



# Multifractal Detrended Cross-Correlation Analysis on Returns of Soybean Futures and Spot Markets

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**Abstract.** Considering that multifractal characteristics widely exists in financial markets, this paper adopts the multifractal detrended cross-correlation analysis (MF-DCCA) method to study the correlation between futures and spot returns in China's soybean market. Based on the data of soybean No. 1 futures and soybean spot price from January 4, 2011 to February 23, 2022, this paper shows that there is a high degree of multifractal cross-correlation between soybean futures and spot returns. Furthermore, this paper proves that the cross-correlation between the two time series are persistent.

**Keywords:** Multifractal Characteristics · Soybean Futures · Soybean Spot · MF-DCCA

## 1 Introduction

In 1990, Zhengzhou Cereal Wholesale Market was established, making it the first market in China with futures trading varieties. Since then, China's futures market has developed rapidly. In 1999, the General Office of the State Council of the People's Republic of China issued *Interim Regulations on Futures*, which stated that only three futures exchanges were retained—Shanghai Futures Exchange, Zhengzhou Commodity Exchange and Dalian Commodity Exchange, and same kind of futures would not be set repeatedly on each exchange. Since then, Dalian Commodity Exchange has become the only exchange where soybean futures are traded.

Then, China's soybean futures market developed by leaps and bounds. Since 2002, the soybean futures market has consistently ranked the second globally, and the non-GMO soybean futures market has held the first place. In recent years, its scale continues to expand so that trading volume increases year by year. According to *China Futures Market Yearbook 2021*, trading volume of soybean No. 1 futures in Dalian Commodity

Exchange was 4955.84 lots in 2021. It ranked No.16 among all agricultural futures and options transactions around the world in 2021, closely following the 15th place of soybean futures in the Chicago Board of Trade.

As the proverb goes that food is the paramount necessity of people, soybeans and other important agricultural products are closely related to the economic development, political stability and social harmony of a country. Therefore, since the birth of agricultural product futures, they have always been the focus of academic circles. Scholars have conducted studies on issues such as characteristics of agricultural product price fluctuations, reasons for fluctuations, interactions between different markets, and risks transmission. It is worth noting that previous studies focus on futures trading-related issues under a linear framework based on the Efficient Market Hypothesis. However, with the progress of computer science, researchers begin to study massive amounts of financial data and reveal the statistical nature of financial systems, finding that distribution of asset prices and returns characterized by “sharp peak and fat tail” goes against Efficient Market Hypothesis. Scholars realize that the market is not as efficient and complete as the efficient market hypothesis suggests, and actual financial system is probably a more complex nonlinear system. Based on this phenomenon, some scholars try to explain the financial system through physical fractals. Specifically, the Fractal Market Theory was proposed by [6] and it could better describe nonlinear characteristics of financial markets. Lots of empirical studies indicate that, compared with traditional efficient market hypothesis, fractal market theory can better explain complex phenomena and behaviors in financial markets.

Meanwhile, the studies on the relationship between soybean futures and spot prices under multifractal framework are limited. Therefore, by using the daily closing price of the main continuous contract of soybean No. 1 futures as an indicator for calculating the return of soybean futures price, and using the daily closing price of soybean spot price as an indicator for calculating the return of soybean spot price, this paper takes Soybean No. 1 futures as the research subject, and uses the multifractal detrended cross-correlation analysis method (MF-DCCA) to investigate the transaction data from January 4, 2011 to February 23, 2022.

## 2 Literature Review

Efficient Market Hypothesis (EMH) is the mainstream of economics, advocating that degree and speed of asset price response to all relevant information are timely, accurate and sufficient. However, more and more studies have shown that many phenomena in the real market cannot be reasonably explained by the EMH, and even appear contradictory. Therefore, some scholars try to repair or reconstruct EMH. Specifically, [6] firstly proposed fractal theory, which won the attention of academic circles, and gradually evolved into a new research direction in economics. Being able to better explain complex phenomena and behaviors in the market, fractal theory effectively introduces nonlinear characteristics in the market.

