



# Modeling and Simulation on Roundabout with Waiting System

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**Abstract.** Roundabouts play a crucial role, especially in the moderately populated areas and places which are largely residential and are in the vicinity of schools where traffic lights are not in place. Though roundabouts can significantly solve the congestion problem on intersection roads, the mobility and time delay depend on the capacity of vehicles. In high traffic moments, the congestion will affect the smooth traffic flow and the queue length on the secondary lane. According to systematic regulations, the cars from the secondary lane need to wait for the cars on the main lane to move forward so as to create a sufficient space to enter the roundabout. In addition, the incoming cars from the main lane have the priority to pass through the arm junction, and the cars on the secondary lane have to make sure these incoming cars are totally driven through the arm junction, before they can be allowed to enter the roundabout. Based on the phenomena mentioned, a long waiting time takes place on the secondary lane. Therefore, a waiting system needs to be installed to facilitate traffic flow on the secondary lane as well as to determine the number of cars crossing the arm junction from the incoming main lane to the outgoing main lane. The plots of Total Travel Time, Total Waiting Time and queue length with different parameters are simulated and discussed. Finally, the results presented in certain simulated situations, produced the tailback on the secondary lane and occasionally, eased the traffic flow due to the mentioned priority regulations.

**Keywords:** Waiting system · Roundabout · Traffic flow · Hyperbolic conservation laws

## 1 Introduction

Nowadays, the congestion problem in moderately populated areas has increased significantly, and it is causing degradation in terms of travel time, traffic safety, fuel consumption and environmental pollution [1]. To overcome this phenomenon, most of the intersections, for instance, T-junctions, across roads or the intersections with traffic lights, have been replaced by roundabouts. This is because roundabouts could convert the intersection into several T-junctions. It significantly helps to ease the traffic flow.

Hence, the construction of roundabouts has also increased. As part of the rules of navigating roundabouts, drivers need to follow the proper flow on the roundabout where cars on the main lane have the priority to pass through the arm junction and towards the desired exit. On the contrary, cars from the secondary lane need to make sure there are no cars from the main lane or only after the cars have passed through the arm junction, then these cars from the secondary lane are allowed to enter the roundabout. Consequently, there is a waiting process on the secondary lane and the tailback may occur.

In 2015, the model of roundabouts was created; however, this model did not take into consideration this issue [2]. In this research, we have included this factor on the secondary lane so that the model can perform in this realistic phenomenon. First of all, the installation of a waiting system is on our four-arm roundabout model where this model is a modification and expansion of their three-arm roundabout model. Our model is more realistic and flexible [3]. The realistic parts of our model are the flow on each arm junction, either the main lane or the secondary lane is different, and the calculation of Total Travel Time and Total Waiting Time involves all the main lanes and secondary lanes. Besides that, our roundabout model is designed to be flexible where the parameters, particularly the incoming flux rate on the secondary lane, the crossing arm junction rate and exiting rate on each arm junction can be set to different scenarios to act like a real phenomenon. Lastly, the waiting time on each arm junction is also distinct.

In order to enable the model to perform in a more realistic setting, especially when the waiting on the secondary lane is concerned, the waiting system needs to be installed into our model. There are many researchers who share similar concerns on this problem and have built delay on their models based on this factor. For example, Flannery presented a renewal-based analytical approach to compute the mean and variance of the time required for an arbitrary in the first position of the approach to enter the roundabout [4]. This model is adopted to real data from several single lane roundabouts in the United States. Moreover, Chang performed the analysis of delay reduction effects on roundabouts based on the entry traffic volume. The study showed that there was a relationship between the frequency of entering the roundabout and the time taken [5].

In 2014, the entrance delay model was created by using the queuing theory [6]. It presented the relationship of delay and queues on the secondary lane. Subsequently, in 2022, Khan modified the HCM 2010 delay model and applied it to the heterogeneous traffic condition in India, which has a mix of vehicle categories with different static and dynamic characteristics [7]. From the model, the HCM model is multiplied by the average control delay in the Indian heterogeneous traffic condition.

