



Analysis on the Performance of FOF Combination for Pension Based on DEA Model

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Abstract. At present, with the global aging becoming increasingly serious, more and more countries and investors attach importance to the construction of pension fund system. FOF fund (fund of funds) is a special fund for the investment target. It does not directly invest in stocks or bonds, but invest in other securities investment funds to indirectly hold stocks, bonds and other securities assets. In the case that the original fund has diversified risks by investing in different assets, the FOF fund uses professional investment technical means to construct fund portfolios to achieve the purpose of diversifying risks again. Therefore, the FOF fund can be widely used in the pension fund market as a form of pension fund operation. In this paper, DEA model is used in the FOF fund's performance analysis, four independent input variables are obtained by principal component method which represent four aspects: management, risk, cost and scale respectively. By comparing the performance of different FOFs, the author found that the efficiency value of pension funds is higher on average. It is able to confirm that the FOF portfolio has unique advantages. In the current situation of global aging, the FOF product portfolio still has development potential. Besides, the application of DEA is able to provide some reference for individual investors.

Keywords: FOF · Pension · Portfolio · Aging · DEA · Fund Performance

1 Introduction

FOF's target-date products or target-risk products are the most suitable products for long-term investment or pension investment management in the market at present, which are in line with the life cycle and the characteristics of long-term stable investment [5]. In recent years, the FOF operation mode has penetrated from European and American fund markets to developing countries represented by China, and achieved remarkable results in the management of pension funds in various countries. Through selection and combination, investors can have a more convenient choice, which is conducive to the participation of public investors in investment and financial management. Although everyone can have different pension investment plans, the pension FOF has a great advantage over other portfolios in terms of product design. This article will conduct a comprehensive discussion on the performance of pension FOF. According to the indicator requirements of the DEA model, the author will select relevant data representing

risk, management, cost, and scale. Through the principal component method and positive management to process the data, the author will compare the performance of other types of pension FOF based on the results. As an important investment method, FOF funds will have advantages in the context of global aging. This article will also provide investors with some new investment ideas.

2 Global Application Categories and Development Status

The FOF fund originated in the United States and the first recognized FOF fund was launched at the end of 1969, which was a hedge fund. From the introduction of public funds to the present decades, FOF funds have been popular in the market. The construction of a portfolio of funds generally includes four steps, which are asset allocation, fund selection, buying and selling timing and risk control [3]. The first step is screening out excellent funds from hundreds of funds in the market. The asset allocation ratio of each type of fund is determined according to the overall economic climate and other market environment as well as various quantitative and qualitative indicators. Specialist agencies will determine the most appropriate asset allocation proportion, according to the quantitative indicators for parameter calculation. After evaluated investment performance indicators, from the fund centralized select eligible sub-fund, fund portfolios final investment targets [6]. In addition, institutions will regularly detect the performance of the sub-fund, observe whether the performance of the fund is in line with the expected result.

With the tendency of aging of the population (see Fig. 1), Funds of funds (FOF) has become an important product category in the United States, Japan and Canada. The scale of the United States reached \$1.72 trillion at the end of 2015, accounting for about 10% of the total size of mutual funds. Since 2015, the FOF of private equity funds in the Chinese market has developed rapidly, with the current scale approaching 300 billion yuan. 92% of mutual fund holders claim that the retirement savings is one of their financial goals, and 73% say retirement savings is their top financial goal, according to the 2020 Annual Book of American Funds (ICI) [1]. As a result, U.S. investors with retirement savings goals naturally hold funds longer. In addition, pension index (see Table 1) can reflect the relationship between household debt and pension assets to some extent, and can reflect the necessity of the existence of pension fund.

In addition, many countries implement deferred pension tax policies. Investors are exempt from personal income tax until they put money into the pension account. However, they have to pay personal income tax in accordance with the current tax rate when they withdraw in retirement, so that they can earn the difference of compound interest. Therefore, pension funds have quickly become a new hotspot in many countries.

