



# Analyzing Mathematics Prospective Teachers' Ability for Higher-Order-Thinking Problem Posing

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**Abstract.** Problem posing is an essential skill for teachers and prospective teachers. The current process and evaluation of mathematics learning in Indonesia must include higher-order thinking skills (HOTS). This study aims to describe the ability of prospective teachers in posing problems of the Higher Order Thinking levels. The prospective mathematics teachers' ability in HOT-level problem posing was analyzed based on three categories of strategy formation, namely: (1) problem reformulation, (2) problem reconstruction, and (3) problem imitation. This research used mix method, with type of research being descriptive quantitative and qualitative, taking 128 students as respondents. The results showed that the proposed HOT-level problems could be categorized as follows: 10.9% are in problem reformulation category, 60.2% in problem reconstruction category, 25.8% in problem imitation category, and 3.1% cannot be categorized.

**Keywords:** Problem Posing · Higher Order Thinking Skills · Reformulation · Reconstruction · Imitation

## 1 Introduction

There are five process skills that must be possessed by students through mathematics learning, namely: (1) problem solving, (2) reasoning, (3) communication, (4) connection, and (5) representation. These five process skills are needed in developing high-level mathematical thinking [1]. Learning mathematics while accessing higher-order thinking skills can help students master 21st-century skills known as 4C skills: critical thinking, creativity, communication, and collaboration [2]. This shows that these five mathematical process skills need to be studied thoroughly and in-depth, in relation to the students' experience. Learning that involves higher-order thinking skills has an impact on the development of thinking processes and can improve students' thinking skills [3].

Research by Prayitno et al. [4] shows that the problem-solving ability of students in mathematics education study program is low. Subsequent research [5] found that there was a high level of geometric misconceptions among students of the mathematics education study program. Meanwhile, students' ability to utilize mathematical representations verbally, visually, and symbolically is in the medium category [6]. The results of this study illustrate that students' higher-order thinking skills need to be improved.

The ability to compose mathematical question instruments is one of the main competencies of prospective mathematics teachers. Considering this competence and the need for the attempts to improve students' higher-order thinking skills, it is necessary for research to try to improve students' ability to submit problems (problem-posing ability) with higher-order thinking (HOT) level. Problem posing competence is the main competency in Mathematics Learning Evaluation course taught by the main researcher. Therefore, the results of this study will have a positive synergy with the improvement of lectures in the Department of Mathematics Education.

The ability to make mathematical problem posing needs to be possessed by teachers and prospective teachers because problem posing is the essence of the nature of mathematical thinking [7]. Teachers are required to be creative in providing a problem that will be done by their students in class. This is because increasing the ability to submit problems will have an impact on the improvement of the quality of learning in the classroom [8].

Problem posing is an approach that requires students to ask questions and make solutions expected to be able to develop thinking skills [9]. The problem posing approach consist of several stages: creating mathematical situations, making math questions, solving math problems, and applying math [7]. Problem posing in learning emphasizes on students' forming or posing questions based on the information or situation provided [10].

Learning mathematics with problem posing has a positive impact on problem solving abilities [8]. Problem posing in mathematics learning is the submission of problems emphasizing the formulation of problems and then solving them based on the situation given to students [11]. Using problem posing, students can develop mathematical abilities or use a mathematical mindset because the questions and solutions are designed by the students themselves. Problem posing includes activities designed by the students themselves and can stimulate all students' abilities so that a better understanding is obtained [12].

Indicators of problem posing ability in this study are classified into 3 categories according to the structure of their formation [13], namely problem reformulation, problem reconstruction, and problem imitation.

## 2 Method

This study used a descriptive quantitative and qualitative research approach. The type of research used was case study research. This research was conducted in the Department of Mathematics Education at one of the universities in Mataram Municipality. The data collection procedure in this study was in the form of written test data that came from the results of student work in solving posing problems. The type of data analysis used in this study was the Milles and Huberman model (in [14]), namely qualitative research data analysis including data reduction, data presentation, and conclusion drawing. At the data reduction stage, the researchers analyzed the results of student work in solving posing problems and their solutions so that they could show students' abilities in posing problems. Furthermore, at the stage of presenting the data, the researchers presented the results of student work for each category. Then in the conclusion stage, the researchers determined the ability of students to pose problems and their solutions.