As for our model, we have modified the demand and computed the number of cars crossing the arm junction from the incoming main lane to the outgoing main lane. From our four-arm roundabout model, the flux of the outgoing main lane is

$$\begin{aligned}
 f(\rho_{n+1}(0+, t)) &= \min((1 - \beta_n)\delta(\rho_n(0-, t)) \\
 &+ d(F_{in}^n(t), l_n(t)), \sigma(\rho_{n+1}(0+, t))), n = 1, 2, 3, 4.
 \end{aligned}
 \tag{1}$$

where  $\beta$  is the split ratio of exiting the roundabout into the secondary lane,  $\delta(\rho_n)$  is the demand function on the incoming main lane, demand  $d(F_{in}^n, l_n)$  is the flux on the incoming road of the secondary lane, supply function  $\sigma(\rho_n)$  is the flux on the outgoing main lane,  $\rho$  is the mean density, and  $n$  is the number of arm junctions.

As in Eq. (1), the demand is without the waiting system and is given by

$$d(F_{in}^n(t), l_n(t)) = \begin{cases} \gamma_{r1,n}^{\max}, & l_n(t) > 0 \\ \min(F_{in}^n(t), \gamma_{r1,n}^{\max}), & l_n(t) = 0 \end{cases} \quad (2)$$

where  $\gamma_{r1,n}^{\max}$  is the maximal flux exiting the lane into the roundabout,  $F_{in}^n(t)$  is the flux entering the lane, and  $l_n(t)$  is the queue length. Based on the Eq. (1), the flux of the outgoing main lane can be the combination of demand function and demand. To be realistic, the demand should include a built-in waiting system as the demand function has priority to pass through the arm junction. Hence, it is closer to reality by incorporating the waiting system into the roundabout model. To accomplish this waiting system, the calculation of the number of cars is needed to trigger the waiting on demand and the confirmation that the car has passed through the arm junction successfully from the incoming main lane to the outgoing main lane.

## 2 Modeling of Roundabout

Firstly, based on our four-arm roundabout model, it is  $J_n, n = 1, 2, 3, 4$  where each arm is designed as a  $2 \times 2$  circuit arm junction and the main lane,  $I$  and the secondary lane,  $r$  are described by arcs, and arm junctions by vertexes [3]. The main lane is partitioned by the arm junction  $[I_n, I_{n+1}], n = 1, 2, 3, 4$ , and the periodic boundary condition is  $I_5 = I_1$  (Figs. 1 and 2).

The traffic flow on the main lane is

$$\partial_t \rho_n + \partial_x f(\rho_n) = 0, (x, t) \in \mathbb{R}^+ \times I_n, n = 1, 2, 3, 4 \quad (3)$$

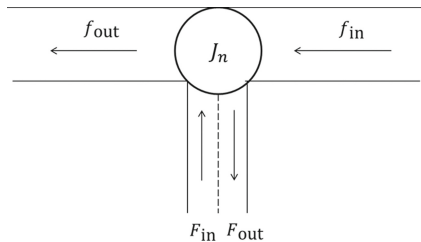


Fig. 1. Arm Junction of Roundabout.

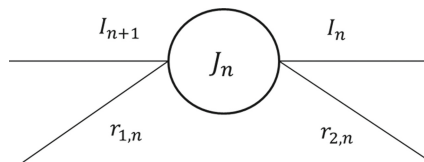


Fig. 2. Corresponding arm junction.

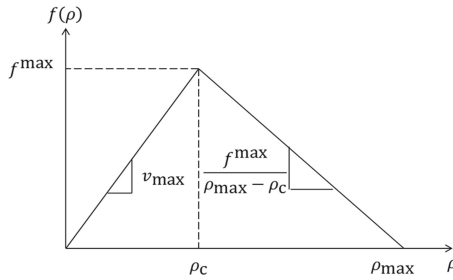


Fig. 3. The rational flux-density relationship graph.

where  $\rho_n = \rho_n(x, t) \in [0, \rho_{\max}]$  is the mean traffic density,  $\rho_{\max}$  is the maximal density on the road and the flux function  $f : [0, \rho_{\max}] \rightarrow \mathbb{R}^+$  is given by the flux-density relation.

$$f(\rho) = \begin{cases} \rho v_{\max}, & 0 \leq \rho \leq \rho_c \\ \frac{f^{\max}}{\rho_{\max} - \rho_c} (\rho_{\max} - \rho), & \rho_c \leq \rho \leq \rho_{\max} \end{cases} \quad (4)$$

In the above equation,  $v_{\max}$  is the maximal traffic speed,  $\rho_c = \frac{f^{\max}}{v_{\max}}$  is the critical density and  $f^{\max} = f(\rho_c)$  is the maximal flux value. For simplicity, we set the fixed constants as  $\rho_{\max} = 1$  and  $v_{\max} = 1$  (Fig. 3).