First, through portfolio investment and constant adjustment of portfolio varieties and asset allocation scheme, FOF not only reduces the threshold, but also alleviates risks, allowing investors to participate in some investment varieties that are not suitable for public investors. Figure 2 shows the recent returns of three typical FOF funds of pension in China, which shows the benefits are growing steadily and the second strength is effective risk diversification.

In pension management, a fund manager or fund strategy that can provide stable and good risk-return characteristics is equivalent to an “artificial asset”. The addition of such

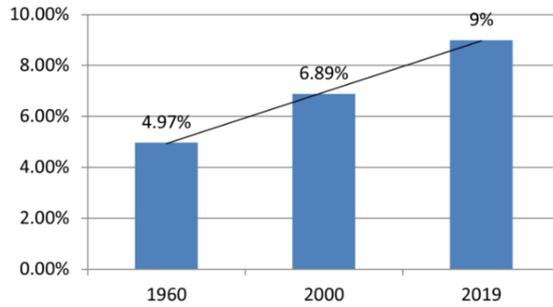


Fig. 1. The proportion of the global aged 65 and over in the total population from 1960 to 2019

Table 1. Mercer CFA Institute Global Pension Index in 2020

Country	Total figure	Index		
		Adequacy	Sustainability	Integrity
Argentina	42.5	54.5	27.6	44.4
Australian	74.2	66.8	74.6	85.5
Austria	52.1	64.4	22.1	74.6
China	47.3	57.4	36.2	46.7
France	60.0	78.7	40.9	57.0
Germany	67.3	78.8	44.1	81.4
India	45.7	38.8	43.1	60.3
Japan	48.5	52.9	35.9	59.2
Korea	50.5	48.0	53.4	50.3
Thailand	40.8	36.8	40.8	47.3
UK	64.9	59.2	58.0	83.7

“artificial assets” is equivalent to adding another asset class and may also have a low correlation with other assets, which can further enhance the effectiveness of portfolio management and risk diversification. In addition, FOF can both filter out some funds and conduct a comprehensive search to avoid missing out on potential funds. When choosing portfolios, FOF can scientifically classify funds and select appropriate funds based on the classification, so as to ensure the overall return and style stability of pension FOF products. Although the FOF funds exist weakness such as short-term investment dilemma. What is more, the products issued by different fund companies are not diversified enough, which will weaken the role of the portfolio fund in risk diversification, generally speaking, the advantages of this fund are irreplaceable.



Fig. 2. Three typical FOF funds (China) yield

3 Data Collection

3.1 DEA Introduction

Using DEA model to evaluate fund performance means that FOF is regarded as a series of production units of the same type and DEA is used to evaluate their performance. These production units are also called decision units, and decisions are based on input variables and output variables.

In this paper, the variable that affects the performance of the fund is the input indicator, and the variable that measures the performance of the fund is the output indicator. Different FOFs choose different portfolios, and these decision units together constitute the effective frontier. Therefore, DEA model can effectively quantify fund performance.

With the development of data envelopment analysis (DEA), many derivative DEA models have emerged, among which gastric CCR model and BCC model are the most widely used. CCR model assumes that the return to scale of the research object is constant, and the output results reflect the comprehensive technical efficiency. The premise of BCC model is that the return to scale is variable, and the output result is pure technical efficiency, that is, the influence of scale economy is excluded from the comprehensive technical efficiency.

3.2 Selection of Samples and Indicators

In consideration of the availability, accuracy and continuity of data, this paper took China as an example to collect a complete year’s historical FOF data, and selected 22 FOF funds with a scale of over 100 million. Among the 22 funds, 10 are general FOFs and 12 are pension FOFs. By summarizing the literature and estimating the investor’s psychology, this paper selects the investment indexes from the aspects of risk control, cost, scale, fund manager’s style, etc.

3.2.1 Annual Fluctuation

$$\sigma = \sqrt{\frac{\sum (x_i - x)^2}{n - 1}} \tag{1}$$

3.2.2 Maximum Pullback Rate

The maximum retracement rate is the net rate of change between two time points when the net value of a fund reaches its maximum spread, reflecting the maximum loss an investor could suffer from holding the fund during this period. $drawdown = \frac{\max(D_i - D_j)}{D_j} D_i$ represents the net value of the fund on day i, D_j represents the net value on a subsequent day.

