Order Reduction of Continuous Time Linear Interval Systems Using Whale Optimization Algorithm

G. Ramesh[1(✉)], M. Siva Kumar[1], B. Dasu[1], and R. Srinivasa Rao[2]

[1] Department of EEE, Seshadri Rao Gudlavalleru Engineering College, JNTUK, Gudlavalleru, A.P., India
ganta.ramesh25@gmail.com
[2] JNTUK, Kakinada, A.P., India

Abstract. The Whale Optimization Algorithm (WOA) is a nature-inspired meta-heuristic optimization algorithm that replicates humpback whale social behaviour. The method of bubble-net hunting inspired the algorithm. In this paper, the decreasing Order Interval System was acquired using WOA from a higher order linear continuous time interval model. The lower order system denominator and numerator polynomials are obtained in this suggested technique by applying WOA to minimise the cost function of Integral Squared Error (ISE). The WOA algorithm outperforms both state-of-the-art meta-heuristic algorithms and traditional approaches in terms of optimization results. The WOA method has been determined to be straightforward, easy to use, and to deliver the best answer. A numerical example from the literature is used to demonstrate the feasibility and usefulness of this WOA.

Keywords: Whale Optimization Algorithm · Reduced Order Interval Model · Integral Square Error · Impulse Response Energy

1 Introduction

Technology, as well as highly complex, large-scale, and unpredictable societal and environmental systems, have created a host of problems. Most large-scale systems fail to achieve centrality due to a lack of centralised computational capabilities or centralised information. Many real-world problems have large scale dimensions by necessity rather than choice. Large scale systems are significant because of their hierarchical (multilevel) and a system of decentralisation, which depict systems dealing with society, business, management, the economy, the environment, energy, power networks, transportation, and aerospace. Many researchers around the world have devoted significant effort to massive systems in the recent past due to the decentralised and hierarchical control properties and potential applications.

Scientists and engineers have frequently had to analyse, design, and synthesise real-world problems. The initial step in such studies is that development of a 'mathematical model' that can be used to simulate the real-world problem.

© The Author(s) 2023
V. E. Balas et al. (Eds.): AITEES 2022, AHIS 3, pp. 192–203, 2023.
https://doi.org/10.2991/978-94-6463-074-9_17

One of the most important subjects is the analysis of higher order systems. It is an unarguable conclusion that the development of a mathematical model of a physical system enabled it to be analysed and designed. When a physical system is represented mathematically, it can produce a transfer function of very high order. When applied to a higher-order system, available methods for analysis and design may become cumbersome. At this point, it is unavoidable that the use of order reduction methods will result in less computational effort and process time.

Simulating and designing complex models is made easier by approximating original higher order processes to decreasing order models. The research area of order reduction techniques has grown tremendously, resulting in the development of a diverse range of techniques. Furthermore, uncertainty is expressed in a variety of ways in engineering and scientific designs, such as convex or ambiguous descriptions to varying degrees, necessitating a study to estimate the least and highest boundaries of the system parameters for a thorough evaluation. So, Interval plants are those that have fixed coefficients but are uncertainty within a specific range.

Interval systems have also been studied in the literature for their stability and transient analysis [1–4]. Routh approximation [5], γ-δ Routh approximation [2], and a straightforward direct method using γ table have been put forward for getting decreasing order continuous time interval systems. To overcome the system's instability, Dolgin and Zeheb [6] modified the Bandoypadhyay [7] method proposed a new hybrid method. Unfortunately, Hwang et al. [8] made a point of saying on Dolgin's method and demonstrated its failure to achieve stability. Aside from these, a number of other mixed methods [9–15] have recently been proposed to reduce system complexity while increasing system accuracy.

This paper proposes and tests an order reduction method for continuous time linear interval systems using WOA.

Seyedali Mirjalili developed WOA [16] by observing the foraging behaviour of Humpback whales. Bubble-net feeding is a unique hunting methodology that are used by whales, in which the whale creates two paths to the prey. WOA was created in response to the whales' unique hunting method.