Meanwhile, the flux on the secondary lane entering the arm junction is assigned with a buffer of infinite size and capacity so as to prevent backward moving shocks on the road. Hence, the queue takes place and the queue length is represented by

$$\frac{dl_n(t)}{dt} = F_{\text{in}}^n(t) - \gamma_{r1,n}(t), t \in \mathbb{R}^+, n = 1, 2, 3, 4 \quad (5)$$

Regarding the queue length equation,  $l_n(t) \in [0, \infty]$  is the queue length,  $F_{\text{in}}^n(t)$  is the flux entering the secondary lane, and  $\gamma_{r1,n}(t)$  is the flux exiting the secondary lane and entering the roundabout.

### 3 Flux on Arm Junction

The flux on the arm junction can be classified by both the incoming and outgoing fluxes. The flux of the incoming main lane is

$$f(\rho_n(0-, t)) = f(\rho_{n+1}(0+, t)) + \gamma_{r2,n}(t) - \gamma_{r1,n}(t), n = 1, 2, 3, 4 \quad (6)$$

In the equation above,  $\gamma_{r2,n}(t)$  is the flux exiting the roundabout and is given by

$$\gamma_{r2,n}(t) = \beta_n f(\rho_n(0-, t)), n = 1, 2, 3, 4 \quad (7)$$

where  $\beta \in [0, 1]$  is the supply ratio of the outgoing secondary lane  $r_{2,n}$ . The flux of the outgoing main lane is referred to as Eq. (1).

According to the traffic flow on the roundabout, the cars from the incoming secondary lane need to ensure the clearance on the arm junction or maintain a certain safety gap,

before the cars are allowed to enter the roundabout. In fact, from the Eq. (2), the demand  $d(F_{in}^n, l_n)$  on the incoming secondary lane needs to be modified with the waiting system, as shown in the Eq. (8) below. There are two modes in this equation: ‘On’ and ‘Off’ modes, where the triggering is totally dependent on the calculation of the number of cars crossing the arm junction.

$$d(F_{in}^n, l_n) = \begin{cases} 0, & \text{on} \\ \gamma_{r1,n}^{\max}, & \text{off and } l_n(t) > 0 \\ \min(F_{in}^n(t), \gamma_{r1,n}^{\max}), & \text{off and } l_n(t) = 0 \end{cases} \quad (8)$$

### 3.1 Triggering of Waiting on Arm Junction

The computation of fluxes on the arm junction can be calculated in two different situations, which are Demand-limited case and Supply-limited case. From this process,  $\Gamma_{1_n}^t$  and  $\Gamma_{r1_n}^t$  are computed, which are the flux of the incoming main lane and the flux entering the roundabout, respectively. Therefore,  $(1 - \beta_n)\Gamma_{1_n}^t$  is the flux coming from the main lane and crossing the arm junction to the outgoing main lane.

The total number of cars can be calculated by

$$\int_{t_n}^{t_{n+1}} (1 - \beta)\Gamma_1 dt \quad (9)$$

In our setting, the triggering of waiting is as below,

$$\text{triggering} = \begin{cases} \text{Off, } N_{car} \in [0, 0.5) \\ \text{On, } N_{car} \in [0.5, 1] \end{cases} \quad (10)$$

In the Eq. (10), the triggering of ‘on’ and ‘off’ modes is based on the calculation of the number of cars, from the Eq. (9). For instance, when the number of cars is between 0 and 0.5, it describes the gap of the car from the incoming main lane and arm junction is big enough, yet it has to be closer to the arm junction, so that it allows the car from the secondary lane to enter the roundabout. In this situation, the ‘off’ mode is triggered, as in the Eq. (10). Inversely, for the total number of cars between 0.5 and 1, it means a car from the incoming main lane is approaching the arm junction and crossing the arm junction. The gap for the car from the secondary lane to enter the roundabout is small enough. At this time, the car from the secondary lane needs to wait for the car to have completely passed through the arm junction, before the car on the secondary lane is allowed to enter the roundabout. Consequently, it will trigger the ‘on’ mode in the Eq. (10), then incorporate it with the Eq. (8) to trigger the ‘on’ mode as well. Once the car has successfully crossed the arm junction, it means the total area integration is 1, which satisfies the Eq. (9). Then the new calculation of the total number of cars will begin, as shown in Fig. 4. In the next calculation, the last point at time,  $t_{n+1}$  in the previous calculation will be treated as  $t_n$ .