3.2.3 Downward Standard Deviation

Downward standard deviation describes the risk in the case of defective returns.

$$DD = \sqrt{\left(\frac{\sum_{t=1}^n (r_i - r)^2}{n} \mid r_i < R\right)} \tag{2}$$

R_i is the rate of return at the moment, \bar{r} is the average rate of return, R is the selected rate of return for reference.

3.2.4 Tracking Error

Tracking error is an index used to measure the deviation between the fund and the underlying index as well as the correlation of return fluctuations. It is calculated on the basis of tracking deviation. Tracking deviation degree and tracking error are also important indicators to measure the deviation degree between fund return and target index.

$$\text{Tracking deviation : } TD_{it} = R_{it} - R_{tm}$$

R_{it} is the return rate of fund i at time t, R_{tm} is the return rate of benchmark portfolio at time t.

$$\text{Tracking error : } TE_{it} = \sigma(R_{it} - TE_{tm}) \tag{3}$$

3.2.5 Information Rate

α_p is the excess return obtained by the fund after eliminating the systemic risk, ε_p is the non-systemic risk measuring the fluctuation of the net value of the fund. The information ratio reflects the excess return brought by the tracking error of fund units.

$$IR_p = \frac{\alpha_p}{\sigma(\varepsilon_p)} \tag{4}$$

3.2.6 Market Timing Ability and Stock Selection Ability

It was calculated from T-M quadratic regression model.

$$R_p - R_f = \alpha + \beta_1(R_m - R_f) + \beta_2(R_m - R_f)^2 + \varepsilon_p \tag{5}$$

R_f is the risk free interest rate, $R_m - R_f$ is the market factor, α measures the stock selection ability of the fund manager, β_2 measures the timing ability of the fund manager.

3.2.7 Unit Fund Expenses

Unit fund cost is used to measure the cost of investment needed by investors to participate in the fund, which mainly includes the cost of buying and redeeming the fund, management cost, custody cost and subscription cost. In this paper, the sum of four kinds of costs as a measure of investment fund costs.

3.2.8 Fund Size

Fund size can reflect whether the fund has scale effect.

3.2.9 Funds Holding of Heavy Camalig Rate

Fund holdings of the top ten heavy warehouse share ratio, reflecting the style of investment characteristics. In addition, this paper takes revenue and scale growth rate as output indicators. The income is reflected by the net growth rate per unit, and the growth rate is measured by the fund size/establishment time.

$$\text{Net growth rate per unit} = \frac{\text{closing net par value} - \text{opening par value}}{\text{opening net par value}}$$

Data were obtained from formula calculation and Wind, as shown in the Tables 2 and 3.