2 Statement of the Problem

Consider the following higher order interval system that is asymptotically stable:

$$G(s,p,q) = \frac{\left[p_{n-1}^{-}, p_{n-1}^{+}\right]s^{n-1} + \left[p_{n-2}^{-}, p_{n-2}^{+}\right]s^{n-2} + \cdots + \left[p_{0}^{-}, p_{0}^{+}\right]}{\left[q_{n}^{-}, q_{n}^{+}\right]s^{n} + \left[q_{n-1}^{-}, q_{n-1}^{+}\right]s^{n-1} + \cdots + \left[q_{0}^{-}, q_{0}^{+}\right]} \tag{1}$$

where $p_i^- < p_i < p_i^+, for\, i = 0, 1, 2, 3, 4 \ldots n-1$ and $q_i^- < q_i < q_i^+, for\, i = 0, 1, 2, 3, 4 \ldots n$ are the numerator and denominator interval polynomial parameters' lowest and upper bounds. The proposed reduction process should be used to obtain the kth decreased Order Interval system to $\mathrm{R_k(s, u, v)}$:

$$R_k(s,p,q) = \frac{\left[p_{k-1}^{-}, p_{k-1}^{+}\right]s^{k-1} + \left[p_{k-2}^{-}, p_{k-2}^{+}\right]s^{k-2} + \cdots + \left[p_{0}^{-}, p_{0}^{+}\right]}{\left[q_{k}^{-}, q_{k}^{+}\right]s^{k} + \left[q_{k-1}^{-}, q_{k-1}^{+}\right]s^{k-1} + \cdots + \left[q_{0}^{-}, q_{0}^{+}\right]} \tag{2}$$

where $p_i^- < p_i < p_i^+, for\ i = 0, 1, 2, 3, 4 \ldots k - 1$ and $q_i^- < q_i < q_i^+, for\ i = 0, 1, 2, 3, 4 \ldots k$ are lower and upper bounds for numerator and denominator polynomial parameters in reduced order interval models, respectively.

It is required to decrease the order of the system (1) into (2). Equation (1) represents the Higher Order Interval System as 4 Kharitonov's transfer functions with constant parameters [9]. They have presented as:

$$\begin{aligned}
G^1(s) &= \frac{P^1(s)}{Q^1(s)} = \frac{p_{n-1}^- s^{n-1} + \ldots + p_2^+ s^2 + p_1^- s + p_0^-}{q_n^- s^n + \ldots + q_2^+ s^2 + q_1^- s + q_0^-} \\
&= \frac{p_{1n-1}s^{n-1} + \ldots + p_{12}s^2 + p_{11}s + p_{10}}{q_{1n}s^n + \ldots + q_{12}s^2 + q_{11}s + q_{10}} \\
G^2(s) &= \frac{P^2(s)}{Q^2(s)} = \frac{p_{n-1}^- s^{n-1} + \ldots + p_2^+ s^2 + p_1^+ s + p_0^-}{q_n^- s^n + \ldots + q_2^+ s^2 + q_1^+ s + q_0^-} \\
&= \frac{p_{2n-1}s^{n-1} + \ldots + p_{22}s^2 + p_{21}s + p_{20}}{q_{2n}s^n + \ldots + q_{22}s^2 + q_{21}s + q_{20}} \\
G^3(s) &= \frac{P^3(s)}{Q^3(s)} = \frac{p_{n-1}^+ s^{n-1} + \ldots + p_2^+ s^2 + p_1^- s + p_0^+}{q_n^+ s^n + \ldots + q_2^+ s^2 + q_1^- s + q_0^+} \\
&= \frac{p_{3n-1}s^{n-1} + \ldots + p_{32}s^2 + p_{31}s + p_{30}}{q_{3n}s^n + \ldots + q_{32}s^2 + q_{31}s + q_{30}} \\
G^4(s) &= \frac{P^4(s)}{Q^4(s)} = \frac{p_{n-1}^+ s^{n-1} + \ldots + p_2^- s^2 + p_1^+ s + p_0^+}{q_n^+ s^n + \ldots + q_2^- s^2 + q_1^+ s + q_0^+} \\
&= \frac{p_{4n-1}s^{n-1} + \ldots + p_{42}s^2 + p_{41}s + p_{40}}{q_{4n}s^n + \ldots + q_{42}s^2 + q_{41}s + q_{40}}
\end{aligned} \tag{3}$$