According to previous studies, nonlinear characteristics exist in the soybean futures market and futures returns. [7] investigated China's soybean and wheat futures from 1999 to 2005 by applying the improved R/S method, finding that the soybean futures

markets have experienced the state persistence and volatility accumulation, and return series display nonlinear characteristics. [1] used the multifractal method to explore four representative agricultural products including soybean No. 1, and found multifractal spectrum in four futures markets. Moreover, [3] stated that all agricultural product markets in China shows distinct multifractal characteristics, which are mainly attributed to non-Gaussian probability distributions, followed by nonlinear time-dependent mechanisms in all markets. Having studied five representative agricultural products, including soybean No. 1, [5] pointed out there is no long memory of the soybean. Subsequently, [4] used MF-DFA method and multifractal spectrum to compare dynamic characteristics of futures price fluctuations of four major agricultural products including soybeans. They summarized that futures price returns are not in simple single fractals but significant multi-fractals. In addition, [2] used improved MF-DFA method to perform multifractal analysis on soybean and aluminum in commodity futures, finding that both soybean and aluminum have multifractal characteristics, and multifractal strength of soybean is greater. At the same time, it is confirmed that the sources of multifractality of commodity market are the correlation of volatility and the fat-tail probability distribution. Fat-tailed distributions have thicker tails and sharper peaks than normal distributions. That is, the probability of extreme values in fat-tailed distribution data is greater than the probability in normally distributed data.

In summary, many scholars have analyzed the linear relationship between soybean futures price and spot price. Moreover, previous studies have also proved multifractal characteristics in single time series of the soybean futures market, such as price and return series. However, in minority literatures is multifractal method used to explain correlation between soybean futures and spot returns. Therefore, based on multifractal theory, this paper shall explore the multifractal relationship between the time series of soybean futures returns and that of spot returns, and reveal their correlation.

### 3 Multifractal Detrended Cross-Correlation Analysis

Massive studies have shown that economic and financial time series data is characterized by sharp peaks, fat tails, asymmetry and non-normality. Under the nonlinear framework, the multi-fractal method has been widely applied in research on nonlinear correlation between markets with fractal characteristics. In this paper, we analyze the relationship between soybean futures and spot returns through multifractal detrended cross-correlation analysis (MF-DCCA). The basic principles of MF-DCCA are briefly introduced as follows.

Given two time series  $x(i)$  and  $y(i)$ ,  $i = 1, 2, \dots, N$ , where  $N$  represents the length of the two time series.

Step 1: Construct a new time series:

$$X(i) = \sum_{k=1}^i (x(k) - \bar{x}), Y(i) = \sum_{k=1}^i (y(k) - \bar{y}), i = 1, 2, \dots, N \quad (1)$$

where,  $\bar{x} = \frac{1}{N} \sum_{k=1}^N x(k)$ ,  $\bar{y} = \frac{1}{N} \sum_{k=1}^N y(k)$

Step 2: Divide series  $X(i)$  and  $Y(i)$  into  $N$  non-overlapping equal-length subintervals of length  $s$ , that is,  $N_s = \text{int}(N/s)$ . Since length  $N$  is usually not an integer multiple of  $s$ , in order to avoid abandonment of the remaining data at the tail, all data in the series  $X(i)$  and  $Y(i)$  can involve in the calculation; repeat this partitioning process from the end of the series. So, for a given  $s$ , a total of  $2N_s$  subintervals can be obtained.

Step 3: Use the least squares method to fit local trends of  $2N_s$  sub-intervals  $v$ ,  $v = 1, 2, \dots, 2N_s$ , and then obtain the fitting equations  $\tilde{X}^v(i)$  and  $\tilde{Y}^v(i)$  respectively of series  $X(i)$  and  $Y(i)$  in the interval  $v$ , use them to remove local trends on each subinterval to calculate covariance of each series.