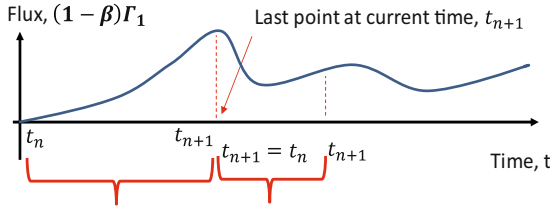


Fig. 4. The area integration for the calculation of the number of cars.

### 4 Numerical Simulations Setting

Several parameters must be considered to conduct the simulation, for instance, the flux on the secondary lane,  $F_{in} \in [0, 1]$ , exiting the roundabout rate,  $\beta \in [0, 1]$  and crossing the arm junction rate from the secondary lane to the main lane or entering the roundabout rate,  $P \in [0, 1]$ . Besides that, the initial conditions, that is, all the roads and the buffers are empty, are reflected as  $\rho_{n,0} = 0$ ,  $F_{in}^n \neq 0$  and  $l_{n,0} = 0$  where  $n = 1, 2, 3$  and  $4$ . It also includes some fixed constants where  $f^{max} = 0.66$ ,  $\rho_c = 0.66$ , and  $\gamma_{r1}^{max} = 0.65$ . Lastly, the simulation on both the three-unit length and four-unit length circumferences of the roundabout with the total time and space step will be  $T = 50$  and  $\Delta x = 0.1$ , respectively [3].

### 5 Optimization on Roundabout

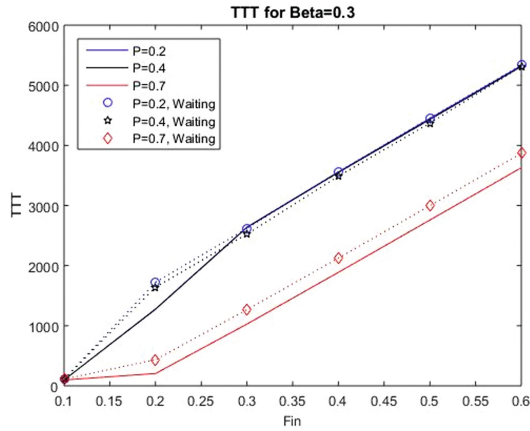
The calculations of Total Travel Time (*TTT*) on the road network and Total Waiting Time (*TWT*) at the entrance of the secondary road are the inputs for analysing the effectiveness of the waiting system. The equations are as follows [3],

$$\begin{aligned}
 TTT &= \sum_{n=1}^N \int_0^T \int_{I_n} \rho(x, t) dx dt + \sum_{n=1}^N \int_0^T l_n(t) dt + T \\
 &\cdot \sum_{n=1}^N \int_{I_n} \rho(x, T) dx + T \cdot \sum_{n=1}^N l_n(T)
 \end{aligned}
 \tag{11}$$

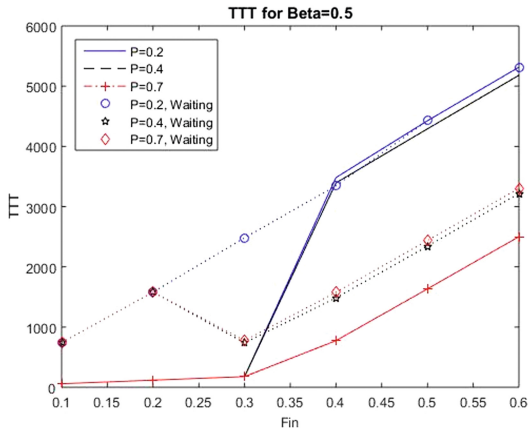
$$TWT = \sum_{n=1}^N \int_0^T l_n(t) dt + T \cdot \sum_{n=1}^N l_n(T)
 \tag{12}$$

### 6 Results and Discussions

In order to observe the difference between the non-waiting and waiting systems, we simulate both results and analyze the effectiveness on them. Figures 5, 6 and 7 and Figs. 8, 9 and 10 represent the comparison between the non-waiting and waiting plots of Total Travel Time and Total Waiting Time for the three-arm roundabout with the 3-unit circumference, respectively. In addition, these plots from Figs. 5, 6, 7, 8, 9 and 10 are also based on the comparison with Obsu. et. al.’s results [2]. Thus, for the rate of entering