After getting parameters of the algorithm, the K$^{\text{th}}$ order and I$^{\text{th}}$ fixed parameter decreasing order model can be constructed as follows:

$$R_k^I(s) = \frac{n_k^I(s)}{d_k^I(s)} = \frac{p^I{}_{k-1}s^{k-1} + p^I{}_{k-2}s^{k-2} + \cdots + p^I{}_0}{q^I{}_k s^k + q^I{}_{k-1}s^{k-1} + q^I{}_{k-2}s^{k-2} + \cdots + q^I{}_0} \tag{4}$$

Using the following equation, this procedure has used to all 4 Transfer functions of Kharitonov, and the decreasing order interval model was obtained with the coefficients of polynomials with numerator and denominator as:

$$R_k(s, p, q) = \frac{\left[\min(p_{k-1}^I), \max(p_{k-1}^I)\right]s^{k-1} + \cdots + \left[\min(p_{k-1}^I), \max(p_{k-1}^I)\right]}{\left[\min(q_k^I), \max(q_k^I)\right]s^k + \cdots + \left[\min(q_{k-1}^I), \max(q_{k-1}^I)\right] + \left[\min(q_0^I), \max(q_0^I)\right]} \tag{5}$$

2.1 Overview of Whale Optimization Algorithm (WOA)

WOA stands for Whale Optimization Algorithm. It is a meta-heuristics algorithm. WOA was inspired by social behaviour as well as humpback whale bubble-net hunting in

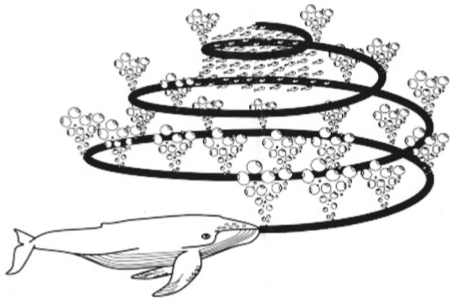

Fig. 1. Humpback whales eating with bubble nets.

the oceans. The bubble-net feeding strategy is a unique hunting mechanism used by humpback whales. Humpback whales prefer to hunt for small fish at the water's surface.

Foraging has been observed to be done by bubbles that are distinct along a '9'-shaped or circle-shaped path, as shown in Fig. 1. Upward spirals and double loops are two bubble net feeding manoeuvres. In 'upward-spirals' maneuver Humpback whales use the 'upward-spirals' move to descend roughly 12 m below the surface and then build a spiral of bubbles around their meal before swimming all the way to the top. The 'double-loops' manoeuvre has three stages: the coral loop, the lobtail, and the catch loop. Only humpback whales are known to engage in bubble-net feeding. The bubble-net spiral feeding maneuver is explained mathematically in the whale optimization algorithm (WOA) in order to accomplish optimization.

2.2 WOA Implementation for Order Reduction

The following is a step-by-step procedure for implementing WOA.

Step 1: Exploration Phase (Searching Model): The investigator (humpback whale) searches for the most effective solution (prey) at random depends on the position of each agent. Agent of discovery position will be updated during this phase by using a rather of using the optimal search agent, a random search agent is used. Following that, if $\{|A| > 1\}$, as defined in Eq. 8, then divert the search agent away from the reference whale.

$$\vec{D} = \left|\vec{C} * \vec{X}_{rand} - \vec{X}\right| \tag{6}$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A} * \vec{D} \tag{7}$$

where $\vec{X}_{rand}$ is a population-based random location vector, and $\vec{A}$, $\vec{C}$ are vectors of coefficients that are being used to discover the most effective search agents:

$$\vec{A} = 2 * \vec{a} * \vec{r} - \vec{a} \tag{8}$$

$$\vec{C} = 2 * \vec{r} \tag{9}$$

where $\vec{r}$ is a range of random vectors [0, 1] and $\vec{a}$ during the iterations, the value decreases linearly from 2 to 0.