Table 2. Input index

	Stock code	Fund size (100million)	Maximum pullback rate	Annual fluctuation	Downward standard deviation	Timing ability	Selection ability	Information rate	Tracing error	Expense per unit	Holding heavy camalig rate
1	005156.OF	5.672	0.0347	4.14%	3.098	-1.4616	0.0017	-1.622	1.934	0.019	0.682
2	005215.OF	4.045	0.0413	4.49%	3.478	-1.5570	0.0017	-1.698	1.886	0.016	0.527
3	005216.OF	2.152	0.0413	4.49%	3.499	-1.5610	0.0016	-2.025	1.886	0.013	0.527
4	005217.OF	4.433	0.0387	3.87%	3.072	-0.8716	0.0011	-2.042	1.921	0.019	0.568
5	005218.OF	6.157	0.0373	4.24%	3.105	-0.9066	0.0016	-1.076	1.938	0.017	0.394
6	005219.OF	1.151	0.0373	4.24%	3.119	-0.8994	0.0015	-1.285	1.939	0.011	0.394
7	005220.OF	6.640	0.1314	16.77%	11.803	-1.8200	0.0033	12.214	0.976	0.019	0.819
8	005221.OF	1.027	0.0402	4.82%	3.215	-1.2737	0.0013	-2.339	1.843	0.013	0.730
9	005809.OF	3.271	0.1014	14.22%	9.776	2.1944	0.0019	12.188	1.291	0.032	0.853
10	006289.OF	4.567	0.0425	5.58%	4.038	-2.1328	0.0020	-1.683	1.921	0.026	0.537
11	006290.OF	2.661	0.0512	6.68%	4.513	-1.9077	0.0025	0.270	1.802	0.021	0.460
12	006292.OF	2.719	0.0130	3.09%	2.021	-1.3180	0.0018	-3.496	2.199	0.018	0.676
13	006294.OF	2.811	0.0075	1.83%	0.957	-0.4703	0.0008	-2.750	2.332	0.012	0.390
14	006295.OF	2.896	0.0372	5.44%	3.880	-1.0683	0.0013	-2.345	1.937	0.014	0.656
15	006296.OF	9.690	0.0197	4.48%	2.711	-0.7868	0.0017	-1.017	2.221	0.010	0.656
16	006297.OF	6.077	0.0042	1.19%	0.649	-0.1983	0.0008	-2.536	2.385	0.013	0.836

(continued)

Table 2. (continued)

	Stock code	Fund size (100million)	Maximum pullback rate	Annual fluctuation	Downward standard deviation	Timing ability	Selection ability	Information rate	Tracing error	Expense per unit	Holding heavy camalig rate
17	006298.OF	2.332	0.0200	3.13%	2.174	-0.8112	0.0010	-4.538	2.139	0.016	0.399
18	006305.OF	2.687	0.0353	5.31%	3.730	-1.2688	0.0017	-1.302	1.997	0.018	0.502
19	006306.OF	4.679	0.0353	5.19%	3.774	-2.0110	0.0014	-3.133	1.842	0.014	0.506
20	006321.OF	3.903	0.0655	6.96%	5.273	-1.4988	0.0010	1.909	1.623	0.026	0.625
21	006507.OF	2.878	0.0613	7.44%	7.478	2.8130	-0.0019	-4.530	2.027	0.024	0.887
22	006763.OF	4.528	0.0710	8.89%	6.211	-3.2045	0.0020	-5.034	1.709	0.025	0.640

Table 3. Output index

Stock code	scale growth rate	growth rate per unit
1	0.0074	10.11%
2	0.0052	10.09%
3	0.0028	9.49%
4	0.0058	8.87%
5	0.0081	11.41%
6	0.0015	11.02%
7	0.0088	30.33%
8	0.0014	9.52%
9	0.0058	37.71%
10	0.0103	10.77%
11	0.0116	15.02%
12	0.0078	9.95%
13	0.0077	6.33%
14	0.0071	11.06%
15	0.0080	11.06%
16	0.0275	5.81%
17	0.0179	2.73%
18	0.0066	11.61%
19	0.0067	8.05%
20	0.0112	17.57%
21	0.0105	8.02%
22	0.0085	10.35%

3.3 Preliminary Data Analysis

It can be seen from the following figure that the average value of the maximum retracement rate is 0.04, among which the maximum retracement rate of pension fund type of FOF is small and its fluctuation range is smaller than that of ordinary FOF. In terms of annualized volatility and downward standard deviation, pension fund is also more stable than ordinary FOF. From the perspective of management, the average value of fund managers' timing ability is negative, indicating that their timing ability is weak. The value of stock selection ability is relatively small, because most FOF funds pursue stable growth rather than high returns, especially pension FOF funds. FOF funds have slightly higher fees than general funds and are smaller. This is because the sample data comes from China, where the FOF fund is still in its early stages of development [7] (Fig. 3).

