Simulation of German DAX Index for the First 20 Years in the 21st Century with Random Walk Model

Shaomin Yan and Guang Wu(✉)

National Engineering Research Center for Non-Food Biorefinery, State Key Laboratory of Non-Food Biomass and Enzyme Technology, Guangxi Biomass Engineering Technology Research Center, Guangxi Key Laboratory of Biorefinery, Guangxi Academy of Sciences, 98 Daling Road, Nanning 530007, Guangxi, China
hongguanglishibahao@gxas.cn

Abstract. The German DAX index is an important indicator not only for following German stock market but also for the health of German economy. The movement of the DAX index is thus the objective of many studies, to which the random walk model does not contribute a lot although this approach has been advocated for years. In the past, the random walk model was mainly done in terms of statistical tests rather than to simulate the real stock data. In this study, we applied the data generated by random walk model to fit the closing price of the DAX index for the first 20 years in this century, which were furthermore divided into five sub-periods. The results show that the simulation generated by random walk works for short sub-periods but has difficulty to mimic the DAX index for a longer period.

Keywords: DAX Index · Random Walk · Simulation · Finance And Trade · Stock Market

1 Introduction

The DAX index is an important indicator for following the German stock market, which is tracked by many investors and funds. Because of its important, the DAX index has been studied intensively [1, 2, 5]. Actually, not only the DAX index itself, but also its derivative securities, for example, its option is the objective of studies [7]. The strategies to predict the trends in index, the price movement, and the volatility in stock market are important.

Some studies suggest that the DAX index follows a random walk pattern while some studies argue against this claim [1, 2, 5, 7]. However, both claims are based on the statistical tests such as autocorrelation test, unit root test, run test, variance ratio test [4, 6]. To our knowledge, not many studies were done by using a certain model to simulate the real stock movement over time and then compare the simulation with the real data.

Thus it appears more important to run a study to test whether the random walk mode can simulate the DAX index, which is designed as the aim of current study.

N. Radojević et al. (Eds.): ICAID 2022, AHIS 7, pp. 1083–1090, 2023.
https://doi.org/10.2991/978-94-6463-010-7_110

2 DAX Data and Random Walk Model

2.1 DAX Data

The daily German DAX index for the first 20years in the 21st century was obtained from German Yahoo Finance (https://de.finance.yahoo.com/). This dataset from 2001 to 2020 includes 5072 daily open, high, low, close, adjusted close, and volume. The daily close is the target for random walk simulation. The simulations were designed to follow five sub-periods with increment of five years for each, 2020 includes 254 trading days, 2016–2020 includes 1262 trading days, 2011–2020 includes 2528 trading days, 2006–2020 includes 3799 trading days, and 2001–2020 includes 5072 trading days.

2.2 Random Walk Model

Random walk by definition [11] is a path that is generated by tossing a fair coin continuously. As one face of coin and the other face are defined as 1 and –1, the addition of results of tossing a coin for a number of times will generate a random walk along the time course. Because the tossing of coin is a random event, the record of a series of tossing is also a series of random events.

2.3 DAX Index as a Random Walk

The DAX index can be simplified in the context of random walk, i.e. a DAX index close in given trading day is higher or lower that in its previous trading day, it marks as 1 or –1, and then these values are added together along the time course, which is exactly a random walk. This rationale is workable because no equal close is found between two consequential days in the DAX index from January 2, 2001 to December 30, 2020.

2.4 Random Walk in Decimal Format

Although the random walk in decimal format is not defined, we could expend the classical random walk, which is the format 1 and –1, into a decimal format. This is because the modern random walk is made of random numbers generated by computer, these random numbers must be rounded to integral to fit the definition that random walk is in the 1/–1 format. If we leave these random numbers as they are, and add these them together along the time course, then their addition will be a random walk in decimal format. In reality, this is the case for any stock index and any individual stock.

