utilization. Assuming that Product 1 is a high-grade product whose wholesale, market price and salvage value are higher, its profit is also higher. So the decision-maker's mental accounting expectation of it is bound to be higher; similarly, Product 2 has lower grade with lower wholesale price, market price and the salvage value, and its profit is lower than that of Product 1, thus, the mental accounting expectation is also lower. According to the research [13], in the same major category, the transfer rate of Product 1 is greater than the transfer rate of Product 2, formulated as $\lambda_1 > \lambda_2$. So we assume that the budget constraints work in the model, i.e., e-commerce is impossible to obtain the optimal solution without the constraints. Referring to the research [4], the corresponded values are selected to determine the parameter values and sensitivity analysis ranges as in Table 2.

This paper utilizes genetic algorithm to solve this model by using sko algorithm base of python 3.7.9 64 bite and basic parameters including initial population size, the maximum number of iterations, the mutation probability, and compares the solutions

Table 2. Algorithm design data

Paradigm	Original Value	Sensitivity Analysis	Paradigm	Original Value	Sensitivity Analysis
f(x)	Normal Distribution, Mean 0.5, Variance 0.1		C	13	[13–16]
p_1	30	____	p_2	16	____
c_1	16	____	c_2	6	____
s_1	10	____	s_2	3	____
λ_1	0.3	[0.3–0.4]	λ_2	0.2	[0.2–0.3]
K_1	13	[13–15]	K_2	3	[2.8–4.4]

comprehensively through crossover, mutation, selection, ranking operator. Finally, we determine the optimal algorithm, and the solution result achieves better convergence after about 200 iterations: the order quantity of the two products are 0.652 and 0.424 respectively, with a total gain of 26.759.

4 Sensitivity Analysis

4.1 Sensitivity Analysis of Product Demand Transfer Rate on Decision Making and E-Commerce Profits

This paper takes Product 1 as an example. When the demand transfer rate of Product 1 gradually increases, the optimal decision change of e-commerce demonstrates as in Fig. 1.

It can be observed that when the demand transfer rate of Product 2 remains unchanged, as the demand transfer rate of Product 1 increases, the order quantity of Product 1 gradually increases while both the order quantity of Product 2 and the overall order quantity decreases. During the process of product sales, the revenue of Product 1 comes from two sources: the sales revenue from satisfying consumer demand for Product 1 and the revenue from satisfying consumer demand transferred from Product 2. The increasing demand transfer rate of Product 1 indicates that the potential unmet demand of Product 2 is transferred to Product 1 in a higher proportion, which equates to the increase in the demand of Product 1. Consequently the revenue increases by satisfying the demand transfer. According to the basic NP theory, the increase in demand is followed by the increase in the order quantity for Product 1. For Product 2, the increase of the order quantity of Product 1 and the demand transfer rate reduces the out-of-stock loss of Product 2 and decreases the shortage cost. So the order quantity of Product 2 decreases according to the basic NP theory. Both products belong to the same major category. Overall, when the demand transfer rate between products increases, it represents

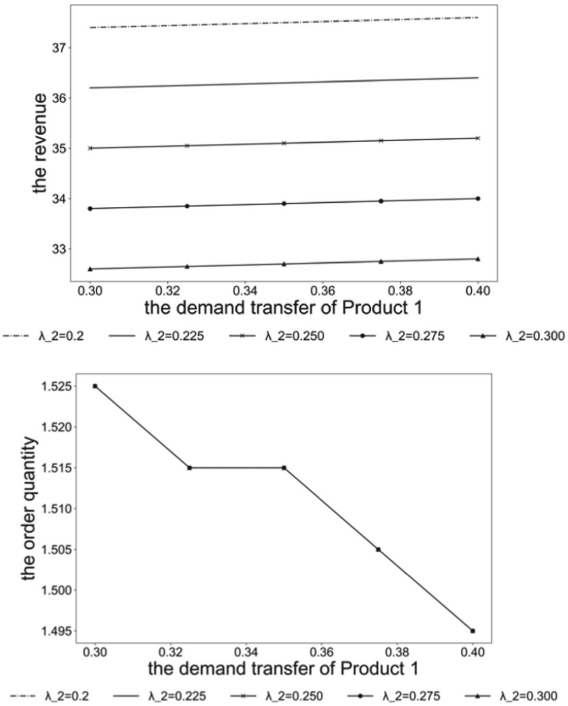

Fig. 1. Sensitivity analysis of demand transfer

better substitution between products, and the e-commerce can respond to the fluctuating market demand with less inventory and less total order quantity. As the total order quantity decreases, the order cost decreases subsequently. The increase in the demand transfer rate allows the products to better meet the transfer demand; the potential revenue rises; the increase in revenue and the decrease in cost directly lead to the increase of e-commerce's profits. When the demand transfer rate of Product 2 increases, similarly, the order quantity of Product 1 decreases; the order quantity of Product 2 increases; the total order quantity decreases; the overall profit increases.

4.2 Sensitivity Analysis of E-Commerce Mental Accounting for Decision Making and Profits

This paper takes Product 1 as an example. When the mental accounting of Product 1 gradually increases, the optimal decision of e-commerce demonstrates changes as in Fig. 2.

