



# Implementation of Adaptive PSODV to Improved Benders Decomposition Based Unit Commitment

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**Abstract.** This paper aims at the latest approach for solving the security-constrained unit commitment problem (UCP) dependent on Improved Benders Decomposition (IBD) to Adaptive Particle swarm optimization with differentially perturbed velocity (APSODV). The proposed IBD determines the optimal unit commitment schedule which includes minimum up/downtime and spinning reserve constraints. APSODV algorithm initializes and updates the Lagrangian multipliers and improves the solution fineness. The accomplishment of the suggested technique is at first examined on a 10-unit system and extended to 100-unit with a 24-h horizon. The results specify that an effective and strong solution for UC can be attained from the proposed technique.

**Keywords:** Unit Commitment · Improved Benders Decomposition · Lagrangian multipliers · adaptive Particle swarm optimization

## 1 Introduction

Unit commitment means determining the state of commitment of power-producing units which is dependent on a set of constraints like power balance constraint, spinning reserve constraint, generating limits, etc. for attaining minimum cost outcome [1–4]. Changing on the size of the power system and longer forecasting time, the complexity of the UCP increases, therefore solving these types of problems is a challenging task.

In the beginning, the UC solution methods used are conventional methods such as the priority list method [5], branch and bound method [6], dynamic programming method [7], and Lagrangian relaxation method [8, 9]. The above-mentioned methods need more computational effort, time and memory requirements when considering many units or a long study period. The conventional methods have the following limitations like non-convex problems and inconsistency in results due to relaxations made while linearizing objective functions and constraints. Benders decomposition is popular in searching for the global optimum of nonconvex problems [10–13].

Heuristic methods like genetic algorithm and improvements, [14–17] particle swarm optimization and modifications [18–20]. Swarm Intelligence [21], and Differential evolution [22] are used to solve various problems in engineering. Apart from these, PSODV possesses a new methodology that is resulting in attaining good outcomes for unit commitment [23, 24]. In this paper, a new method is implemented called as Adaptive PSODV Hybrid Algorithm to IBD to solve the unit commitment problem.

## 2 Problem Formulation

Relying on a set of constraints in determining the cost of production of generating units to attain the minimal cost criteria is the essential goal of any unit commitment problem [9].

The objective function of UC to be minimized is

$$F(P_i^t, G_{i,t}) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P_i^t) + ST_{i,t}(1 - G_{i,t-1})] G_{i,t} \quad (1)$$

Subject to the following constraints Power balance

$$\sum_{i=1}^N P_i^t G_{i,t} = P_D^t \quad (2)$$

Spinning reserve

$$\sum_{i=1}^N P_{i,\max} G_{i,t} \geq P_D^t + R^t \quad (3)$$

Generator limit

$$P_{i,\min} G_{i,t} \leq P_i^t \leq P_{i,\max} G_{i,t}, \quad i = 1, \dots, N \quad (4)$$

Minimum up and downtime

$$MiG_{i,t} = \begin{cases} 1, & \text{if } T_{i,\text{on}} < T_{i,\text{up}} \\ 0, & \text{if } T_{i,\text{off}} < T_{i,\text{down}} \\ 0 \text{ or } 1, & \text{otherwise} \end{cases} \quad (5)$$

Start-up cost

$$ST_{i,t} = \begin{cases} HSC_i & \text{if } T_{i,\text{down}} \leq T_{i,\text{off}} \leq T_{i,\text{cold}} + T_{i,\text{down}} \\ CSC_i & \text{if } T_{i,\text{off}} > T_{i,\text{cold}} + T_{i,\text{down}} \end{cases} \quad (6)$$

## 3 Improved Benders Decomposition Method

Large-scale mixed integer programming problems can be easily solved by Benders decomposition [11]. An improved Benders decomposition technique is blended with the APSODV algorithm. Mixed integer programming (MIP) problem, in general, can be written as [10]:

$$\begin{aligned} & \text{Min } F(x, y) \\ & \text{s.t. } x \in Z^{n_1}, y \in R^{n_2} \\ & \quad h_1(x) \geq b \\ & \quad h_2(x, y) \geq c \end{aligned} \quad (7)$$

### 3.1 Benders Decomposition

The multi objective function is as given in equation In this benders decomposition topology, an apt. division of the large-scale problem (which has to be optimized) is taken place into a master problem (integer program) and several sub problems (linear programs) dealing with real variables.