2.5 Simulation

With the random walk in both 1/–1 and decimal formats, we can use a computer to generate a series of random numbers, whose addition is a random walk simulation. The simulation should be as similar as possible to the DAX index. The difference between a random work and the DAX index is measure to evaluate the performance of simulation.

3 Results and Discussion

Table 1 shows how to conduct the random walk simulation in both 1/–1 and decimal formats. Columns 1and 2 are the date and its corresponding DAX index close for the beginning of 2020. Column 3 records whether the DAX index is larger or smaller than its preceding day in terms of the 1/–1 format. For example, 13761.38, the DAX index close on January 3, 2020 is larger than 13718.78, the DAX index close on January 2, 2020, so 1 was given to the second cell in column 3. Column 4 is the addition of each cell in column 3, and constructs a DAX index random walk in the 1/–1 format. Column 5 is the first step to build a random walk, i.e. to generate random numbers as many as the number in column 3. Column 6 records whether the generated random number is larger or smaller than its preceding random number in the 1/–1 format. Column 7 is the addition of each cell in column 6, and constructs a random walk simulation for comparison with column 4. The last two columns show how to construct a random walk simulation in the decimal format. Column 8 is a series of random numbers generated according to the standard deviation of the DAX index close in 2020 because the command for generation of random numbers usually includes 4 terms, i.e. seed, number to be generated, and upper and lower ranges. In our previous studies [8–10], we found that the standard deviation is more suitable for upper and lower ranges. Column 9 is the random walk simulation by adding each random number in column 8 to the corresponding DAX index close value in column 2. Finally comparison can be made between columns 2 and 9.

According to the procedure in Table 1, Fig. 1 uses the data from column 4 (black line) and column 7 (red line) as a part of line. Figure 2 uses the data from column 2 (black line) and column 9 (red line) as a part of line.

Figure 1 demonstrates that the random walk simulation very similarly fits the DAX index. Because this random walk operates on 1 or –1 for each step, theoretically, it has a $(½)^{254}$ chance to exactly fit the DAX index for 254 trading days in 2020.

Table 1. Procedure to Construct A Random Walk Simulation

Date	DAX index Close	Compare Preceding Close	Random Walk in 1 or –1 Format	Generated Random Number	Compare Preceding Random Number	Random Walk in 1 or –1 Format	Generated Random Number	Random Walk in Decimal Format
Jan 2, 2020	13718.78							
Jan 3, 2020	13761.38	1	1	–0.85949	−1	−1	124.47243	13507.40243
Jan 6, 2020	13790.29	1	2	0.32574	1	0	−4.77235	13502.63008
Jan 7, 2020	13587.23	−1	1	–0.33913	−1	−1	23.5523	13526.18238
Jan 8, 2020	13418.11	−1	0	0.39213	1	0	−21.60509	13309.5773
Jan 9, 2020	13246.30	−1	−1	0.08469	−1	−1	−0.19514	13309.38216
Jan 10, 2020	13630.51	1	0	0.03815	−1	−2	−71.29754	13237.58452
Jan 13, 2020	13667.25	1	1	0.63435	1	−1	2.59234	13240.17696
Jan 14, 2020	13565.98	−1	0	0.41228	−1	−2	−54.12555	13185.05141
Jan 15, 2020	13362.87	−1	−1	0.96162	1	−1	136.81254	13322.86395

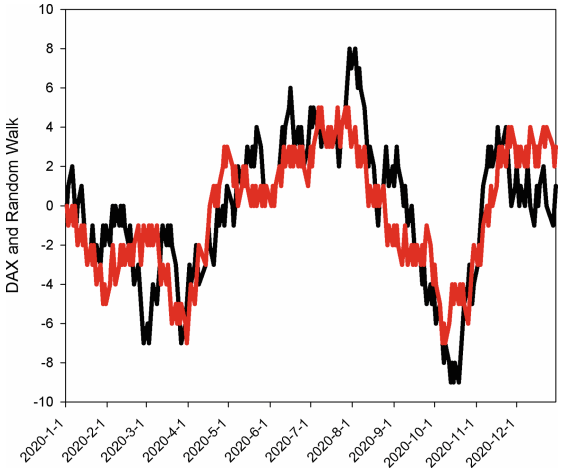

Fig. 1. The DAX indexed in 2020 in 1 or –1 format (black line) and its simulation (red line) generated by random walk in 1 or –1 format using seed of 0.65599.