3.4 Principal Component Analysis

As the indicators consider multiple aspects, there may be a strong correlation between the indicators. However, the advantages of DEA model cannot be exerted if the number of indicators is reduced. Therefore, the principal component method is adopted for dimensionality reduction, and the principal component with the largest contribution will be used as the final input indicator. In addition to the unit fund cost as the cost index, this paper will reduce the dimension of the remaining 9 indicators to generate independent principal component factors. In this paper, SPSS software was used to conduct principal component analysis on the data. Then the correlation test of 22 input indexes was carried out. KMO and Bartley spherical test were used here. According to the SPSS output results, $KMO = 0.674$ and $P < 0.01$ showed significant correlation, so the principal component calculation was reasonable (Table 4).

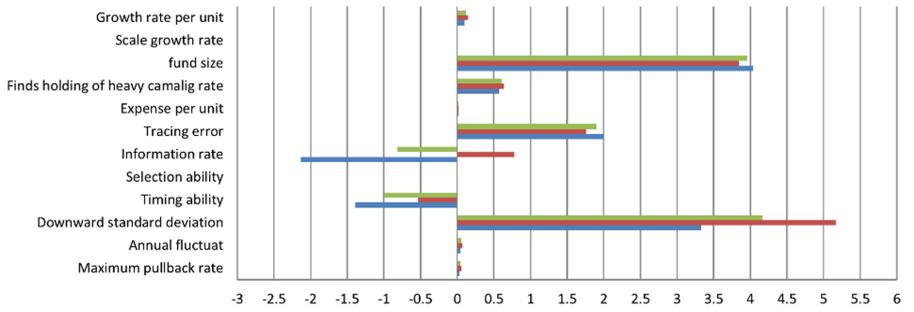
1) KMO and Bartlett's Test

According to the results in the following Table 5, there are three factors with eigenvalues greater than 1, and the cumulative variance contribution reaches 89.259%, which can reflect the information of most of the original indicators. Therefore, the first three indicators are selected as the main components, and the variance values are 5.008, 1.967 and 1.058 respectively.

2) Eigenvalues and cumulative variance

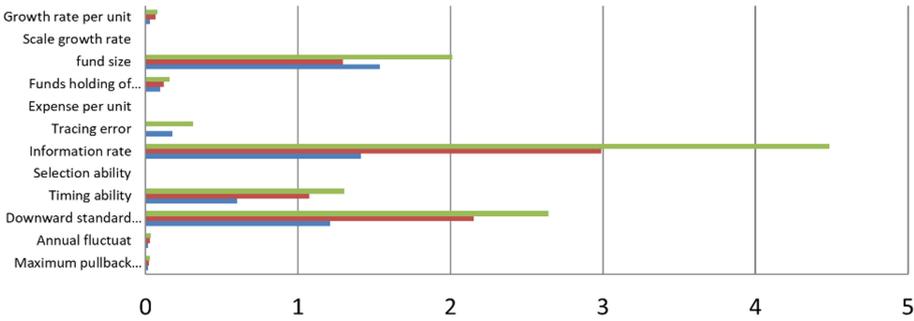
Under the principal component factor 1, the annual fluctuation, downward standard deviation, information ratio, tracking error and maximum retracement rate have higher loads, indicating that the factor is closely related to these indicators. Here factor 1 is named risk factor X1. Under the principal component factor 2, timing ability and stock selection ability have a high load. Here, factor 2 is named as management factor X2. Under the principal component factor 3, the fund size has a relatively high load, and factor 3 is named as size factor X3 here (Table 6).

Finally, the values of the three principal components were obtained according to the coefficient matrix Table 7. The values of the coefficient matrix were multiplied by the



	Maximum pullback rate	Annual fluctuat	Downward standard deviation	Timing ability	Selection ability	Information rate	Tracing error	Expense per unit	Finds holding of heavy camalig rate	fund size	Scale growth rate	Growth rate per unit
Total	0.044	0.0571	4.1625	-1.0009	0.0015	-0.8122	1.8977	0.0179	0.603	3.9535	0.0086	0.1213
ordinary type	0.0565	0.0687	5.1644	-0.5344	0.0014	0.7784	1.7642	0.0183	0.6388	3.8451	0.0057	0.1466
pension type	0.0335	0.0475	3.3276	-1.3898	0.0016	-2.1378	2.0089	0.0175	0.5736	4.0437	0.0109	0.1003