### 3.2 Master Problem

In the master problem

$$\begin{aligned}
 & \text{Min } F(x, \hat{y}) \\
 & \text{s.t. } x \in Z^{n1} \\
 & \quad h_1(x) \geq b \\
 & \quad W(x) \geq 0
 \end{aligned} \tag{8}$$

Here the solution ( $\hat{x}$ ) to the master problem is only required to be feasible [11].

### 3.3 Sub Problem

The master problem should be checked whether all constraints of Eq. (8) is satisfied. In this regard, the corresponding sub problem is formulated as:

$$\begin{aligned}
 & \text{Min } F(\hat{x}, y) + W(\hat{x}), \\
 & \text{s.t. } y \in R^{n2} \\
 & \quad h_2(\hat{x}, y) \geq c
 \end{aligned} \tag{9}$$

The sub problem solution replicates  $y \in R^{n2}$ . For evaluating the extent of violation of the variable limits in the sub problem,  $w(x)$  function is utilized. In the case of zero violation its outcome is zero, but if there is the presence of any violation its outcome shows a positive number. For bypassing these violating factors, we introduce Benders cut into the master problem, so that is feasible solution is determined. In this process, an iterative procedure is essential between the master problem and the sub problem for obtaining the final solution which is dependent on the Benders decomposition algorithm.

### 3.4 Improving Benders Decomposition Method Using Metaheuristics

For improving the Benders decomposition method, we add heuristic methods to it, i.e., during iterative operations few optimality cuts are summed up to the master problem.

The detailed implementation of benders decomposition to unit commitment makes clear in [11].

## 4 APSODV Hybrid Algorithm

In recent years, combining other methods with PSO is evolving into popularity. Among these combinational methods, evolutionary computation techniques having Evolutionary operators are one of them. The best-performed particles are put forward for future generations by combining selection operations [20]. Effective interchange of information is preceded among two individuals by including crossover operation in PSO [21]. Lastly, the property of diversification is enhanced along with increased local minima escaping characteristics is taken place by including mutation operator in PSO [22–24], Particle swarm optimization with differentially perturbed velocity (PSODV) instigates a differential operator in the velocity-modification manifesto of PSO. Here the mutation operation of DE is converged with the velocity part of PSO [19]. A brief discussion about APSODV is mentioned here:

### Step 1: Initialization

The production of starting population is progressed in randomly for  $N$  generators as follows:

$$X_i^0 = X_{i,\min} + \text{rand}() \cdot (X_{i,\max} - X_{i,\min}), i = 1, \dots, N_p \quad (10)$$

where  $\text{rand}()$  is uniformly distributed random number having its range as  $[0, 1]$ . This produces  $N_p$  individuals of  $X_i^0$  randomly.

**Step 2:** Run power flow and evaluate the fitness value of everyone.

### Step 3: Mutation operation

The choice of taking two particles is preceded in a random way to include the mutation operator in the velocity updating part of PSO; the mutation operator is shown as:

$$\delta_d = F(X_k - X_j), \quad i \neq j \neq k \quad (11)$$

### Step 4: Crossover operation

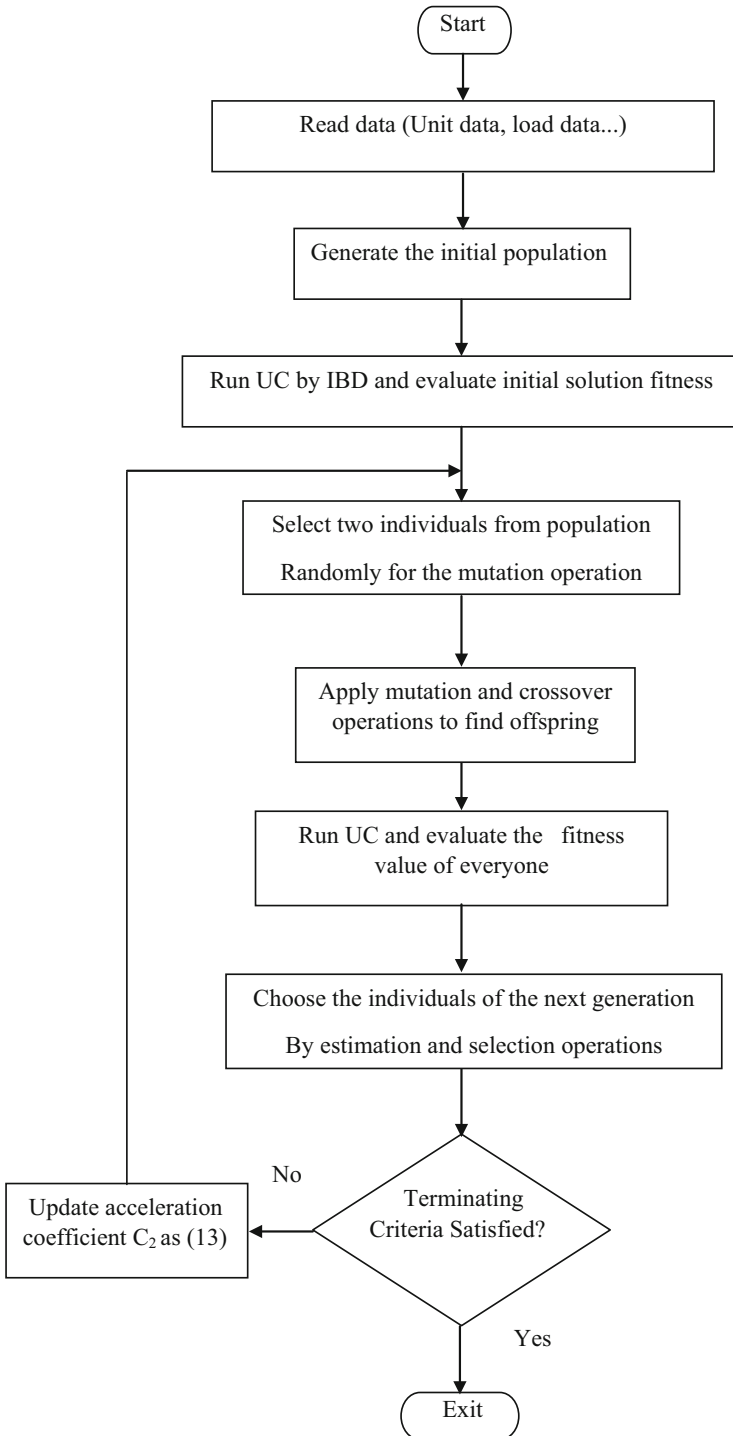
Crossover or recombination operation provides a strong support by producing child individuals out of existing individuals before a successful operation so as to make better progress in diversifying individuals in future generations; the perturbed individual  $\hat{X}_i^{G+1}$  is generated from the existing individual by summing the differentially perturbed velocity to  $X_i^G$ . The recombination constraints of freshly attained individuals are known from the crossover constant (CR). Velocity wise parameter  $j$  of  $i$ -th individual is attained from the perturbed individual velocity  $V_i^{G+1}$  and the pre-existing individual velocity  $V_i^G$  by:

$$V_{ij}^{G+1} = \begin{cases} \varpi V_{ij}^G + \delta_d + C_2 \varphi_2 (P_{gj} - X_{ij}^G), & \text{if } \text{rand}(0, 1) < CR \\ V_{ij}^G, & \text{Otherwise} \end{cases} \quad (12)$$

where  $i = 1, \dots, N_p$ ;  $j = 1, \dots, n$ ;  $n$  = number of parameters,  $\varpi$  is the weighting factor,  $\varphi_2$  is a random number (0, 1),  $C_2$  is social parameter or acceleration coefficient and  $\delta_d$  is inferred through (11). The adaptive acceleration coefficient  $C_2$  [18] is written as:

$$C_2 = (C_{2f} - C_{2i}) \frac{\text{gen}}{\text{gen max}} + C_{2i} \quad (13)$$

Here  $C_{2i}$  and  $C_{2f}$  are constants.