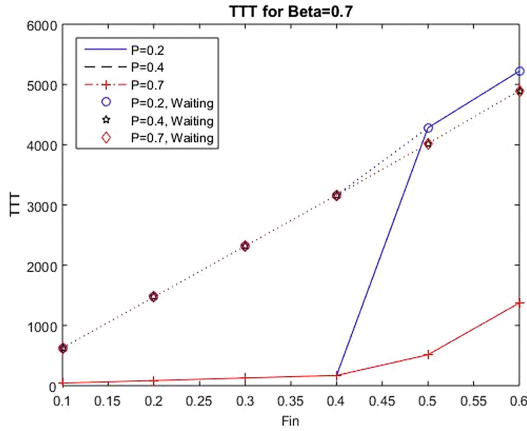


**Fig. 5.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.3$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.

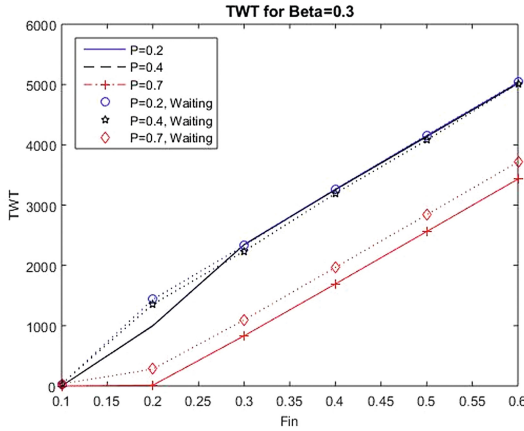


**Fig. 6.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.5$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.

the roundabout,  $P$  we have set it according to the setting from Obsu et. al.’s simulation. We can see that the plots increased smoothly as  $F_{in}$  increased. The results show the reasonableness where the waiting system results are higher than that of the non-waiting system. Another point of view is that the Total Waiting Time values are not totally zero as the  $F_{in}$  is low. This is because the waiting is guaranteed to take place although there are just a few cars on the secondary lane. It fulfils the traffic flow regulations that the car on the secondary lane needs to wait until the car from the incoming main lane has passed through the arm junction, and only then can the car from the secondary lane be allowed to enter the roundabout. In fact, there is a guarantee that the waiting time takes place (Figs. 11, 13, 14, 15 and 16).



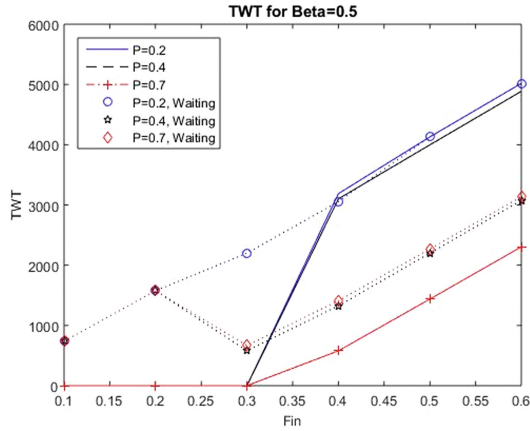
**Fig. 7.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.7$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.



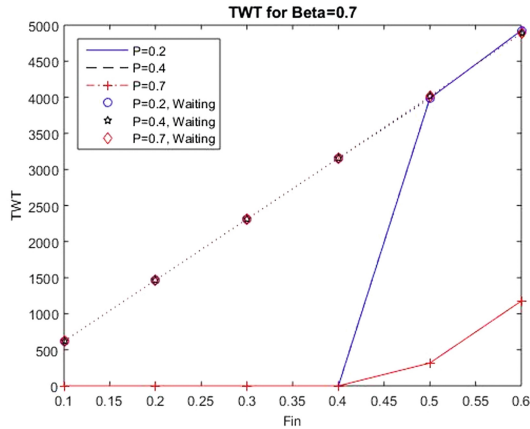
**Fig. 8.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.3$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.

For some cases, an example of the Total Travel Time for both three-arm and four-arm roundabouts, is set as  $\beta = 0.5$  and  $P = 0.4$  in Fig. 6 and Fig. 12, respectively. It shows the traffic flow is smoother than the non-waiting one. This is because the waiting system assumes drivers obey the regulations, in which the incoming main lane drivers have priority to cross the arm junction. Thus, it avoids the heavy density on the roundabout.



