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A Novel Model Reduction Approach for Linear Time-Invariant Systems via Whale Optimization Algorithm

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Abstract. For the determination of accurate and stable decreasing-order model, Heuristic search method is used. Whale optimization algorithm is used to optimize stable higher order systems. By lowering the objective function (E) value, this approach builds the best reduced-order model. The First function determines measure of integral squared error between the original HOS step response and the decreasing-order model. The second term in the objective function assesses the reduced-order model's ability to retain the original system's full impulse response energy. By reducing objective function 'E', the suggested method ensures that the original system's correctness, stability, and passivity are preserved in the decreasing-order model. The method's validity is verified using eighth-order and ninth-order SISO systems. The results of integral squared error show that the suggested technique is superior to the existing decreasing methods that have been existing in literature.

Keywords: Whale optimization algorithm · Model order reduction · integral squared error · single-input single-output systems

1 Introduction

The creation of a mathematical model would be required to undertake virtual experimentation. For the purposes of analysis, design, and control, this will operate as representative of complicated engineering system.

Based on the modeling of large-scale systems, the MOR approaches [1–3] accessible in the literature are divided into two types: (1) time domain model order reduction methods and (2) frequency-domain methods for SISO linear time-invariant (LTI) systems. There are three types of frequency domain model order reduction techniques documented in literature, those are (1) MOR methods of the traditional type [4–7], (2) stochastic search–based MOR approaches [8–13], and (3) merging MOR techniques based on standard and stochastic search algorithms [14–18].

The standard-type MOR approach suggested by Prasad and Sikander [14] was utilized to reduce SISO systems. Viswakarma et al. [6] employed improved Pade' approximation and improved pole clustering methods to decrease higher order SISO systems

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with large gain. To keep the system stable in the ROM, Narwal and Prasad [7] adopted the logarithmic pole clustering technique.

The primary goal of the paper is to suggest a frequency domain model order reduction approach that falls into the second category. Any of the stochastic search methods in the existing second category of MOR techniques can be used to find numerator and polynomial denominator coefficients. In the suggested method, the ROM transfer function is determined using the whale optimization (WO) algorithm. The suggested method validity is demonstrated by apply it to both eighth and ninth order power system models.

2 Generalization of the Method

Problem Statement

In the frequency domain, the SISO dynamic HOS is represented by the n^{th}-order transfer function as follows

$$G_n(s) = \frac{C(s)}{R(s)} = \frac{b_m s^m + \ldots + b_1 s + b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_o} \tag{1}$$

where $m \leq n$.

It is needed to determine a r^{th}-order decreasing order model transfer function (i.e. $r < n$) for the actual LTI higher order system. The user can choose the reduced order 'r' they want. The ROM's temporal and frequency domain responses must be approximated to the original HOS, while passivity and stability features must be preserved.

As a result, for the SISO system, the decreasing order model's transfer function is as follows:

$$R_r(s) = \frac{C_r(s)}{R(s)} = \frac{d_{r-1} s^{r-1} + \ldots + d_1 s + d_o}{S^r + c_{r-1} s^{n-1} + \ldots + c_1(s) + c_0} = \frac{N(s)}{D_r(s)} \tag{2}$$

In order to conserve the passivity and stability qualities of actual HOS [24, 25] in the decreasing order model, the properties listed below must be fulfilled.

Property of Stability of Stability Preservation: *Definition 1*: For BIBO stability, all poles of an LTI system have a negative real portion. In Corollary 1, Nie and Xie16 provide a novel simple criterion for polynomial stability, which is considered.

Corollary 1: The polynomial $h(s) = Z_0 s^n + Z_1 s^{n-1} + \ldots + Z_{n-1} s + Z_n$ is stable if it meets both the essential and sufficient criteria.