**Fig. 1.** Flow chart of the proposed IBD-APSODV algorithm.

The weighting factor is given by

$$\varpi = 1 - \frac{gen}{gen\ max} \quad (14)$$

Here  $gen$  and  $genmax$  are the number of current generation and the maximum number of generations respectively.

Previous position  $X_i^G$  is summed up with updated velocity to create a revised trail location  $\hat{X}_i^{G+1}$ :

$$\hat{X}_i^{G+1} = X_i^G + V_i^{G+1} \quad (15)$$

**Step 5: Estimation and selection**

In case of existence of better fitness in offspring than their parent, they are replaced with their fittest counterpart.

Or else they are left as it is. These two forms are demonstrated as:

$$X_i^{G+1} = \arg \max \left\{ f(X_i^G), f(\hat{X}_i^{G+1}) \right\} \quad (16)$$

$$X_b^{G+1} = \arg \max \left\{ f(X_i^{G+1}) \right\} \quad (17)$$

Here  $\arg \max$  means the argument of the maximum. Since the fitness function i.e.,  $f = 1/OF$ , which represents an addition function that can be minimized, showing that  $\arg \max$  can be employed in this case.

**Step 6:** Steps 2 to 5 are carried out again and again till; we attain the maximum generation quantity.

This process is put to hold when the required numbers of generations are attained respectively. Figure 1 depicts the flowchart of the suggested algorithm.

## 5 Implementation of the Proposed Method

The execution of the proposed approach starts with encoding the parameters of variables  $\lambda$  and  $\mu$ . Unit commitment is progressed within the generators in a system to satisfy load requirements of the hour. An appropriate generator is selected naturally for attaining a successful UC, at a reasonable cost. The number of base generators remains constant in each hour regardless of load variations. Different generators from the system are respectively selected for meeting the load in each hour. After the completion of successful iterations, a pattern is proposed. The fitness value associated with this pattern is calculated and the most suitable value among these patterns is chosen, which a required optimum value is having a minimal cost function.

The fitness function of the proposed method is:

$$f = \sum_{t=1}^T \left( \sum_{i=1}^N [F_i(P_i^T) + ST_{i,t}(1 - U_{i,t-1})]U_{i,t} + k_s \sum (s_p - s_{p\lim})^2 + k_u \sum (T_U - T_{u\lim})^2 + k_d \sum (T_D - T_{d\lim})^2 \right) \quad (18)$$

Here  $k_s$ ,  $k_u$  and  $k_d$  are the weights associates with spinning reserve, uptime, and down-time constraints respectively. The values of  $k_s$ ,  $k_u$ , and  $k_d$  are 0.5, 0.6 and 0.9 respectively. The main computational procedure of IBD-APSODV is depicted using Fig. 1.

The suggested technique mainly involves cost effective unit commitment (selection of appropriate units), with the application of IBD-APSODV method. The computation reaches out an effective unit commitment with reduced cost.

## 6 Simulation Results

The primary goal of the IBD-APSODV to UC application is to prove the well-versed outcomes of the suggested algorithm on a system of 10 to 100 units. The APSODV on modified benders decomposition has been tested initially to the 10-generator system and later extended to 100 generator systems. 10 runs were preceded for each set from 10 to 100 generators for proposed approach. The best of the runs is evaluated as optimum solution. Results are verified with the pre-existing techniques for verifying robustness of proposed methods.

The test system is of 10 to 100 units and a time horizon of 24 h is taken from [6]. For implementing the requisite outcome, the population is set to be 50 and iterations are 100. Unit commitment schedule of proposed 20 generator system given in Table 1 and 100 generator approaches is given in Table 2. Figure 2 and Figure 3 shows the convergence of total cost for 20 and 100-unit system respectively. Comparison of results is given in Table 3. Simulation results of IBD-APSODV are tabulated in Table 4. Figure 4 shows variation of execution time (10–100 units).