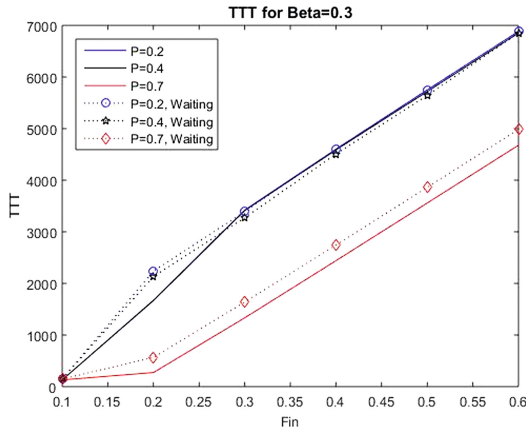


**Fig. 9.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.5$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.

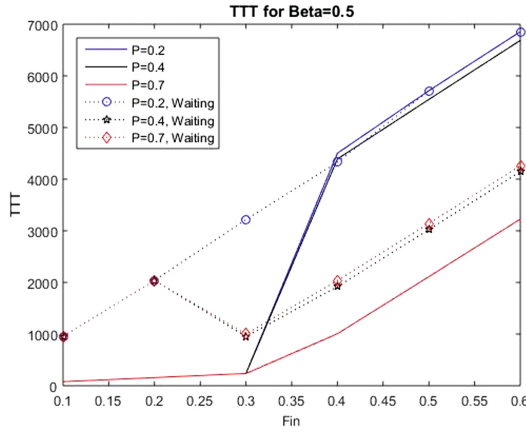


**Fig. 10.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.7$  and various values of  $P$  for the three-arm roundabout with a 3-unit circumference.

Meanwhile, Fig. 17 and Fig. 18 represent the plot of Total Travel Time and Total Waiting Time versus the various values of  $\beta$ . According to the results, the plot is decreasing while the value of  $\beta$  is increasing, which indicates the flux exiting the roundabout rate is increased. In addition, the higher the value of  $P$ , the higher the flux entering the roundabout. Based on the two criteria mentioned, the smoothness of traffic flow is high.

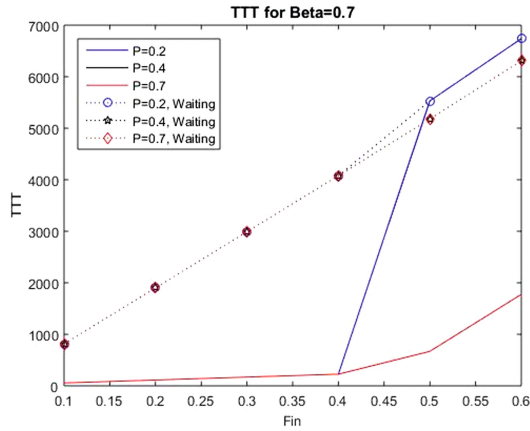


**Fig. 11.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.3$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

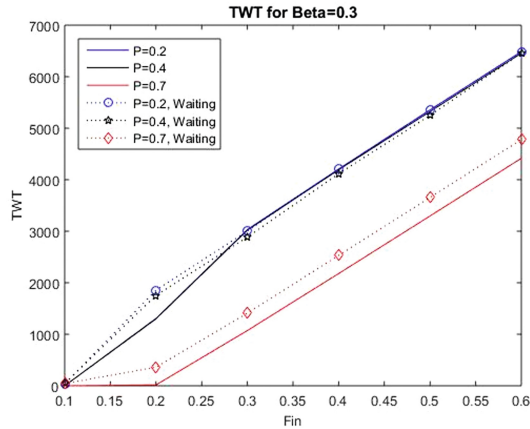


**Fig. 12.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.5$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

The Fig. 19 demonstrates the plot of density on the main lane segment of the roundabout whereas Fig. 20 represents the Queue Length versus Time on the secondary lane. For this four-arm roundabout with a 4-unit circumference simulation, we set the distinct parameters for  $F_{in}$ ,  $\beta$  and  $P$  on each arm junction in order to perform in a realistic situation. In the setting, we set the  $F_{in}^2 = 0.8$ ,  $P_2 = 0.2$  and  $\beta_3 = 0.8$ , which means the more fluxes from the secondary lane of the second arm, the lower the rate of entering the