Necessary Condition

$$Z_i > 0 \text{ for } (i = 0, 1, 2, 3 \ldots n) \tag{3a}$$

Sufficient Condition
Find out the coefficients
$\alpha_i = (Z_{i-1} \cdot Z_{i+2}) = (Z_i \cdot Z_{i+1})$ for i = 0, 1, 2, 3…n − 2 utilising the polynomial h(s) coefficients. A sufficient condition determines the stability of h(s), which is

$$\left.\begin{array}{ll}\alpha_i < 1\, for & n = 3\\ \alpha_i < 0.46557\, for & n > 3\end{array}\right\} \text{where } i = 1, 2, \ldots, n-2 \tag{3b}$$

Corollary 2: The proposed method always produces a decreasing order model that maintains the original higher order system's stability.

Necessary Condition

$$c_i > 0 \text{ for } i = 0, 1, 2, 3, \ldots, r \tag{4a}$$

Sufficient Condition
Find out the coefficients
$\alpha_i = (c_{i-1} \cdot c_{i+2}) = (c_i \cdot c_{i+1})$ (for i = 1, 2, 3, 4 … r − 2) for the polynomial $C_r(s)$. A sufficient condition determines the stability of $C_r(s)$ is

$$\left.\begin{array}{ll}\alpha_i < 1\, for & r = 3\\ \alpha_i < 0.46557\, for & r > 3\end{array}\right\} \text{where } i = 1, 2, 3, 4 \ldots, r-2 \tag{4b}$$

Property of Passivity Preservation. *Definition 2*: The impulse response energy of a strictly output passive system is finite, which is $\int_0^\infty (p(t))^2\, dt < \infty$, where p(t) is the supplied system's impulse response.

Objective Function: The suggested method is an iterative strategy in which the objective function is minimised as the method continues (F). The process continues until the objective function (F) hits its global minimum value (or) the maximum iteration (G_{max}). The objective function (F) is defined as

$$\text{Objective function (F)} = \text{ISE} + \left(\frac{IRE_s}{IRE_D}\right) \tag{5}$$

ISE stands for integral squared error, which is defined as $\text{ISE} = \int_0^\infty (c(t) - c_r(t))^2\, dt$. The original HOS and the decreasing order model's impulse response energy is given as $IRE_s = \int_0^\infty (p(t))^2\, dt$, where p(t) represents the higher order system's impulse response and $IRE_D = \int_0^\infty (y(t))^2\, dt$, where y(t) represents the decreasing order model impulse response.

Hence, finding the decreasing order model by optimizing the objective function (F) strengthens the suggested Model Order Reduction method in constructing the accurate, stable and passivity-preserving decreasing order models for the higher order linear time invariant system under consideration.

3 Performance Analysis of Proposed WOA

From the literature, two examples are considered and the observations are analysed for checking the usefulness of the suggested method

Problem I: Consider the SISO LTI system transfer function of 9^{th} order discussed in [19]

$$G_9(s) = \frac{s^4 + 35s^3 + 291s^2 + 1093s + 1700}{s^9 + 9s^8 + 66s^7 + 294s^6 + 1029s^5 + 2541s^4 + 4684s^3 + 5856s^2 + 4620s + 1700} \tag{6}$$

By applying the Whale optimization algorithm with the mentioned parameters which are depicted in Table 1, the decreasing third order model is presented as follows:

$$R_3(s) = \frac{-0.34823s^2 - 1.525038s + 2.8641}{s^3 + 2.82821s^2 + 4.50748s + 2.8601} \tag{7}$$

with ISE(F) = 0.001494. The IRE of $R_3(s)$ is given by $IRE_3 = \int_0^\infty (r_3(t))^2\, dt = 0.47028$.

The simplified second-order model is obtained as

$$R_2(s) = \frac{-0.62148s + 0.9689}{0.8s^2 + 1.58s + 0.969} \tag{8}$$

The step responses of HOS and reduced 2^{nd} and 3^{rd} order models obtained are shown in Fig. 1. From the graphs, it's worth noting that the proposed solution closely resembles the original system's key properties in the ROMs. In addition, the Bode responses are shown in Fig. 2 (Table 2).

