# Commognition Framework: Types of Students' Difficulty in Solving Fractional Task

Andika Setyo Budi Lestari[1(✉)], Nonik Indrawatiningsih[2], Ani Afifah[1], and Supriyo[1]

[1] Universitas PGRI Wiranegara, Pasuruan, Indonesia
andikalestari123@gmail.com
[2] Universitas Negeri Surabaya, Surabaya, Indonesia

**Abstract.** The fraction is substantial material because it supports understanding the concepts of other mathematical materials and is used in daily life. However, many students have a weak ability to find a solution to fraction tasks. Also, the specific researches focused on fraction issue based on commognition framework are very limited. Therefore, this descriptive research investigates the types of students' difficulty in solving fractional tasks based on the commognition framework. The study used a qualitative approach. The commognition framework includes visual mediators, routine, narratives, and word use. The knowledge was gathered through a written examination and an interview. The participants were 36 fifth-grade students. The triangulation process was used to ensure the quality and reliability of the testing findings by reviewing the outcomes of video recorded interviewing and the results' written analysis. The research findings revealed that the students' problems were mostly in the case of visual mediators, narratives, and routine mistakes. These kinds of failures will influence the next mission approach. In solving fraction tasks, the students used visual mediators very rarely. Thus, solving the fractional tasks made it difficult for them.

**Keywords:** Commognition · Fraction · Students' difficulty

## 1 Introduction

A fraction is an exciting topic in mathematics education, which is studied by many researchers. The students must follow the computational procedure by reversing the numerator with the denominator without knowing the concept to solve the problem of fractional division and multiplication. Solving division only by following the concept procedure will impact the confusion and lack of understanding based on multiplication and fractions division [1, 2].

Fractions in primary schools are deemed necessary as they are the basis for advanced materials such as algebra when they are in high school once they are in problem-solving matters. Fractional concepts are often used in everyday life, so students need to learn and understand them. Students feel difficulty understanding fractions [6, 11]. Fractions are a concept that is considered difficult by elementary school students and many teachers to have a deep understanding of the fractional concept [12, 13].


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Researches related to students' difficulties in making precise and accurate guesses and interpretations, memorizing and using facts, focus and logic have been widely practiced [3, 4]. The research has shown that students' characteristics of errors in constructing their thinking in mathematical concepts include pseudo right & pseudo incorrect, analogical thinking, conceptions, and logical thinking [5].

Problem-solving is a form of student's communication that can be done verbally and non-verbally. To analyze students' communication in solving the task, the commognition framework initiated by Sfard is applied. The commognition framework is a combination of communication and cognition. In the analysis using the commognition framework, it can be described in four categories i.e. visual mediators, endorsed narratives, routine, and word use. The phenomenon above has led the researcher to investigate the difficulties elementary school students face in solving fractions using the commognition framework.

The mechanism of interpersonal and cognitive-communication is two distinct manifestations within a common phenomenon, such that these two concepts (communication and cognition) are merged into a new concept called commognition. She introduces four interrelated characteristics of mathematical discourse: words, visual mediators, narrative support, and routines. Commognition is used to evaluate the learning process of mathematics. Diagrams and symbols of mathematical objects are referred to by visual mediators, as are physical objects used as teaching aids. Endorsed narratives: refers to a set of propositions agreed in a specific mathematical community (such as theorems and definitions), proof, and calculation laws. As well as a set of meta-rules (e.g., how to quantify, how to prove, how to generalize), routines mean frequently patterned repetitive behaviors (e.g., addition, proof, generalization). Word usage refers to the way in the conversation and debate.

Students also need to consider the rational number density because there is an infinite number of fractions between two successive integers, e.g., between zero and one [6, 7]. When students are asked what lies between zero and one, they will answer their questions ½, 1/3, and ¼. It showed that the responses offered by the students were less diverse. It is proof that the students' presumption about the number between zero and one is just a few numbers. They have not understood that there are simply indefinite numbers between zero and one there. The problem was the low skill of students' problem solving in fraction. Therefore, it is necessary to examine more in-depth research on student difficulties. To achieve this goal, the problem in this study is how students' difficulties in solving fraction problems using a commognition framework.

## 2 Methods

The goal of this analysis is to examine the difficulties faced by learners in solving fractions. The qualitative method used in this descriptively exploratory analysis was [8]. This research was conducted explosively with students about what students display through students' answers, in the form of specific images or symbols when writing the answers of fractional material in word problems, so that the research is descriptive. Besides, the data reveals what their knowledge is, through interviews, to understand fractional ideas in a given time and continuously. Qualitative data has more benefits than quantitative data since qualitative data interpretation and clarification are richer.