When  $v = 1, 2, \dots, 2N_s$

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s |X((v-1)s+i) - \tilde{X}^v(i)| * |Y((v-1)s+i) - \tilde{Y}^v(i)| \quad (2)$$

When  $v = N_s + 1, N_s + 2, \dots, 2N_s$

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s |X(N - (v-1)s+i) - \tilde{X}^v(i)| * |Y(N - (v-1)s+i) - \tilde{Y}^v(i)| \quad (3)$$

Step 4: Calculate  $q$ -order correlation coefficient for two series:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s, v)]^{q/2} \right\}^{1/q}, q \neq 0 \quad (4)$$

$$F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln [F^2(s, v)] \right\}, q = 0 \quad (5)$$

If there is long-range power-law correlation in the two series  $\{X(i)\}$  and  $\{Y(i)\}$ , then:

$$F_q(s) : s^{H_{xy}(q)} \quad (6)$$

or

$$\log F_q(s) = H_{xy}(q) \log(s) + \log C \quad (7)$$

where  $H_{xy}(q)$  is the generalized Hurst exponent, which is the power-law correlation of two time series. When  $q = 2$ ,  $H_{xy}(2)$  is known as the standard Hurst exponent. If  $H_{xy}(2) = 0.5$ , it means two series are not correlated. If  $H_{xy}(2) > 0.5$ , it indicates the cross-correlation is persistent, and intensity gets stronger with the increase of  $H_{xy}(2)$ . If  $H_{xy}(2) < 0.5$ , it implies the cross-correlation is anti-persistent, and the intensity increases as  $H_{xy}(2)$  decreases.

## 4 Empirical Analysis

The price of the futures market is the future price prediction of the spot market, so theoretically, spot price and futures price are related, and their final prices will converge. Besides, existing researches show that the futures and spot markets in China exhibit certain multi-fractal characteristics with sharp peaks and thick tails. Therefore, this paper assumes that there is a multifractal cross-correlation between soybean futures and spot returns. And based on multifractal theory, this section investigates the correlation between soybean futures and spot returns using MF-DCCA method.

### 4.1 Variable Selection and Data Processing

The year 2001 witnessed the introduction of national agricultural genetically modified policies. As a response, Dalian Commodity Exchange split original soybean contract into soybean contract No. 1 and soybean contract No. 2, respectively in the following year, with non-GMO soybeans and genetically modified soybeans as objects. These contracts differ in terms of underlying purpose, contract pricing and delivery regulations.

Soybean futures have two types, Yellow Soybean No. 1 and Yellow Soybean No. 2. Since the open interest and trading volume of the former are much higher than those of the latter, this paper uses the daily closing price and the soybean spot price of the most traded futures contract of Soybean No. 1 to calculate returns of soybean futures and spot price, respectively. The return is calculated as follows, where  $R_t$  is return,  $P_t$  denotes the price in day  $t$ .

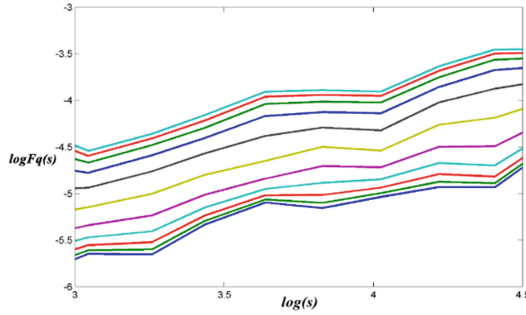
$$R_t = \ln(P_t/P_{t-1}) \quad (8)$$

The daily closing price data and spot price data of the most traded futures contract of Soybean No. 1 come from Wind Financial Terminal. Considering availability and consistency of closing price data, this paper sets data ranging from January 4, 2011 to February 23, 2022, with a total of 2,698 sets of transaction data for 2,698 trading days.