### 3 Result and Discussion

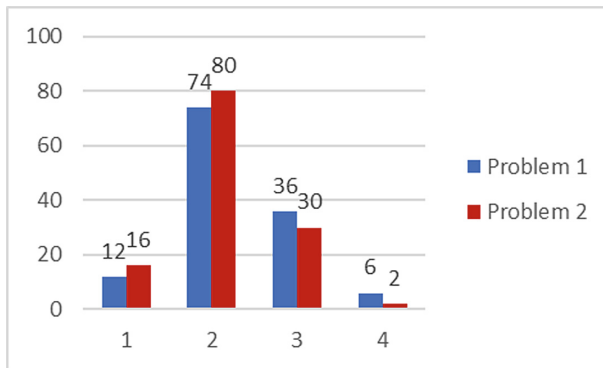
Data collection was taken through an essay test. The test consisted of two questions: geometry and social arithmetic. Geometry problems contained information on two circles and their radii, and the students were asked to arrange based on the problem based on the available information. The social arithmetic problem contained information about the prices of several items and discounts on some items, and the students were asked to make problems based on the available information.

Respondents were 128 students, so the number of answers analyzed was 256. Based on the indicator criteria in Table 1, student answers could be grouped into 4 categories which are presented in Fig. 1.

Based on the data in Fig. 1, the percentage of students in each category is obtained. It is presented in Table 2.

**Table 1.** Indicators of problem posing ability

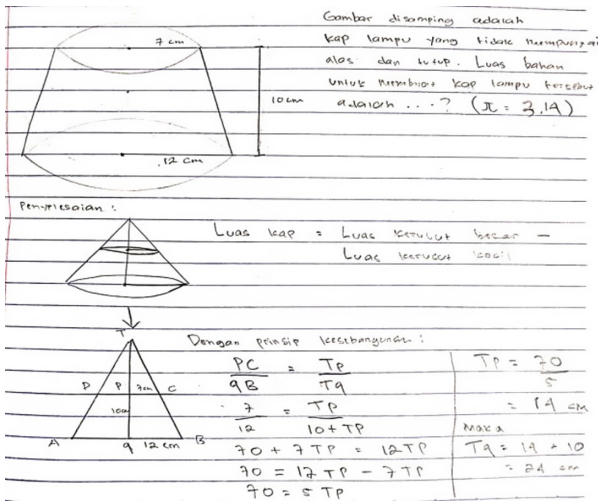
No	Problem posing ability	Indicator
1	Problem Reformulation	<ul style="list-style-type: none"> <li>• Rearranging or directly using the information contained in the original problem</li> <li>• Not changing the information provided</li> </ul>
2	Problem Reconstruction	<ul style="list-style-type: none"> <li>• Modifying the original problem or information provided</li> <li>• Changing the nature of the original problem but not changing the purpose of the problem</li> </ul>
3	Problem Imitation	<ul style="list-style-type: none"> <li>• Formulating problems by adding to the structure of the information provided</li> <li>• Changing the intent of the problem</li> <li>• Relating to other materials and real-life or combining several of these strategies</li> </ul>



**Fig. 1.** Distribution and grouping of problem posing abilities into reformulation (1), reconstruction (2), imitation (3), and uncategorized (4) categories.

**Table 2.** Percentage of student problem posing abilities in each category

Category	Number of Answers (Maximum 256)	Percentage (%)
Reformulation	28	10,9
Reconstruction	254	60,2
Imitation	66	25,8
Uncategorized	8	3,1
Total	256	100



**Fig. 2.** M1 student’s answers to geometry questions in the imitation category

The results of the work of the first student (M1) in solving the first problem are presented in Fig. 2.