A qualitative approach was used because data processing was standard and rooted in sign language, movements and facial expressions, gestures and facial expressions, and written responses in the form of words both spoken and used. In the context of terms extracted from observations, interviews, or records, the general form of qualitative evidence is.

### 2.1 Participants

The subjects in this study were elementary school students in the $5^{th}$ grade. The students were selected because fraction is the initial material in $5^{th}$ grade. Also, the students have been able to be invited to communicate well when later interviews are conducted. The selection of subjects in this study is based on the mathematical abilities possessed by students. The subjects in this study were 36 fifth graders students of primary school in Pasuruan, Indonesia. There were 14 males and the rest were 22 females. Scientists also regard input from a school math teacher. Afterward, the criteria of chosen characters are the ability to adapt and engage with others based on insights in the classroom and teacher knowledge in the classroom. This is because the teacher is more aware of the character of his pupils. Subsequently, researchers selected one male student and one female student.

### 2.2 Data Collection

In the form of questions and interview guides, the researcher gathered data using supporting instruments as the main instrument. The study's main data are the results of student work and responses when they were being interviewed. Data related to students' works were taken from the students' assignments, which were collected through observation, field notes, and closed questionnaires. Data associated with responses and complete information regarding the work of the students is collected through interviews. The interviews are important in collecting data so it needs to be well-prepared to obtain the expected data. For this reason, the researcher should ensure that participants know that they have no intervention during the interview. It should be pointed out that while participants should speak loud enough so that the audio can capture their voice clearly, the participants must be ensured that they are comfortable during the interview process when they are asked to clarify the questions about the task. The interviewer should be able to take inquiring questions.

The method for gathering data is carried out using the following steps: (1) given test about fraction in the form word problem. (2) After the student completes the fractional story test, the subject is asked to read and understand the question that has been given. (3) The next stage of conducting semi-structured interviews with spoken language and gestures and observations are carried out. Interview results, observations, field notes, and written answers are used to explore students' understanding (4). Triangulating. Through combining data from written responses and data from the second interview with various time ranges, the triangulation used in this analysis was time triangulation. (5) All subject activities at the time of data collection are recorded using audiovisual instruments.

The interview's essence is to get authentic answers from students in regard to the thinking processes and capture related commognitive processes. In the case of open questions, it should be the main way of encouraging student responses. Recorded video

**Table 1.** The distribution of students' answers within commognition framework

| Characteristics | Word Use | Visual Mediators | Endorse Narratives | Routines |
|---|---|---|---|---|
| Female | correct | Correct | Correct | correct |
| | incorrect | incorrect | incorrect | incorrect |
| Male | correct | Correct | Correct | correct |
| | incorrect | incorrect | incorrect | incorrect |

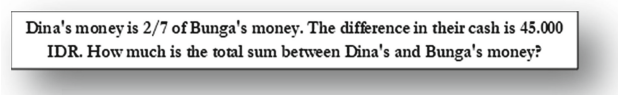



**Fig. 1.** The Problem Test for Student

on the interactions during the discourse is also another essential part of the data collection process. To capture the classroom interactions, we employed two cameras - one was from the back part of the class, and another one was from the front so that student interactions could all be captured. To capture the interaction during the interview, one camera was used to take pictures of the interviewee, while another camera was used to capture the process of how the participants completed the instrument. Camera batteries must be checked, and where necessary, the use of a microphone to capture crystal clear audio was also required. Another important thing was to run the audio taping along with recording video in case of camera damage.

In other words, this study is also seen as a case study and this is applied in the area of education. It is focusing on the work and what students responded when they were interviewed. Here is the distribution of students' answers within commognition framework shown in Table 1.

Based on the four components in commognition, two students were selected to be interviewed. Selection of subjects to be interviewed based on students' good communication skills and experienced the most errors based on the commognition component in working on the problems.

In this study, data collection was based on test results. The following Fig. 1 is the problem used in this study. 2 experts validated the problem test (instrument) used in this study. Based on an average of each aspect (concept, content, and language), an overall average of 86.45%, and established validity criteria (valid if the overall average is more than 75%). Hence, the test prototype meets the validity criteria and does not need to be revised so that it can be directly used for research.