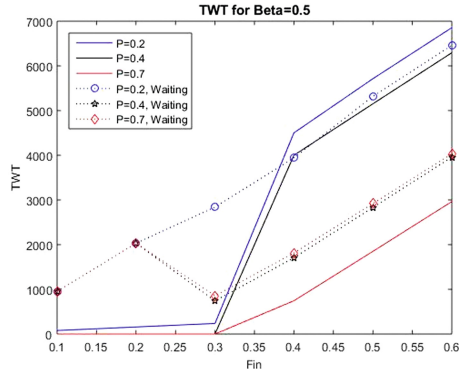


**Fig. 13.** Plot of Waiting System and Non-Waiting System of Total Travel Time versus  $F_{in}$  with  $\beta = 0.7$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

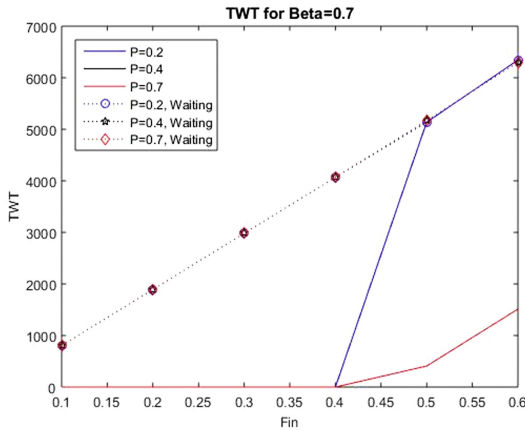


**Fig. 14.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.3$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

roundabout on the second arm junction, making fewer fluxes from the secondary lane of the second arm to enter the roundabout even though the rate of exiting the roundabout on the third arm junction is high. It caused the longest tailback on the incoming road of the second arm and the low density on the second segment of the main lane at the roundabout, as shown in Fig. 19. Conversely, the queue length on the first arm was the shortest because the flux from the secondary lane was just 0.3. In addition, the rate of entering the roundabout was 0.5, higher than the flux on the secondary lane; thus, it had the shortest queue length. Furthermore, for the third arm, the settings were  $F_{in}^3 = 0.7$ ,  $P_3 = 0.3$  and  $\beta_4 = 0.2$ , and the flux supplied from the secondary lane was lower and the rate of crossing the arm junction was higher than what we applied on the second junction. However, the rate of exiting the roundabout was low, hence the heaviest density took

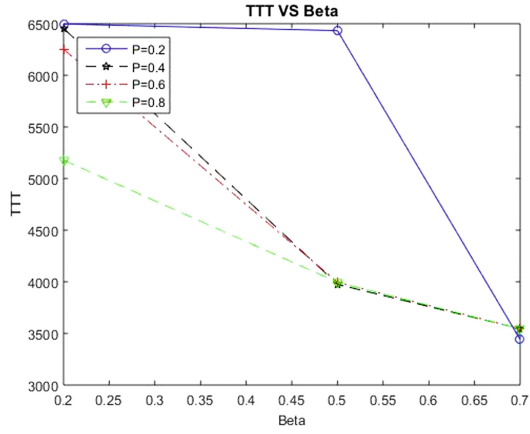


**Fig. 15.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.5$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

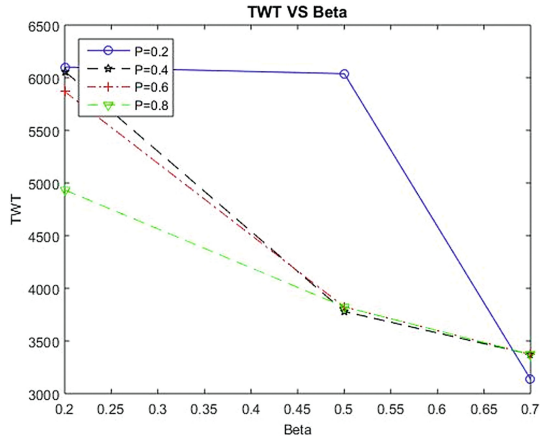


**Fig. 16.** Plot of Waiting System and Non-Waiting System of Total Waiting Time versus  $F_{in}$  with  $\beta = 0.7$  and various values of  $P$  for the four-arm roundabout with a 4-unit circumference.