Figure 2 shows that the random walk can approximately simulate the decline in the DAX index from January to April. The simulation becomes better from April to July, and then cannot fit the valley in October. Nevertheless, one could argue that the fear on Covid-19 and economic slowdown lead the fast decline in the DAX index.

Comparing Fig. 1 with Fig. 2, we can find that they are completely different although they present the DAX index in 2020. This is the difference between 1/–1 format and decimal format. The simulation in Fig. 1 is better than that in Fig. 2 because the random walk simulation using the 1/–1 format does not care the upper/lower range of generated random numbers. This also leads the difficulty to use the decimal format random walk to simulate the DAX index. Indeed, two valleys (black line) in Fig. 2 cannot be followed by the random walk simulation (red line).

Figure 3 illustrates the random walk simulation on DAX index for five years. When looking at the volatility in the DAX index, it did not fluctuate too much until the Covid-19 pandemic. Thereafter, the simulation almost has the similar tendency to follow the DAX index until the Covid-19 pandemic except for two periods in 2016 where the simulation tendency is opposite to the DAX index. Although the simulation goes down three times in 2019 and 2020, it cannot generate a rapid decline to follow the DAX index, especially to follow the impact of Covid-19 in 2020.

Figure 4 depicts the random walk simulation on DAX index for ten years. With increase of time involved in simulation, the sharp decline in the DAX index in March 2020 appears less visible, which actually is due to the increased range for the DAX index because it climbs from 5072.33 points up to 13790.29 points from 2011 up to 2020 (Fig. 4) whereas the DAX index ranges from 8441.71 points up to 13790.29 points for both Fig. 2 (2020) and Fig. 3 (2016–2020), from 3666.41 points up to 13790.29 points for Fig. 5 (2006–2020), and from 2202.96 points up to 13790.29 points for Fig. 6 (2001–2020). Yet, the simulation from 2011 to 2013 goes opposite to the movement of

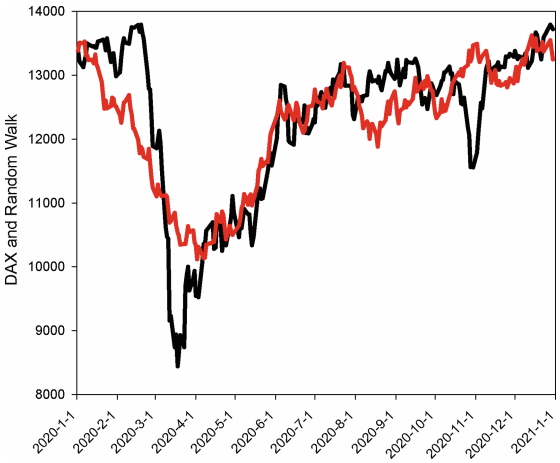

Fig. 2. The DAX index in 2020 (black line) and its simulation (red line) generated by random walk in decimal format using any of five seeds from 5.53532 to 5.53536.

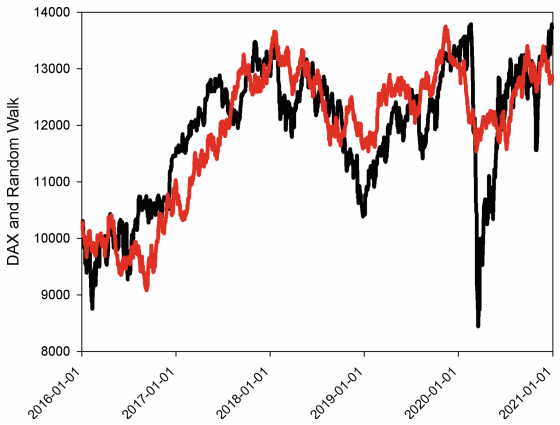

Fig. 3. The five-year DAX index from 2016 to 2020 (black line) and its simulation (red line) generated by random walk in decimal format using a seed of 0.48775.

the DAX index. The simulation cannot catch the peak in 2015. Also we can see a certain lag time to follow this peak in 2015.