### 2.3 Data Analysis

The researcher must have an analytical framework that is in line with the research's objectives and questions. Sfard's commognitive framework is a useful framework for the analysis of mathematics discourse. Data on students' oral activities were analyzed by encoding at each meeting the findings and field notes from two observers. The data's

reliability indicates that the observations and field reports of the two observers are 95% identical. The next step is time and source triangulation by paying attention to the accuracy and similarities of students' responses over different periods. Data review was performed by triangulation to ensure the study's validity and reliability by viewing video data, transcribing the interview, and comparing it with the student's written test data. The knowledge was then analyzed using an interactive model that includes data reduction, data presentation, and concluding/verification [9].

### 2.4 Procedure

The procedure is the flow of activities carried out from research preparation to the writing of research reports. The flow of this activity is described as 1) The preparation stage. The preparation stage consists of: (a) the selection and determination of the research site, conducting a survey of the research site and applying for research permission (b) compiling and validating instruments (c) preparing collection tools and tools for recording data); 2) Implementation stage: a. Researchers explain the purpose of study. b. Application of instruments, c. Selection of research subjects d. Data collection; 3) Data analysis stage: at the data analysis stage, the process is: planning the entire data, making transcripts, analyzing all recordings, describing the data, presenting in the narrative, and interpreting the data. 4) Writing of research report: the activities carried out at this stage are composed in research articles.

## 3 Findings and Discussion

Fraction products used at the primary school level in the fifth grade are related to the fraction operation. In the form of scenarios, the function given to students is. To be completed classically, the task is given to all students. In addition, the effects of the work of the students were analyzed. The results of the written examination of students grouped into four criteria were summarized in Table 2.

One student with incorrect responses was chosen to represent each group in the commognition system to examine the students' difficulties in solving fractions' assignments. The Sfard method of commognition is a powerful framework for exploring the mathematical vocabulary of students' challenges in conducting fractional tasks. Here, we discussed how the study uses Sfard's commognitive form in each division. The research findings were obtained in detailed interviews using commognitive structure that can be seen from the following transcript of the interview.

**Table 2.** The answers within commognition framework

| Characteristics | Word Use | Visual Mediators | Endorse Narratives | Routines |
|---|---|---|---|---|
| Female | 81,81% | 9,09% | 13,63% | 22,72% |
| | 18,19% | 90,91% | 86,37% | 77,28% |
| Male | 85,71% | 14,28% | 28,57% | 28,57% |
| | 14,29% | 85,72% | 71,43% | 71,43% |

### 3.1 Visual Mediator

The written test answers of S1 and S2 can be seen in Fig. 2

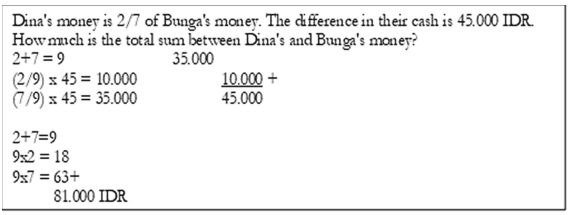
From an interview with S1, it was known that Visual Mediators in this study are visible objects that are used as communication media, the manifestation of word problems of fraction. The following are students' answers to fraction word problems.

From an interview with S1, it was known that Visual Mediators in this study are visible objects used as communication media, the manifestation of word problems of fraction. The following are students' answers to fraction word problems. Data was collected based on the interview's extract that students' visual mediators were affected in the sentence by word usage. The visual mediator that is used often adjusts when students realize the error and replace it. The initially used symbol was plus, and it was changed to minus.

Based on the excerpt from the interview, information was gathered that students' visual mediators were influenced by word usage in the sentence. Once students realize the mistake and replace it, the visual mediator that is used also changes. The symbol used initially was plus, and it was changed to zero, Based on the answers written by students (Fig. 3), it can be seen that the visual mediator used by students is symbolic.

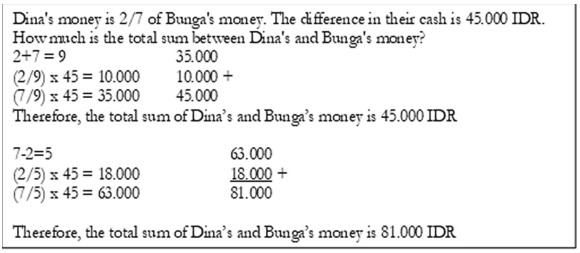
Students use statistical terminology or keywords such as percentages, sums, and denominators in resolving a given phrase. It indicates that keywords have a role in the problem-solving process, based on knowledge gathered from student work and interviews.