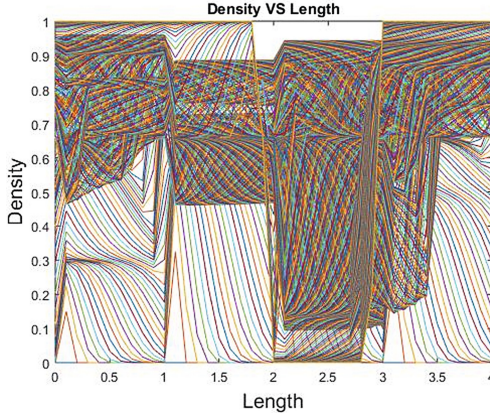
place on the third segment of the main lane at the roundabout. Lastly, Table 1 presents the comparison of queue lengths between the non-waiting system and waiting system for the parameters of the traffic situation set as similar parameters in Fig. 19 and Fig. 20 at  $T = 50$ . As a result, the queue length on all four arm junctions were reduced; it has reflected the effectiveness of the waiting system.



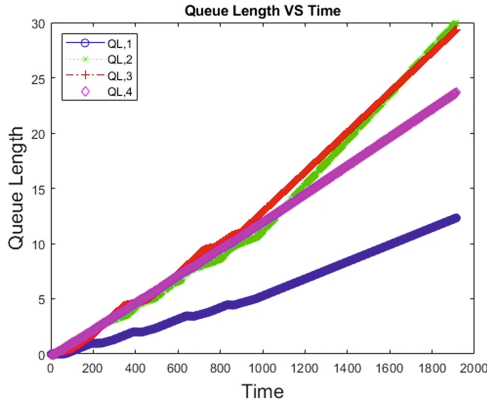
**Fig. 17.** Plot of Waiting System of Total Travel Time versus  $\beta$  with various values of  $P$  for the four-arm roundabout with a 4-unit circumference.



**Fig. 18.** Plot of Waiting System of Total Waiting Time versus  $\beta$  with various values of  $P$  for the four-arm roundabout with a 4-unit circumference.



**Fig. 19.** Plot of Density versus Length with  $F_{in}^1 = 0.3, F_{in}^2 = 0.8, F_{in}^3 = 0.7, F_{in}^4 = 0.5,$   
 $\beta_1 = 0.3, \beta_2 = 0.3, \beta_3 = 0.8, \beta_4 = 0.2, P_1 = 0.5, P_2 = 0.2, P_3 = 0.3, P_4 = 0.8$  for the  
four-arm roundabout with a 4-unit circumference.



**Fig. 20.** Plot of Queue Length versus Time with  $F_{in}^1 = 0.3, F_{in}^2 = 0.8, F_{in}^3 = 0.7, F_{in}^4 = 0.5,$   
 $\beta_1 = 0.3, \beta_2 = 0.3, \beta_3 = 0.8, \beta_4 = 0.2, P_1 = 0.5, P_2 = 0.2, P_3 = 0.3, P_4 = 0.8$  for the  
four-arm roundabout with a 4-unit circumference.

**Table 1.** Effectiveness of the Waiting System.

Arm of Roundabout	Queue Length (Unit)	
	Non-Waiting System	Waiting System
1 <sup>st</sup> – Arm	<b>14.0177</b>	<b>12.3153</b>
2 <sup>nd</sup> – Arm	<b>37.3566</b>	<b>29.9766</b>
3 <sup>rd</sup> – Arm	<b>32.9144</b>	<b>29.3398</b>
4 <sup>th</sup> – Arm	<b>24.1601</b>	<b>23.6530</b>

## 7 Conclusions

In this paper, we have created a waiting system on our roundabout model with the installation on demand from the non-waiting system model. This model involves the calculation of the number of cars which compute the integration of flux rather than a delay differential equation. Thus, this model is more realistic. The simulation results show the comparison and effect of non-waiting and waiting models. The plot of waiting model is set in a more realistic situation, and this enhances the smoothness of traffic flow. Furthermore, in the Total Waiting Time plots, there is supposed to be a waiting system even though the  $F_{in}$  rates are low. Therefore, the waiting system on demand is needed and should be applied.

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**Author's Contributions.** The model of the roundabout with a waiting system is created to adapt the real traffic flow on the roundabout. It is possible to study the effectiveness of traffic flow by the calculation of Total Travel Time and Total Waiting Time because in certain conditions, the traffic flow is either smoother or heavier.

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