Figure 5 pictures the random walk simulation on DAX index for fifteen years. There is a big lag time in the random walk simulation for the first peak of the DAX index in 2007. Then the simulation cannot climb up for three peaks of the DAX index from 2017 to 2018, and then the simulation seems to completely ignore the sharp fall due to the Covid-19 pandemic. Overall, the simulation follows the DAX index along its upward trend. This could be useful to forecasting of the general DAX index tendency.

Figure 6 describes the random walk simulation on DAX index for twenty years. With the increment of time scale, it appears that both DAX index and its random walk

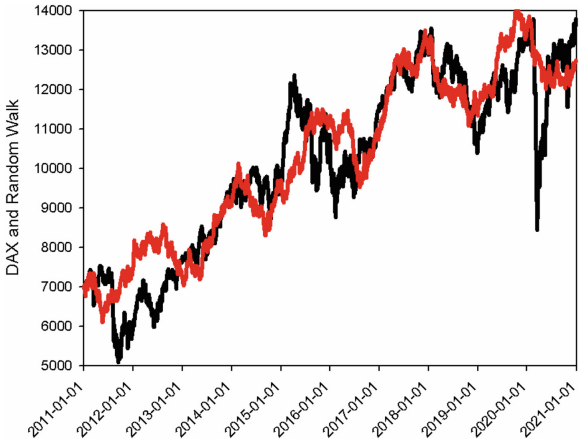

Fig. 4. The ten-year DAX index from 2011 to 2020 (black line) and its simulation (red line) generated by random walk in decimal format using any of four seeds from 4.86905 to 4.86908.

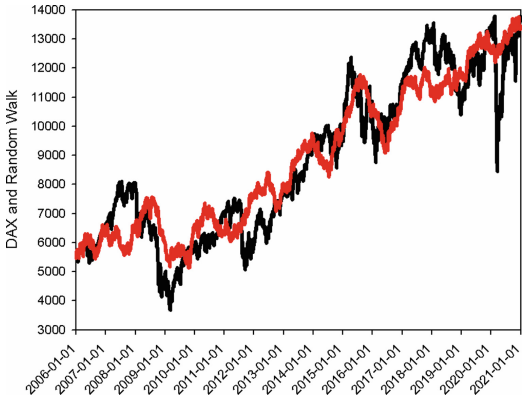

Fig. 5. The fifteen-year DAX index from 2006 to 2020 (black line) and its simulation (red line) generated by random walk in decimal format using a seed of 2.26416.

simulation become flattened. Therefore, the exciting moments in the DAX index become weakened. Although the curve (black line) becomes flattened, the random walk simulation appears more flattened than the DAX index. Meanwhile, the random walk simulation often goes to opposite directions. Therefore, the simulation fails to catch up with any significantly exciting moments in the DAX index. Clearly, the longer the time is involved in simulation, the more the difficulty the simulation is.

Although the simulation seems to follow the trend of the DAX index in a large-scale, it requires different seeds for different periods as shown in these figures. If these simulations would come from the same seed, then it could be possible to use this seed to predict the movement of the DAX index in future.