(a)



	Maximum pullback rate	Annual fluctuat	Downward standard deviation	Timing ability	Selection ability	Information rate	Tracing error	Expense per unit	Funds holding of heavy camalig rate	fund size	Scale growth rate	Growth rate per unit
Total	0.0292	0.0359	2.6409	1.3054	0.001	4.4856	0.3132	0.0056	0.156	2.0125	0.0055	0.0776
Ordinary type	0.0217	0.0307	2.1512	1.0734	0.0008	2.9896	0.0053	0.0041	0.1195	1.2961	0.002	0.0681
Pension type	0.0157	0.0151	1.2127	0.6015	0.0004	1.4124	0.1782	0.0038	0.0948	1.5362	0.0044	0.0289

(b)

Fig. 3. (a)(b) Average amount and variance amount

arithmetical square root of the corresponding variance to obtain the final data of the three principal components respectively in the following Table 7.

Since DEA model is a model describing the relative efficiency value, the model requires that the data should not appear negative number, so the negative value should be processed forward according to the following formula: $X^1 = 0.1 + 0.9 \times \frac{(X - MIN)}{(MAX - MIN)}$. The result as follows in Table 8.

Table 4. KMO and Bartlett’s Test

Kaiser-Meyer-Olkin Measure of Sampling	.674
Adequancy	290.154
Bartlett’s Test of Sphericity Approx. Chi-Square	36
Df	.000
Sig	

Table 5. Eigenvalues and cumulative variance

Component	Initial eigenvalue			Extract the sum of square and loading		
	Total	Variance %	Accumulated%	Total	variance %	accumulated%
1	5.008	55.647	55.647	5.008	55.647	55.647
2	1.967	21.851	77.498	1.967	21.851	77.498
3	1.058	11.761	89.259	1.058	11.761	89.259
4	.494	5.483	94.742			
5	.386	4.292	99.034			
6	.055	.608	99.642			
7	.021	.238	99.880			
8	.008	.094	99.974			
9	.002	.026	100.000			

Table 6. Component Matrix

	Component		
	1	2	3
Fund size	.096	.161	.959
Annual fluctuation	.986	.027	-.086
Downward standard deviation	.958	.139	-.127
Selection rate	.438	-.825	.199
Timing rate	.130	.907	-.105
Information rate	.887	-.030	.128
Tracing error	-.951	.214	.083
Funds holding of heavy camalig rate	.514	.608	.181
Maximum pullback rate	.973	-.017	-.088

Table 7. Coefficient Matrix

	Component		
	1	2	3
Annual fluctation	.197	.015	-.082
Downward standard deviation	.191	.072	-.121
Timing ability	.026	.461	-.100
Selection ability	.087	-.419	.189
Information rate	.177	-.014	.121
Tracing error	-.190	.108	.078
Fund size	.019	.082	.906
Funds holding of heavy camalig rate	.102	.310	.171
Maximum pullback rate	.194	-.011	-.082

3.5 Efficiency Value Calculation

In terms of DEA model calculation, this paper adopts DEAP2.1. This program only needs to set the number and model of indicators to obtain the efficiency value of each FOF fund, and the results under CCR model and BCC model can be obtained (Table 9).

CRSTE stands for comprehensive efficiency, which is the efficiency value calculated in the context of CCR model; VRSTE is pure technical efficiency which is the efficiency value calculated in the BBC context; Scale represents the sample's economies of Scale under the background of changes in returns to Scale. IRS stands for increasing returns to scale and DRS for diminishing returns to scale. According to the scale efficiency value = comprehensive efficiency value/technical efficiency value, the output results can be divided into the following three categories:

(1) $CRSTE = 1$, $VRSTE = 1$, $SCALE = 1$. All efficiency values are 1. According to the validity definition, these funds belong to relatively effective funds in the samples, that is, they believe that the input indexes are fully utilized to achieve the maximum output. According to the table, funds 7, 9, 11, 13, 15, 16, 17, 21, 22 fall into this category.