As a result of the preceding comparison, the proposed method's enhancements enable it to obtain the best solution considerably faster and with less computing time than the other reduction methods.

Problem II: Consider the seventh-order SISO system transfer function investigated in [21].

$$G_7(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \tag{9}$$

Table 1. Parameters used WO algorithm for Example 1.

Factors	Levels
Size of Population (Np)	20
Number of iterations (G_{max})	200
[X_{min}, X_{max}]	[0.001, 1]

Table 2. Comparison of the ISE and IRE performance indices acquired by the WOA and some other existing algorithms

Order reduction method	Reduced-order models ($R_3(s)$)	ISE_S	IRE
Proposed method (WOA)	$\frac{-0.34823s^2-1.525038s+2.8641}{s^3+2.82821s^2+4.50748s+2.8601}$	0.001494	0.47028
MOR method in Singh et al. [6]	$\frac{-0.915677s+2.713}{s^3+3s^2+4.713s+2.713}$	0.009807	0.392852
MOR method in Nasirisoloklo et al. [20]	$\frac{-0.4121s^2-2.9431s+5.2356}{s^3+4.2602s^2+7.7705s+5.2356}$	0.0098	0.50183
MOR method in Narwal and Prasad [7]	$\frac{-0.569097s^2-0.16713s+2.023}{s^3+3.008s^2+4.03s+2.023}$	0.021134	0.481268
MOR method in Sikander and Thakur [4]	$\frac{-0.001935s^2+0.005725s+1.073}{s^3+1.681s^2+2.183s+1.073}$	0.02252	0.34653
MOR method Desai and Prasad [4]	$\frac{-0.0789s^2+0.3142s+0.493}{s^3+1.3s^2+1.34s+0.493}$	0.0252	0.268282
MOR method in Sikander and Prasad [4]	$\frac{-0.12026s^2+03172s+0.493}{s^3+1.494s^2+1.34s+0.493}$	0.034126	0.323108

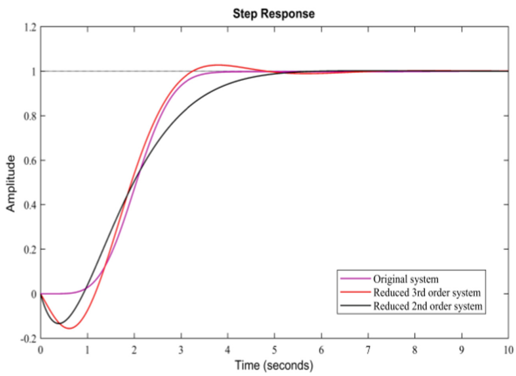

Fig. 1. Step responses of the proposed ROMs

The impulse response energy of $G_7(s)$ is given by $IRE_7 = \int_0^\infty (g(t))^2\,dt = 0.182569$.

Using the Whale optimization algorithm and the parameters listed in Table 3, the second-order reduced model is obtained, as follow:

$$R_2(s) = \frac{16s + 5.032}{0.99s^2 + 6.549s + 5.036} \tag{10}$$

The value of objective function (F) is = 0.10447. The impulse response energy of $R_2(s)$ is given by $IRE_2 = \int_0^\infty (r_2(t))^2\,dt = 0.17587$ (Table 4).

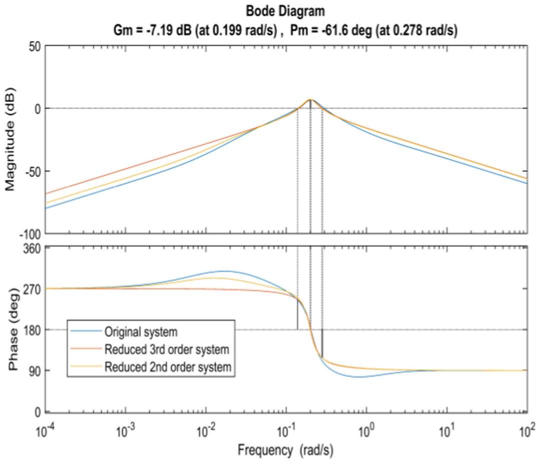

Fig. 2. Bode plots of ROMs with the original HOS system

Table 3. Parameters used WO algorithm for Example 2.