Based on the interview quotation, knowledge was gathered that percentages, totals, amount of discrepancies and denominators were the terms use chosen by students in fraction word problems. The terms "total sum" and "sum" are the prevalent word use


Dina's money is 2/7 of Bunga's money. The difference in their cash is 45.000 IDR.
How much is the total sum between Dina's and Bunga's money?
2+7 = 9 35.000
(2/9) x 45 = 10.000 10.000 +
(7/9) x 45 = 35.000 45.000

2+7=9
9x2 = 18
9x7 = 63+
81.000 IDR

**Fig. 2.** Translation of S1 Written Test Answer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Therefore, the total sum of Dina's and Bunga's money is 45.000 IDR

7-2=5 63.000
(2/5) x 45 = 18.000 18.000 +
(7/5) x 45 = 63.000 81.000

Therefore, the total sum of Dina's and Bunga's money is 81.000 IDR

**Fig. 3.** Translation of S2 Written-test answer

that influences problem-solving. Students write two answers when solving the problem by presenting inaccurate knowledge about the solution. After seeing what was asked about the question, learners understand the error at the end of the completion. Students attempted to correct the error after recognizing the mistake that was made, but the solution was still incorrect since the previous completion step was not correct. This illustrates that in solving the dilemma, the use of word usage plays an important role.

### 3.2 Endorse Narrative

Endorse Narrative are a series of sentence that describe objects, interactions, and procedures, such as definitions, theorems, and evidence. It is important to remember that, while not articulated explicitly, supported narratives can exist in student responses.

Q: Why do you add them to solve it?
S1: Because there are clue words, namely "total sum" and "sum".
Q: Why do you have two solve it?
S2: Because there are clue words, namely "total sum", "sum" and "difference".
Q: When you using sum and difference to solve it?
S2: Em,… I am still confused.

Students have not provided the correct explanations for the fractional issue, based on the findings of the extracts from the interview. The pupil should be told that in solving responses to fractional word problems, students can have interpretations and reasoning for making choices. But since the approach used is incorrect to solve the problem, the reason offered is also false.

### 3.3 Routine

In proposing solutions to address the challenges they face, the routine is word selection or representation selection. Students' job effects demonstrate their patterns when presented with a narrative to fix problems. Students illustrate multiple exercises about how they solve problems. There were two-word problems associated with fraction operations in this report. By combining two and seven ($2 + 7 = 9$), they solve them. For number two, where the first answer has been crossed, there are two responses. Researchers perform interviews with students to gain more knowledge relevant to the behaviors of students. The above are the consequences of interview extracts that have been completed.

Data was collected to solve the issue based on the findings of student work and interviews. Without understanding the cause, students are familiar with the quick path and understand the problem. When reading questions in which it impacts the comprehension of word issues by learners, the consistency of learners is another aspect that affects the response. When students face a slightly different question, the assignment's answer is worked out in the same manner as the previous question.

Based on the excerpt from the interview, it can be seen that by writing down the known items, asking questions and then responses, the routine that is carried out by students while solving the story dilemma. Students do this based on guidance given

by the instructor. However, at the response point, students still find it hard to write the solution to the problem.

Fractions are one of the basic mathematics materials for the level of elementary school. Fractions in the primary school level do not only study about fractions but also the operation of fractions. Fractional materials need to be given to math at school because fractions and fractional operations are often used in everyday life. At the elementary school level, fractional material is considered as one of the hardest materials. This is obtained from students' work results; 88.9% of students have incorrect answers to solving fractional tasks. Based on the questionnaire the students have completed, it showed that 91.6% of the students stated that they have difficulty in solving fractional tasks.

Students' mistakes in solving fraction problems are in the algorithm, errors based on intuition, and their formal knowledge [10, 11]. In solving word problems, students have difficulty in understanding the operation of fractions [12–15]. Therefore, it is necessary to do more in-depth research on student difficulties and scaffolding.

To evaluate the challenges of students in solving fraction tasks, a commognition method consisting of word usage, visual mediator, supported narratives, and routine can be used [16–19]. Fractional concepts are the basis for students to learn fractional operations, advanced fractional concepts, and fractional applications. Fractions can be interpreted as divisions, as ratio measurements, as operators, as a measure of [20, 21].

It is understood that most students are only at the procedural level of knowledge, not even at the level of conceptual understanding, based on the outcomes of the student's work. If students focus on procedural skills only, it results in less developed conceptual abilities. This is evident from the student's work, when interviewed, which students only remember the solution by summing up the known question element's denominator without understanding its meaning. Simply applying the procedure in the absence of a concept leads to solving the wrong question.