### 4.2 Multi-fractal Test Based on MF-DCCA

According to MF-DCCA method, this section will examine relationship between soybean futures and spot returns. We assume that the value range of the time scale  $s$  is ten to  $N$ , and  $N$  is the length of time series,  $q = 10, 8, 6, \dots, -6, -8, -10$ . Finally, according to Eqs. (1)–(7), the log-log plots of  $F_q(s)$  versus  $s$  on soybean futures and spot return series is shown in Fig. 1.

In Fig. 1, the horizontal axis  $\log(s)$  is the logarithm of the time scale  $s$ , its unit is the logarithm of the number of days. And the vertical axis  $\log F_q(s)$  is the logarithm of the  $q$ -order correlation coefficient  $F_q(s)$  which is calculated by Eq. (4). As show, the fluctuation function  $F_q(s)$  increases nonlinearly as the size of the scale  $s$  increases. And for different values of  $q$ , a good power-law relationship is found between fluctuation function  $F_q(s)$  and scale  $s$ , that is, returns of soybean futures and spot price are cross-correlated. This



**Fig. 1.** The log-log plots of fluctuation function  $F_q(s)$  versus  $s$

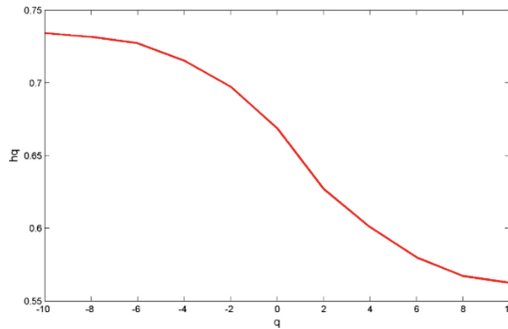
**Table 1.** Generalized Cross-Correlation Exponents  $H_{xy}(q)$  of Soybean Futures and Spot Returns

$q$	$H_{xy}(q)$
-10	0.7342
-8	0.7316
-6	0.7273
-4	0.7154
-2	0.6975
0	0.6689
2	0.6271
4	0.6007
6	0.5800
8	0.5673
10	0.5626

shows that the two series have the same tendency to change, one increases and the other also increases.

Furthermore, the least squares fitting is used to obtain the slope of the double logarithmic fluctuation curve function graph of soybean futures and spot returns time series—generalized Hurst exponent  $H_{xy}(q)$ . The results are shown in Table 1, with  $q = 10, 8, 6, \dots, -6, -8, -10$ .

Table 1 shows that, when  $q$  changes from  $-10$  to  $10$ ,  $H_{xy}(q)$  gradually becomes smaller, which indicates that the cross-correlation between soybean futures and spot returns is highly multifractal, and also proves that the relationship between two time series is nonlinear. In particular, when  $q = 2$ , the value of  $H_{xy}(q)$  is  $0.6271$ , greater than



**Fig. 2.** Generalized Cross-Correlation Exponents  $H_{xy}(q)$  with  $q$  Varying from  $-10$  to  $10$

0.5, which means two time series are persistent, that is, they are non-linearly positively correlated. This signifies soybean futures return rises with the increase of spot return, and vice versa. In order to present the data content in Table 1 intuitively, we have drawn a graph, as shown in Fig. 2.

## 5 Conclusion

Based on the multifractal nonlinear framework, this paper analyzes the cross-correlation between returns of soybean futures and spot price through MF-DCCA method. Specifically, the first-order logarithmic difference of daily closing price data of most traded futures contract of soybean No. 1 is used as an indicator to measure the returns of soybean futures price, and that of soybean spot price to calculate spot return. The results show that there is a highly multifractal cross-correlation between soybean futures and spot returns. The returns of soybean futures and spot returns are persistent, that is, as soybean futures yields increase (decrease), spot yields also increase (decrease). In summary, this paper reveals the nonlinear correlation between soybean futures and spot returns, which supplements academic research in this field, to some extent.

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