Parameters	Values
Size of Population (Np)	80
Number of iterations (G_{max})	300
[X_{min}, X_{max}]	[0.01, 1]

Table 4. Observations of the performance index of $R_2(s)$ investigated by WOA and the other available techniques in terms of the integral squared error and impulse response energy

Order reduction method	Reduced-order models ($R_2(s)$)	ISE_S	IRE
Proposed method (WOA)	$\frac{16s+5.032}{0.99s^2+6.549s+5.036}$	0.10447	0.17587
ROM of DE [21]	$\frac{18s+5.669}{0.98s^2+7s+5.681}$	0.15906	0.17998
ROM of PSO [22]	$\frac{15s+3.499}{0.98s^2+6.383s+3.684}$	0.16004	0.18235
ROM of Jayanthapal [21]	$\frac{151776s+40320}{65520s^2+75600s+40320}$	0.69498	0.14387
ROM of Eigen spectrum [22]	$\frac{499.3s+221}{s^2+81.78s+221}$	1.13618	0.07508
ROM of Shamesh [23]	$\frac{1.99s+0.4318}{s^2+1.174s+0.43118}$	1.49125	0.07431

Figures 3 and 4 shows the step responses and Bode plots to show the functionality of the suggested method.

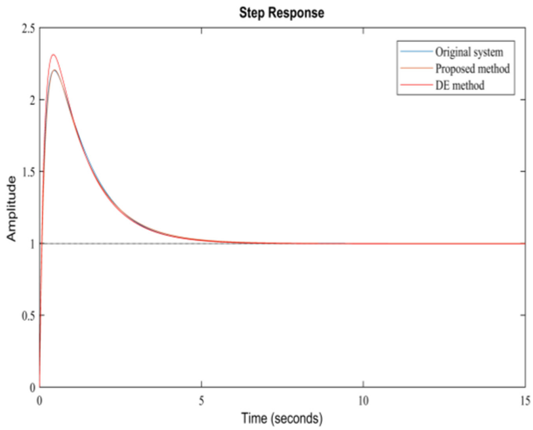

Fig. 3. Comparison of Step responses

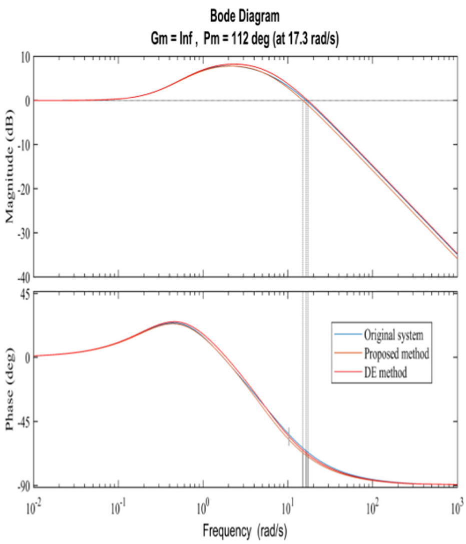

Fig. 4. Comparison of Bode plots

Hence, the above comparison demonstrates that the proposed method allow it to obtain the finest solution significantly faster than the other reduction methods with less computational effort.

4 Conclusion

Using the Whale optimization algorithm, a unique and effective MOR method for the LTI system is proposed in this research to get a stable and accurate ROM. The approach has the features of passivity and stability preservation built in. The ISE values of the proposed decreasing order models are lower than the ISE values of decreasing order models achieved by many notable reduction methods in the literature. The capability of the proposed method is demonstrated by applying to single input single output system models. Comparison of results shows the strength of the proposed method. As a result, the suggested method's reduced order model could be highly useful in validating linear time invariant HOS's analysis, design, and development of continuous control systems.

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