Among the major category products, as the unit value of Product 1 is relatively higher with higher profit margins, e-commerce will prefer to order Product 1. Therefore, when the mental accounting of Product 1 increases within a reasonable range, it will not have an impact on the original order strategy, and the optimal order quantity and e-commerce revenue will not change as well. When the mental accounting of Product 2 increases, e-commerce will increase the order quantity of Product 2 to meet its expectation. Though

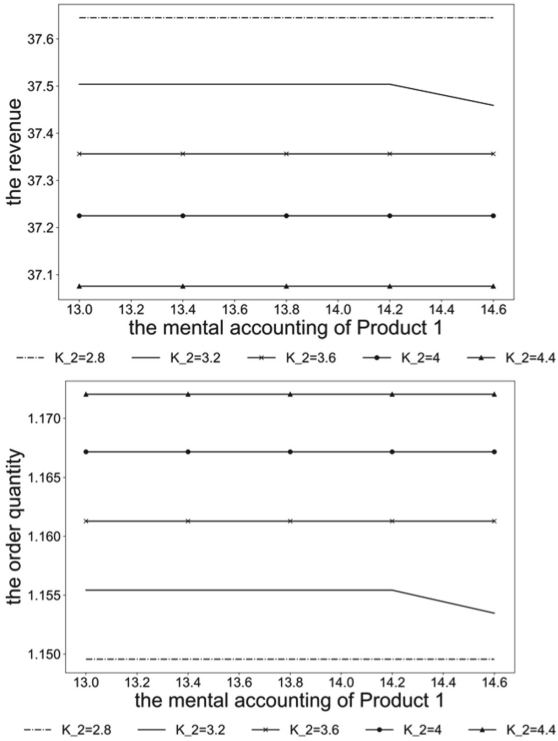

Fig. 2. Sensitivity analysis of mental accounting

the budget constraints undermine the order quantity of Product 1 whose unit cost is higher, there will be more order in Product 2 which is transferred from Product 1, and the total order quantity will increase. The mental accounting of Product 2 can deviate e-commerce decision from rational optimal solution, resulting in profit loss. The more influence of mental accounting, the more serious loss there will be on e-commerce profits.

4.3 Sensitivity Analysis of Budget Constraints on Decision Making and E-Commerce Profits

As the available capital held by retailers increases, retailers can have more choices in order strategies and purchase more Product 1 with higher unit cost, selling price and profit. So the order quantity of Product 1 gradually increases, which exerts a crowding-out effect on the order of Product 2 leading to the decrease in the order quantity of Product 2. Since the unit revenue of Product 1 is higher, the overall revenue of e-commerce will increase along with the increase of budget constraints within a certain interval (Fig. 3).

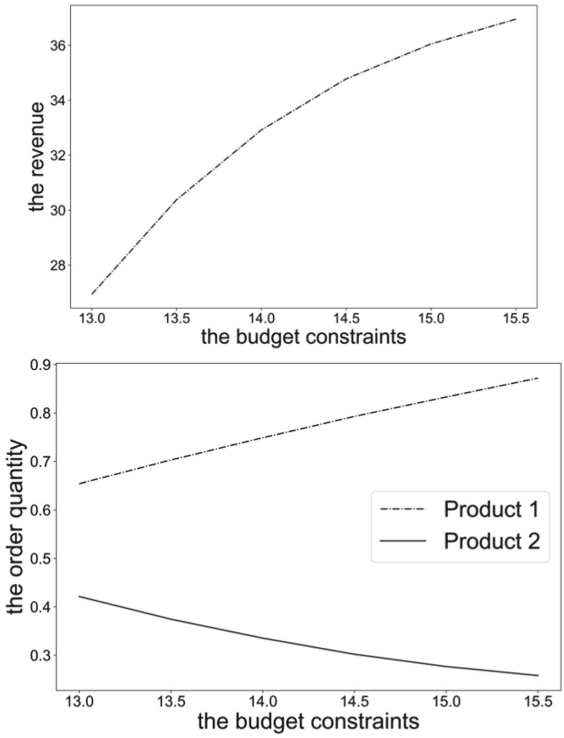

Fig. 3. Sensitivity analysis of budget constraints

5 Conclusion and Advice

5.1 Research Conclusion

In this paper, to study consumers' demand transfer among substitutes, we build MPNP model with budget constraints and mental accounting and analyze the optimal ordering decision and revenue made by e-commerce under the influence of mental accounting and budget constraints when they manage different grades of products under a major category with demand transfer. The study concludes that for premium products, e-commerce should always purchase more than the demand. The greater the transfer probability is, the higher the retailer's profits will be. And the budget constraints have the greatest impact on e-commerce profits and constrain purchasing decisions of the e-commerce.

5.2 Advise to E-Commerce Based on MPNP

With continuous development of digital intelligence technology and e-commerce, the increasing diversity of products and demands brings great uncertainties and operational challenges to enterprises. How to solve these difficulties like reducing cost and increasing profits is one of the keys to gain competitive advantages for enterprises. This paper

optimizes the order quantity decision of enterprises by considering MPNP model with mental accounting, and budget constraints and proposes the following recommendations:

First, e-commerce need to improve the product demand transfer rate. A higher product demand transfer rate is conducive to reducing shortage cost, inventory cost, and better meeting consumers' demand and responding to the market changes. Thus, the revenue of e-commerce is enhanced. In this sense, to improve the demand transfer rate between products, e-commerce companies can improve the similarity of core functions among different products, strengthen product publicity, and create intelligent recommendations on similar products when the products are out of stock. Then, the overall benefits will be enhanced.

Second, overcome the influence of mental accounting. The more impact of personal mental accounting is, the greater extent of e-commerce deviation from rational decision will be. This can undermine the optimal order quantity, resulting in profits loss. Therefore, e-commerce should recognize the impact and logic of mental accounting and try to overcome its side-effect in the decision making to maximize profits.

Third, the e-commerce should try their best to improve the budget. The higher the budget they have, the higher the revenue they will gain. So the enterprises can increase the order budget ratio, or reasonably use bank loans, mortgages, agreement deeds and other ways to finance, so as to increase the funds for order and enhance the benefits of e-commerce.

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