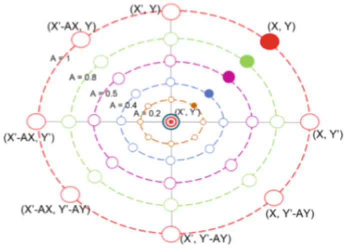

Fig. 2. Different placements from {(X, Y)} to {({X′}, {Y′})}

Step 2: Surrounding Prey: The best candidate solution available right now is believed to be close to the prey on the hunt, while other solutions adjust their positions in relation to the best agent.

$$\vec{D} = \left|\vec{C} * \vec{X}_{best}(t) - \vec{X}(t)\right| \tag{10}$$

$$\vec{X}(t+1) = \vec{X}_{best}(t) - \vec{A} * \vec{D} \tag{11}$$

where t is the current iteration, $\vec{X}_{best}$ is the best solution's position, $\vec{X}$ refers to the vector of position of a solution.

Step 3: Attacking with a bubble net (exploitation phase): Two techniques to statistically modelling humpback whale bubble-net behaviour are as follows:

i) The value of $\vec{A}$ in this mechanism is a random value within the [−a, a] interval, and over the course of iterations, $\vec{a}$ is reduced from 2 to 0. Let define $\vec{A}$ values at random in [−1, 1]. A new search agent position can be created anywhere between the current best agent's position and the agent's original position. The graph depicting the different placements from {(X, Y)} to {({X′}, {Y′})} (Fig. 2).
ii) Position updating spiral: The separation between both the whale {(X, Y)} and the prey {({X′}, {Y′})} is calculated first in this method.

The spiral equation underlying humpback whales' helix-shaped movement to describe the position among both whale and prey is:

$$\vec{X}(t+1) = \vec{D}'' * e^{bl} * \cos(2\pi l) + \vec{X}' \tag{12}$$

where $\vec{D}'' = \left|\vec{X}'(t) - \vec{X}(t)\right|$ represents the distinction between the whale and its prey (best solution found here so far), l is a number at random between [−1, 1], and b is a constant that defines the logarithmic spiral's shape.

The mathematical model behind the humpback whale's swimming style around the prey using a shrinking circle and also following a spiral-shaped path at the same time:

$$\vec{X}(t+1) = \begin{cases} \vec{X}'(t) - \vec{A} * \vec{D} & if\ p < 0.5 \\ \vec{D}'' * e^{bl} * \cos(2\pi l) + \vec{X}' & if\ p \geq 0.5 \end{cases} \tag{13}$$

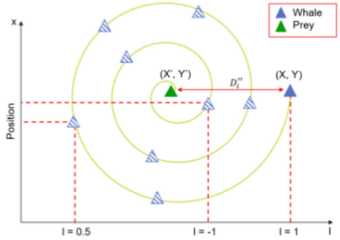

Fig. 3. The spiral updating position

Table 1. WOA settings

Size of the group	20
Variable limitations	[0–10]
Maximum no of Iterations	50

where p represents the probability of selecting one of these two methods to keep track of the whereabouts of whales. Let's say that the probability of choosing between the two approaches is 50%. Then, p is a random number in [0, 1]. The graph that shows the spiral updating position (Fig. 3).

The decreasing order model is created by minimising ISE with WOA and Table 1 summarizes the parameters.

3 Error in Integral Square

The integral square error (ISE) among both higher order interval system model and decreasing order interval system transient responses is given by:

$$ISE = \int_0^\infty \left[y(t) - y_r(t)\right]^2 \tag{14}$$

where $y(t)$ and $y_r(t)$ are the original higher order interval model and decreasing order interval system unit step responses, respectively.

4 Application of Proposed Method

This section contains an example to demonstrate the method.