Based on the answers in Fig. 2, the researchers analyzed that the M1 student posed a problem by adding an information structure related to the information provided. It can be seen from the problem posed by adding high-value information. In addition, the M1 student also changed the purpose of the problem by adding a high value to the initial information which could eventually be used to find a surface area. M1 student could also relate the initial problem to other materials, namely the initial problem of the plane figure material associated with the geometry material. The formula used by the M1 student in the completion process and the obtained final results were correct. The problems submitted are also categorized as HOTS. Based on the results of this work, the researchers can conclude that the problems posed by the M1 student belong to the category of problem imitation.

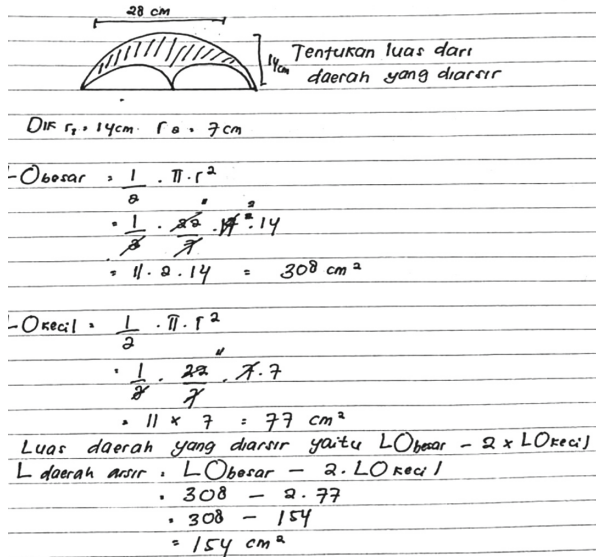


Fig. 3. M2 student's answers to geometry questions for the reconstruction stage

Furthermore, the results of the work of the second student (M2) on the first problem are also presented in Fig. 3.

Based on the answers of the M2 student, the researchers analyzed that M2 posed a problem by modifying the initial problem or the information provided. In addition, M2 also changed the nature of the initial problem but did not change the purpose of the problem, namely linking circle 1 and circle 2, the area of the bounded area in Figure. The formula used by M2 in the completion process and the obtained final results were correct. The problems posed by the M2 student are in the HOTS category. The work of the M2 student in posing problems in the reconstruction category because the proposed problem contains elements of modifying the problem or initial information, and changing the nature of the initial problem but not changing the purpose of the problem.

Furthermore, the example of problem submission by the third student (M3) to the first problem is analyzed in Fig. 4.

Based on the M3 student, the researchers analyzed that M3 did not directly use the initial problem or the information provided in posing a problem. It can be seen in the problem posed by M3 that to determine the value of a number related to the center of a circle, there was incomplete information about the radii of two circles, so the problem posed could be considered incomplete. The formula M3 used in solving process; the process and results were correct, but the problem was less meaningful. Based on the results of M3's work, the researchers conclude that the problem proposed by M3 does not fall into the categories of reformulation, reconstruction, or imitation because the proposed problem does not meet several indicators of the three problem submission categories.

lingkaran yang berpusat di  $(a,b)$ , dengan  $a,b > 5$ , menyinggung garis  $3x + 4y = 12$ . Jika lingkaran tersebut berjari-jari  $12\text{ cm}$

• • maka  $3a + 4b = \dots$

⇒

• Jarak pusat  $(a,b)$  ke garis  $3x + 4y - 12 = 0$  adalah  $12$ .

yaitu jari-jarinya ( $r = 12\text{ cm}$ ) •  $d = 12 \cdot 2 = 24$

$$d = \frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

$$24 = \frac{|3a + 4b - 12|}{\sqrt{3^2 + 4^2}}$$

$$24 = \frac{3a + 4b - 12}{\sqrt{25}}$$

$$24 = \frac{3a + 4b - 12}{5}$$

$$120 = 3a + 4b - 12$$

$$3a + 4b = 120 + 12$$

$$3a + 4b = 132$$

Fig. 4. Student's answers to uncategorized geometry questions

Next, the problem posed by the fourth student (M4) is presented in Fig. 5 in dealing with the second problem concerning social arithmetic.

Based on the work of the M4 student, researchers analyzed that M4 used initial information and did not change the purpose of the problem. The problem includes HOTS, solutions made, processes, and results was correct. Based on this, the problem posed by M4 can be categorized as problem reformulation.