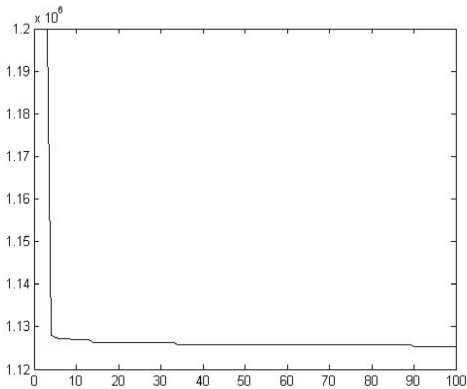
**Table 1.** UC schedule of the 20-generator system

Hour	20-Unit System
1	11110000000000000000
2	11110000000000000000
3	11110000010000000000
4	11110000110000000000
5	11110001110000000000

(continued)

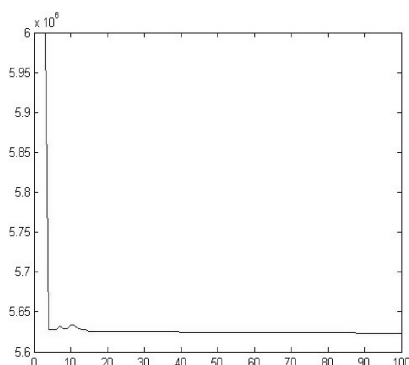
**Table 1.** (continued)

Hour	20-Unit System
6	11111011110000000000
7	11111011110000000000
8	11111111110000000000
9	1111111111110000000
10	111111111111110000
11	111111111111111100
12	111111111111111111
13	1111111111111111000
14	1111111111110000000
15	11111111110000000000
16	11111111110000000000
17	11111111110000000000
18	11111111110000000000
19	11111111110000000000
20	1111111111100111101
21	1111111111100100000
22	11110001111100000000
23	11110000010000000000
24	11110000000000000000



**Fig. 2.** Convergence of total cost of the 20-unit system





**Fig. 3.** Convergence of total cost of the 100-unit system

**Table 2.** Unit commitment schedule of the 100-generator system[illegible]**Table 3.** Comparison of various methods

No of generators	Total cost (\$)						
	LR [9]	GA [9]	DPLR [9]	ALR [9]	ELR [9]	BD [11]	IBD-APSODV
10	565,825	565,825	564,049	565,508	563,977	565,537	563,977
20	1,130,660	1,126,243	1,128,098	1,126,720	1,123,297	1,123,619	1,123,357
40	2,258,503	2,251,911	2,256,195	2,249,790	2,244,237	2,243,646	2,243,402

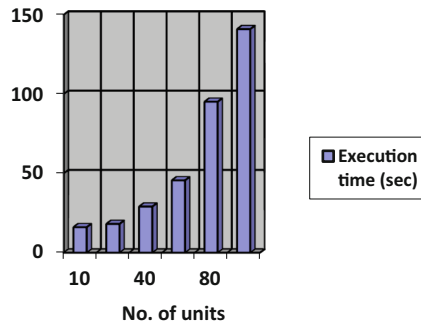
(continued)

**Table 3.** (continued)

No of generators	Total cost (\$)						
	LR [9]	GA [9]	DPLR [9]	ALR [9]	ELR [9]	BD [11]	IBD-APSODV
60	3,394,066	3,376,625	3,384,293	3,371,188	3,363,491	3,363,876	3,362,921
80	4,526,022	4,504,933	4,512,391	4,494,487	4,485,633	4,485,443	4,485,352
100	5,657,277	5,627,437	5,640,488	5,640,488	5,605,678	5,603,496	5,603,258

**Table 4.** Simulation results of the proposed IBD-APSODV method

No. of Units	Best cost (\$)	Average cost (\$)	Worst cost (\$)
10	563,977	565,843	570,222
20	1,123,357	1,126,528	1,129,323
40	2,243,402	2,250,725	2,254,525
60	3,362,921	3,372,450	3,378,524
80	4,485,352	4,492,682	4,505,854
100	5,603,258	5,621,854	5,630,999

**Fig. 4.** Variation of execution time (10–100 units)

## 7 Conclusions

This paper proposes new methods using improved benders decomposition to APSODV algorithm for solving the problem of UC. The feasibility of the suggested technique has been executed with the system of 10 to 100 units in respect to demand. The total production costs over the schedule of 24 h horizon by IBD based APSODV are reduced when verified with other optimization methods reported in the literature.

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