Example: Consider an asymptotically stable Higher Order Interval System [4]

$$G(s) = \frac{[1.9,\ 2.1]s^6+[24.7,\ 27.3]s^5+[157.7,\ 174.3]s^4+[542,\ 599]s^3+[930,\ 1028]s^2+[721.8,\ 797.8]s^1+[187.1,\ 206.7]}{[0.95,\ 1.05]s^7+[8.779,\ 9.703]s^6+[52.23,\ 57.73]s^5+[182.9,\ 202.1]s^4+[429.1,\ 474.2]s^3+[572.5,\ 632.7]s^2+[325.3,\ 359.5]s^1+[57.35,\ 63.93]}$$

Using the procedure given in (3), the four-fixed parameter Kharitonov transfer functions are obtained as follows

$$G^1(s) = \frac{2.1s^6 + 24.7s^5 + 157.7s^4 + 599s^3 + 1028s^2 + 721.8s + 187.1}{1.05s^7 + 9.703s^6 + 52.23s^5 + 182.9s^4 + 474.2s^3 + 632.7s^2 + 325.3s + 57.35}$$
$$G^2(s) = \frac{2.1s^6 + 27.3s^5 + 157.7s^4 + 542s^3 + 1028s^2 + 797.8s + 187.1}{0.95s^7 + 9.703s^6 + 57.73s^5 + 182.9s^4 + 429.1s^3 + 632.7s^2 + 359.5s + 57.35}$$
$$G^4(s) = \frac{1.9s^6 + 27.3s^5 + 174.3s^4 + 542s^3 + 930s^2 + 797.8s + 206.7}{1.05s^7 + 8.779s^6 + 57.7s^5 + 202.1s^4 + 429s^3 + 572.5s^2 + 359.5s + 63.39}$$
$$G^3(s) = \frac{1.9s^6 + 24.7s^5 + 174s^4 + 599s^3 + 930s^2 + 721.8s + 206.7}{1.05s^7 + 8.779s^6 + 52.23s^5 + 202.1s^4 + 474.2s^3 + 572.5s^2 + 325.3s + 63.39}$$

The 2nd order reduced models are given by

$$R_2^1(s) = \frac{3.823s + 5.709}{s^2 + 4.347s + 1.75}$$

$$R_2^2(s) = \frac{4.626s + 2.649}{s^2 + 3.045s + 0.8118}$$

$$R_2^3(s) = \frac{4.142s + 3.967}{s^2 + 3.265s + 1.216}$$

$$R_2^4(s) = \frac{3.961s + 2.178}{s^2 + 2.425s + 0.668}$$

The decreasing order interval system can then be constructed using (5) and is provided by

$$R_2(s) = \frac{[3.823, 4.626]s^1 + [2.178, 5.709]}{[1, 1]s^2 + [2.425, 4.347]s^1 + [0.668, 1.75]}$$

Integral Square Error (ISE) values of original higher order Interval system and decreasing order interval system that are generated by suggested method are calculated and Table 2 contains the results.

And similarly Impulse Response Energy (IRE) values of original higher order interval model and decreasing order interval system that are generated by suggested Method are calculated and Table 3 contains the results.

4.1 In Comparison to Another Method

To demonstrate the effectiveness of the suggested method, numerical examples are used and compared to other methods (Gamma Delta [2], Mixed Method [3], and Mixed Evolutionary Method [5]) presented in the literature.

i) Method in Gamma Delta [2] yields the 2nd order reduced interval model.

$$R_{2b}(s) = \frac{[1.61, 1.84]s^1 + [0.27, 0.53]}{[1, 1]s^2 + [0.52, 0.83]s^1 + [0.08, 0.16]}$$