Based on the results of student work in posing mathematical problems, it can be discussed how to categorize the proposed problems based on the structure of their formation. The first category in category posing a problem is problem reformulation. In this category, what the research subject did was posing a problem by directly using the information contained in the initial problem, not changing the information provided. The skills of mathematical problem submission in reformulating problems carried out by students are identifying what is known, what is being asked, and related to concepts in the initial problem correctly [15]. Next, in preparing a problem-making plan, what students do when they are going to create a problem (questions) is that the problem (question) made must be able to be solved, easy questions, and related to the concepts in the existing information.

Problem reconstruction category. In the problem reconstruction category, the research subject can pose a problem by modifying the initial problem, changing the

Ani sedang berbelanja kemeja dan celana panjang dengan masing-masing harga Rp 180.000 dan Rp 225.000. Jika Ani membeli di toko A, Ani akan mendapatkan diskon 30% untuk kemeja. Sedangkan di toko B akan mendapatkan diskon 20% untuk celana panjang. manakah total biaya yang lebih murah jika Ani akan membeli sepuluh pasang? (10 baju dan 10 celana panjang)?

Jawab

Dik:  $P_a$  = harga kemeja = 180.000  
 $P_b$  = harga celana = 225.000

Toko A = Diskon 30% utk baju kemeja  
Toko B = Diskon 20% utk celana panjang

Jawab: Toko A =  $10a + 10b$   
 $= 10 \cdot (180.000 \cdot \frac{70}{100}) + 10 \cdot 225.000$   
 $= 1260.000 + 2.250.000 = 3.510.000$

Toko B =  $10a + 10b$   
 $= 10 \cdot 180.000 + 10 \cdot (225.000 \cdot \frac{80}{100})$   
 $= 1.800.000 + 1.800.000$   
 $= 3.600.000$

maka  $3.600.000 > 3.510.000$   
Jadi Ani harus berbelanja di Toko A.

Fig. 5. M4 student's answers to arithmetic problems in the reformulation category

nature of the initial problem but not changing the purpose/objective of the problem. In line with [15], problem reconstruction is a submission problem strategy when posing a problem that is generated by modifying the initial problem and at the time of modifying it, that is by changing the nature of the problem. Problem imitation category. In this category, the research subject can pose a problem by adding a new information structure and changing the intent/purpose of the problem. Imitation of problems is posing problems resulting from adding a structure related to the information provided by changing the intent and purpose [15]. In this case, students often feel afraid of making mistakes if they create problems by adding new information. Therefore, when the test is given, students tend to create problems by using the existing information directly [16].

There are several research subjects whose problem submissions cannot be categorized, because they do not meet several indicators from the three problem categories.

Quantitative analysis from Table 2 shows that 60.2% of students, equal to a good category, could compile questions in the problem reconstruction category, and 25.8% of students, equal to a very good category, could arrange questions in the imitation problems category. In other words, as many as 86% of students could compose questions in the problem reconstruction and problem imitation category. It shows that the problem posing ability of prospective teacher-students is in the good category.

In the research, the problem posing ability obtained by students is much affected by critical thinking-oriented learning. The results of this study strengthen the view that the development of problem posing requires strong conceptual knowledge that can be

obtained through critical thinking based on the experience of individuals who compose the questions [17]. Problem posing ability for prospective teachers will be very useful in learning because it will have an impact on innovation and improve the quality of learning [18]. Learning mathematics with problem posing is one way to prepare for future mathematics learning [19, 20], so that the problem posing ability of prospective teachers needs to be constantly considered.

## 4 Conclusion

The results showed that the problems posed by prospective mathematics teachers could be categorized as follows: (1) 10.9% reformulation category, (2) 60.2% reconstruction category, (3) 25.8% imitation category, and (4) 3.1% that cannot be categorized. These results indicate that prospective teachers' ability in problem posing is good because 86% of them can develop problem-posing skills in the categories of reconstruction and imitation. The problems posed by them are also categorized as HOTS at the levels of analyzing, evaluating, and creating.

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