## 4 Conclusion

Interactions between students and between students and teachers should be considered for discourse analysis in mathematics. Other factors, including interpersonal dynamics in the learning environment and social expectations, are also to be studied, including ways in which students are encouraged to participate in dialogue and communicate their ideas. Based on a cognitive approach, we argue that the essence of discourse analysis lies in the discourse analysis of students who consider different communicative, verbal and nonverbal languages, explore past and present meaning through participation, and explain the discourse between text, context, and culture.

The findings of the commognition-based student difficulty research indicate that students still have difficulty solving fraction problems related to everyday life, which is called word problems. The process of elucidating the problem and finding the strategy of solution is the crucial difficulty. This is evident from the routine stage that the student does. Students only use word use as the key to selecting the procedure used. When clarified on the endorse narrative the student was unable to explain the reason. The form of student error in the form of a visual mediator is only a symbol of algebra. The study results showed that the students' difficulties were mainly in the case of errors in visual

mediators, assisted narratives, and routine. These types of errors will impact the next task's solution. In solving fractional tasks, the students used visual mediators very rarely. Thus, solving the fractional tasks made it difficult for them.

## References


1. de Castro, B. (2004). Pre-service teachers' mathematical reasoning as an imperative for codified conceptual pedagogy in Algebra: A case study in teacher education. *Asia Pacific Education Review, 5*(2), 157–166.
2. National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. The National Council of Teachers of Mathematics Inc.
3. Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: A general or specific deficit? *Journal of Experimental Child Psychology, 96*(3), 197–228. https://doi.org/10.1016/j.jecp.2006.10.001
4. Bryant, D. P. (2009). Working with your child teachers to identify and address Maths disabilities. *LD Online*, pp. 197–228.
5. Nusantara, T. (2013). Karakterisasi Kesalahan Berpikir Siswa dalam Mengonstruksi Konsep Matematika. *Jurnal Ilmu Pendidikan, 19*(2), 208–217.
6. Ahmad, A., Tarmizi, R. A., & Nawawi, M. (2010). Visual representations in mathematical word problem solving among form four students in Malacca. *Procedia Social and Behavioral Sciences, 8*, 356–361. https://doi.org/10.1016/j.sbspro.2010.12.050
7. Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology, 51*(2), 101–140.
8. Creswell, J. W. (2012). *Research design qualitative, quantitative, and mixed* (2nd ed.). Sage Publications.
9. Miles, M. B., Michael Huberman, A., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.).
10. Tirosh, D., & Tsamir, P. (2020). Intuition in mathematics education. In *Encyclopedia of mathematics education* (pp. 428–433).
11. Babai, R., Brecher, T., Stavy, R., & Tirosh, D. (2006). Intuitive interference in probabilistic reasoning. *International Journal of Science and Mathematics Education, 4*(4), 627–639. https://doi.org/10.1007/s10763-006-9031-1
12. Cramer, K. A., Post, T. R., Robert, C., Cramer, K. A., Post, T. R., & Robert, C. (2002). Initial fraction learning by fourth- and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education, 33*(2), 111–144.
13. Ni, Y., Zhou, Y., & Ni, Y. (2010). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychology, 40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001
14. Irfan, M., Sudirman, S., & Rahardi, R. (2018). Characteristics of students in comparative problem solving. *Journal of Physics: Conference Series, 948*, 1–11. https://doi.org/10.1088/1742-6596/948/1/012007
15. Irfan, M., Nusantara, T., Subanji, S., & Sisworo, S. (2020). Students know the concept but are incorrect in solving the proportional problem how does it happen? *The International Journal of Science, Mathematics and Technology Learning, 27*(2), 1–12.
16. Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity, 8*, 42–76.

17. Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.
18. Viirman, O. (2015). Explanation, motivation and question posing routines in university mathematics teachers' pedagogical discourse: A commognitive analysis. *International Journal of Mathematical Education in Science and Technology, 46*(8), 1165–1181. https://doi.org/10.1080/0020739X.2015.1034206
19. Sfard, A. (2006). Participationist discourse on mathematics learning. In *New mathematics education research and practice.*
20. Clarke, D., Clarke, B., & Roche, A. (2011). Building teachers' expertise in understanding, assessing and developing children's mathematical thinking: The power of task-based, one-to-one assessment interviews. *ZDM - International Journal on Mathematics Education, 6*(43), 901–913.
21. Purnomo, Y. W., Kowiyah, K., Alyani, F., & Assiti, S. S. (2014). Assessing number sense performance of Indonesian elementary school students. *International Education Studies, 7*(8), 74–84. https://doi.org/10.5539/ies.v7n8p74