(2) $CRSTE \neq 1$, $VRSTE = 1$, and $SCALE \neq 1$. The value of comprehensive efficiency is not equal to 1, indicating that the fund is not a relatively effective fund. However, the pure technical efficiency value is 1, indicating that the comprehensive efficiency is not optimal due to the investment scale of the fund. In this case, we should analyze whether there are any input factors that have not been fully utilized from each input index. As can be seen from the table, funds 6, 18 and 19 fall into this category.

(3) $CRSTE \neq 1$, $VRSTE \neq 1$, and $SCALE \neq 1$. None of the three efficiency values is equal to 1, indicating that the fund is in poor operation condition, with insufficient input factors or is not fully utilized. All funds except those mentioned in (1) and (2) fall into this category. The investors of these funds should consider the risk control of the fund, the performance of the fund manager and the change of the fund size and other factors

Table 8. Input index after principal component analysis

Number	Scale growth rate Y1	Net growth rate per unit Y2	Risk factor X1	Management factor X2	Scale factor X3	Cost amount X4
1	0.0074	0.0721	0.3127	0.2971	0.5875	0.0193
2	0.0052	0.0728	0.3121	0.2159	0.3966	0.0160
3	0.0028	0.0675	0.3042	0.2094	0.2200	0.0126
4	0.0058	0.0608	0.2862	0.3340	0.4145	0.0189
5	0.0081	0.0796	0.2837	0.2358	0.5568	0.0172
6	0.0015	0.0756	0.2704	0.2051	0.1000	0.0188
7	0.0088	0.2612	1.0000	0.1714	0.6077	0.0107
8	0.0014	0.0675	0.3260	0.3212	0.1447	0.0132
9	0.0058	0.1042	0.8483	0.5667	0.2579	0.0324
10	0.0103	0.0673	0.3358	0.1641	0.4573	0.0260
11	0.0116	0.0433	0.3978	0.1000	0.4434	0.0208
12	0.0078	0.0590	0.2022	0.2861	0.3463	0.0179
13	0.0077	0.0433	0.1000	0.3219	0.2659	0.0119
14	0.0071	0.0590	0.3213	0.3314	0.2838	0.0140
15	0.0080	0.0774	0.2529	0.3298	0.3491	0.0095
16	0.0275	0.0458	0.1542	0.5721	1.0000	0.0133
17	0.0179	0.0499	0.1698	0.3064	0.5405	0.0164
18	0.0066	0.0751	0.3006	0.2217	0.2371	0.0184
19	0.0067	0.0439	0.2971	0.1914	0.2522	0.0143
20	0.0112	0.1295	0.4941	0.1756	0.4745	0.0225
21	0.0105	0.0266	0.3912	1.0000	0.1851	0.0242
22	0.0085	0.0584	0.4413	0.1161	0.2678	0.0246

when choosing the fund, and analyze the reasons for the failure to achieve the optimal efficiency.

3.6 Comparison Between Pension Type and Ordinary Type

In terms of types, 9 of the 22 funds are effective funds, and 6 of them are pension funds. In addition, no matter the comprehensive efficiency or pure technical efficiency, the average value of pension fund is higher than that of ordinary fund. On the whole, the value of pension FOF is the highest both in terms of the mean efficiency and the proportion of effective funds. This indicates that pension type has great development potential and obvious advantages. The main reasons for the relatively high performance

Table 9. Output Result

Fund number	crste	vrste	scale	
1	0.747	0.767	0.974	Irs
2	0.781	0.894	0.873	Irs
3	0.73	0.935	0.781	Irs
4	0.655	0.728	0.899	Irs
5	0.931	0.946	0.984	Irs
6	0.906	1	0.906	Irs
7	1	1	1	-
8	0.648	0.807	0.803	Irs
9	1	1	1	-
10	0.783	0.957	0.819	Irs
11	1	1	1	-
12	0.991	0.998	0.993	irs
13	1	1	1	-
14	0.795	0.881	0.902	Irs
15	1	1	1	-
16	1	1	1	-
17	1	1	1	-
18	0.926	1	0.926	Irs
19	0.826	1	0.826	Irs
20	0.982	0.991	0.991	Drs
21	1	11	1	-
22	1	1	1	-