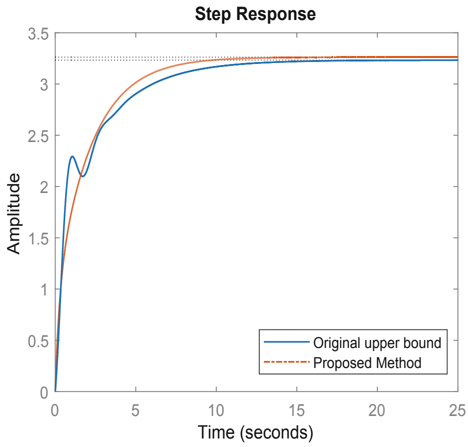

Fig. 4. Upper Bound Step Response

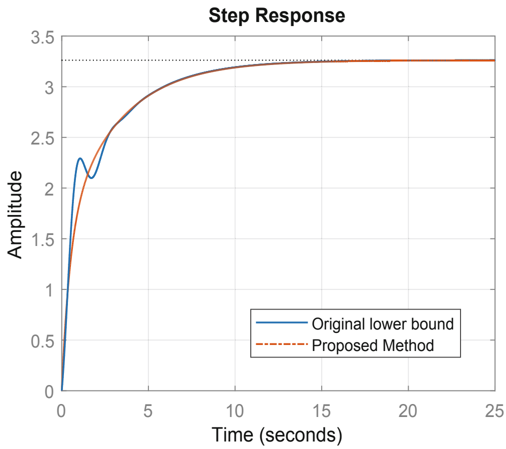

Fig. 5. Lower Bound Step Response

Table 2. ISE Values in Comparison

Transfer function	1st Kharitonov Transfer function	2nd Kharitonov Transfer function	3rd Kharitonov Transfer function	4th Kharitonov Transfer function
Proposed method	0.55202	0.25644	0.14808	0.6586

Table 3. IRE Values in Comparison

Transfer function	1st Kharitonov Transfer function	2nd Kharitonov Transfer function	3rd Kharitonov Transfer function	4th Kharitonov Transfer function
Proposed method	0.772	0.771	0.759	0.750

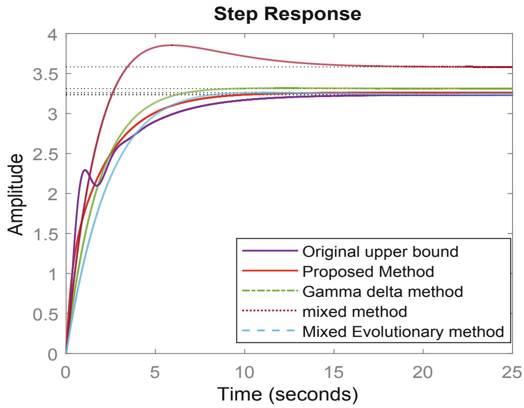

Fig. 6. Comparison with Upper Bound Step Responses

ii) Method in Mixed Method [3] is used to obtain the 2nd order reduced interval model.

$$R_{2s}(s) = \frac{[260.955, 861.331]s^1 + [175.232, 218.581]}{[364.72, 366.62]s^2 + [281.08, 282.35]s^1 + [59.74, 61]}$$

iii) Method in Mixed Evolutionary Method [5] is used to generate the 2nd order reduced interval model.

$$R_{2s}(s) = \frac{[562.4, 555.6]s^1 + [181.6, 205.4]}{[319.49, 406.63]s^2 + [259.89, 300.36]s^1 + [57.352, 63.389]}$$

Comparison with other methods:

Figures 4, 5, 6 and 7 show comparison of step responses of decreasing order interval model obtained by the suggested method and interval system getting from Gamma Delta [2], Mixed Method [3], and Mixed Evolutionary Method [5]. The simulated results show that the ROIM responses obtained using the suggested technique closely approximate the HOIS.

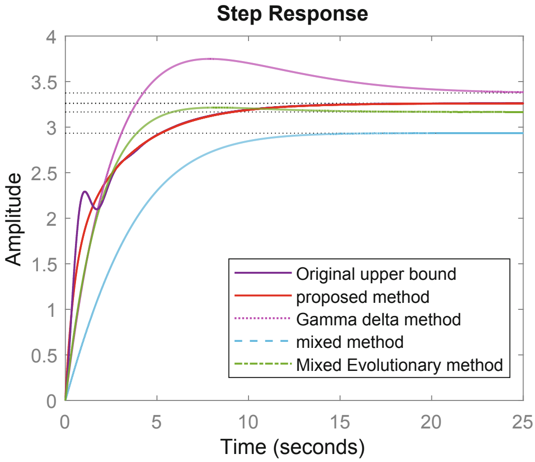

Fig. 7. Lower Bound Step Response Comparison

5 Results and Discussion

The decreasing order interval model coefficients in the denominator and numerator were obtained by minimization of a cost function ISE between the higher order interval model and the decreasing order interval system in this suggested method. Because this method avoids interval arithmetic, the computation complexity associated with interval arithmetic is reduced. The acquired ISE and IRE values for original higher order interval systems and decreasing order interval models are shown in Tables 2 and 3. The proposed method's transient and steady state responses of a lower order interval model are closely matched. In order to solve complicated issues, the suggested method employs a new optimization strategy based on nature-inspired meta-heuristics from social behaviour and humpback whale bubble-net hunting in oceans. This approach tunes with a single parameter, reducing computing time and making it simple and quick to apply.