The random walk simulation presented in each figure is the one from 100,000 simulations, i.e. we generate 100,000 series of random numbers for each figure, and choose

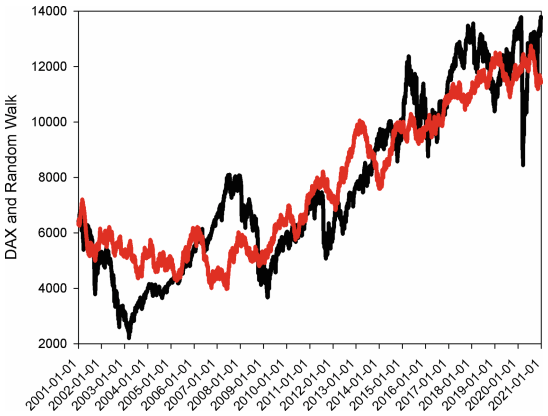

Fig. 6. The twenty-year DAX index from 2001 to 2020 (black line) and its simulation (red line) generated by random walk in decimal format using any of three seeds of 0.29753, 3.36101 and 3.36102.

the simulation with minimal difference between simulation and the DAX index. Naturally, 100,000 simulations come from 100,000 seeds, which range from 0 to 10. It is of interest to note that sometimes only one seed can generate a series of random numbers, whose addition makes a random walk simulation similar to the given period of the DAX index, but sometimes several seeds can do the same. Because of good-fitness for random walk in the 1/–1 format, it necessarily simulates the DAX index in a longer period rather than 2020 in future.

In a broad sense, the issue of whether a stock index follows a random walk model is closely related to the efficient market hypothesis (EMH), in particular, the weak form market efficiency. If a stock index does not follow a random walk, then it is not efficient in the weak-form over a certain period of time. This study shows that the random walk simulation, especially, in the 1/–1 format, i.e. the classical random walk can fit the DAX index, hence the DAX index would satisfy with the market efficiency in the weak-form. Theoretically, there would always a chance, which is equal to $(½)^{\text{trading days}}$, to find a perfect simulation for any longer period of the DAX index although such a chance is extremely small.

The issues of whether the choice of time period affects the market efficiency in the weak-form and whether the market efficiency improves over time are topics that need to address. In this context, we would be better to divide our samples into equal length periods in our future studies.

4 Conclusion

This study attempts to answer the question of whether the DAX index behaves randomly, which was approved and disapproved using the statistical tests in the past. Our results show that the movement of the DAX index can be considered completely randomly if the random walk simulation in its 1/–1 format. But the random walk has difficulty to

simulate the very fluctuating DAX index in the decimal format because of the difficulty in setting the upper/lower range in random number generation.

Acknowledgment. The authors are grateful to the Scientific Development Fund of Guangxi Academy of Sciences (2021YFJ1203).

References

1. Aktan C, Iren P, Omay T (2019) Market development and market efficiency: evidence based on nonlinear panel unit root tests. Euro J Finan 25:979–993
2. Ren F, Zheng B, Lin H, Wen LY, Trimper S (2005) Persistence probabilities of the German DAX and Shanghai Index. Phys A 350:439–450
3. Claessen H, Mittnik S (2002) Forecasting stock market volatility and the informational efficiency of the DAX-index options market. Euro J Finan 8:302–321. https://de.finance.yahoo.com/
4. Chow KV, Denning KC (1993) A simple multiple variance ratio test. J. Econometrics 58:385–401
5. Reutter M, Von Weizsäcker J, Westermann F (2002) SeptemBear - a seasonality puzzle in the German stock index DAX. Appl Finan Econ 12:765–769
6. Deo RS, Richardson M (2003) On the asymptotic power of the variance ratio test. Economet Theor 19:231–239
7. Muzzioli S (2010) Option-based forecasts of volatility: an empirical study in the DAX-index options market. Euro J Finan 16:561–586
8. Yan S, Wu G (2021) Fit Hang Seng index for 21st century with random walk model. In: Conference Publication Series: CBFD, pp 66–69
9. Yan S, Wu G (2020) Fitting of impact of COVID-19 pandemic on S&P 500 Index using random walk. In: Conference Publishing Services: ICEMME, vol 4, pp 1007–1011
10. Yan S, Wu G (2011) Fitting of SSEC index (Shanghai composite) from January 2000 to July 2010 using random walk model. Guangxi Sci 18:92–96
11. Feller W (1968) An introduction to probability theory and its applications, 3rd edn. Wiley, New York