Table 10. Comparison between pension FOF and common FOF

	Number of effective funds	Combined efficiency		Pure Technical Efficiency	
		Numbers of sample	Average amount	Numbers of sample	Average amount
Pension type	6	12	0.9419	12	0.9856
Ordinary type	3	10	0.8398	10	0.9077
total	9	22	0.8955	22	0.9502

of pension FOF funds are as follows: the establishment of the individual pension account system supported by national policies, and the stability brought by the stipulated period (Table 10).

In terms of scale, among the 9 effective funds, the average fund size is 4.7335, larger than the average of 3.9535 in all the samples. But only four of the nine funds were better than the group, so that does not mean that smaller FOFs outperform the average. In general, when the fund size is small, the fund manager will have stronger control over the fund and can respond quickly in the face of market fluctuations. Such performance means better performance. When the fund scale reaches a certain level, the fund itself will have a certain stability, in the face of market fluctuations can also achieve steady growth [4].

In terms of expenses, the maximum value of the 9 funds is consistent with the situation of the total sample, with a mean value of 0.0191, a difference of 0.1% from the overall mean. Since China's FOF funds are at the initial stage and investors have limited choices, in order to reduce investment costs and attract investors, many FOF funds choose the funds of the same company as the investment target, so as to avoid double fees.

For funds which technical efficiency not exceed 1, this paper will summarize the reasons by analyzing their slack variables. Through calculation, it can be concluded that, for example, when the risk factor is reduced by 0.07, the 005156.OF fund has higher relative efficiency which means the efficiency can be improved by controlling risks. However, the redundancy of the scale factor of the fund is 0.3, which proves that the economies of scale not be reflected [2]. It can be improved by increasing the flexibility of the fund operation or change the investment strategy. In addition, the slack variables of the input index are almost all negative, and the slack variables of the output factor are basically 0. Generally speaking, the fund efficiency is not high. The improvement of risk control ability and capital use efficiency is more conducive to the development of FOF Fund.

4 Discussion

In this paper, DEA model is adopted. Considering the development speed of aging trend, FOF funds in China are selected as samples to analyze the investment hot spot FOF, which bring reference significance for investors and fund companies in portfolio management. DEA model is a multi-output-multi-input model. This paper considers the input index from four dimensions and the output index from two aspects, using the comprehensive efficiency value to measure the performance of the fund. In addition, there are deficiencies in this paper. For the accuracy of the sample, the data are selected from 22 FOF funds in China. However, the development time of FOF funds in China is relatively short and the data are not comprehensive. The input index and output index of DEA model have great subjectivity and deviations may occur.

5 Conclusion

As the population ages and life expectancy increases, how to obtain a sustainable and stable income after retirement is an important issue for every investor. Taking into account

inflation, interest rate changes, longevity risk and so on, low-yielding money or bond funds will not meet investors' retirement income goals. Pension FOF as hot investment in recent years is analyzed in this paper. In this paper, a large number of recent data were selected as samples. Through correlation analysis, principal component reduction and positive processing, the four independent variables of risk, management, scale and cost were taken as input indexes, and the rate of change of scale and the rate of change of net value were taken as output indexes. The efficiency value of the samples was calculated by DEA model. Through the analysis of efficiency value, it can be found that the number of effective funds of pension FOF is significantly higher than that of ordinary FOF, and the average value of efficiency value of pension fund is generally higher than that of ordinary fund. As the focus of future FOF fund development, pension type FOF can be expected in the future.

Acknowledgment. Thanks to Prof. Biffis' help for promoting my understanding of Investment and Portfolio Management. His course has completely broadened my horizon and aroused my great interest in the direction of Investment, thus giving me the inspiration for the topic of this paper. Zelin Ye taught me the methods of data collection and model building, Tracy and Jane gave me guidance and modification in the writing process of my paper. Thank you for your patience and kindness to me.

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