6 Conclusion

This paper describes how to obtain a decreasing order linear time invariant interval system from a higher order interval model by minimising ISE with WOA. This optimization technique has been demonstrated to produce a robustly stable decreasing order interval system. It has the benefit of being mathematically simple. This technique produces an acceptable step response by reducing a higher order interval model to a decreasing order interval system. This method is simple and produces reliable decreasing order interval systems. The efficiency of the suggested strategy is demonstrated in this paper through a numerical example.

References

1. Selvaganesan, N.: Mixed method of model reduction for uncertain systems. Serb. J. Electr. Eng. **4**(1), 1–12 (2007)
2. Bandyopadhyay, B., Upadhye, A., Ismail, O.: $\gamma - \delta$ Routh approximation for interval systems. IEEE Trans. Autom. Control **42**(8), 1127–1130 (1997)
3. Vijaya Anand, N., Siva Kumar, M., Srinivasa Rao, R.: A novel order reduction procedure for linear time invariant interval systems using SGO algorithm. Int. J. Eng. Technol. **7**(8), 118–122 (2018)
4. Saini, D.K., Prasad, R.: Mixed evolutionary techniques to reduce order of linear interval systems using generalized Routh array. Int. J. Eng. Sci. Technol. **2**(10), 5197–5205 (2010)
5. Bandyopadhyay, B., Ismail, O., Gorez, R.: Routh Pade approximation for interval systems. IEEE Trans. Autom. Control **39**, 2454–2456 (1994)
6. Dolgin, Y., Zeheb, E.: On Routh Pade model reduction of interval systems. IEEE Trans. Autom. Control **48**(9), 1610–1612 (2003)
7. Ismail, O., Bandyopadhyay, B.: Model order reduction of linear interval systems using Pade approximation. In: IEEE International Symposium on Circuit and Systems (1995)
8. Hwang, C., Yang, S.F.: Comments on the computation of interval Routh approximants. IEEE Trans. Autom. Control **44**(9), 1782–1787 (1999)
9. Sastry, G.V.K., Raja Rao, G.R., Rao, P.M.: Large scale interval system modelling using Routh approximants. Electron. Lett. **36**(8), 768 (2000)
10. Saraswathi, G.: A mixed method for order reduction of interval systems. In: International Conference on Intelligent and Advanced Systems, pp. 1042–1046 (2007)
11. Singh, V.P., Chandra, D.: Routh approximation based model reduction using series expansion of interval systems. In: IEEE International Conference on Power, Control & Embedded Systems (ICPCES), vol. 1, pp. 1–4 (2010)
12. Singh, V.P., Chandra, D.: Model reduction of discrete interval system using dominant poles retention and direct series expansion method. In: IEEE 5th International Power Engineering and Optimization Conference (PEOCO), vol. 1, pp. 27–30 (2011)
13. Choo, Y.: A note on discrete interval system reduction via retention of dominant poles. Int. J. Control Autom. Syst. **5**(2), 208–211 (2007)
14. Kranthi Kumar, D., Nagar, S.K., Tiwari, J.P.: Model order reduction of interval systems using modified Routh approximation and factor division method. In: 35th National System Conference (NSC), IIT Bhubaneswar, India (2011)
15. Kranthi Kumar, D., Nagar, S.K., Tiwari, J.P.: Model order reduction of interval systems using Mihailov criterion and Routh approximations. Int. J. Eng. Sci. Technol. (IJEST) **3**(7), 5593–5598 (2011)
16. Mirjalili, S., Lewis, A.: The whale optimization algorithm. Adv. Eng. Softw. **95**, 51–67 (2016)

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (http://creativecommons.org/licenses/by-nc/4.0/), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.