Neutrosophic Semi-alpha Continuous Mappings in Neutrosophic Topological Spaces

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Abstract. In 2017, Qays Hatem Imran, F. Smarandache, Riad K. Al-Hamido, and R. Dhavaseelan presented the concept of neutrosophic open sets called neutrosophic semi-α-open sets and studied their fundamental properties in neutrosophic topological spaces. He also presented neutrosophic semi-α-interior and neutrosophic semi-α-closure and studied some of their fundamental properties. In this paper, we introduce the concepts of neutrosophic semi−α−continuous mappings, neutrosophic contra semi−α−continuous mappings, and contra semi−α−irresolute mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.

Keywords: Neutrosophic topological space · Neutrosophic *semi−α−open* set · Neutrosophic *semi−α−closed* set · Neutrosophic *semi−α−continuous* mapping · Neutrosophic *semi−α−irresolute* mapping · Neutrosophic contra *semi−α−continuous* mapping · Neutrosophic contra *semi−α−irresolute* mapping

1 Introduction

The following studies presented some of the researches based on the neutrosophic mapping topological spaces. First of all, Abbas and Khalil [1] investigated neutrosophic continuous and contra mappings while Arokiarani et al. [2] added new notions and functions. Then, Atkinwesley et al. [3] introduced neutrosophic g*-closed sets and g*-maps while Banu and Chandrasekar [4] investigated neutrosophic αgs-continuous and neutrosophic αgs-irresolute maps. Further study was done by Damodharan et al. [5] for $N_{\delta *g\alpha}$ – continuous and $N_{\delta *g\alpha}$ – irresolute functions as well as Maheshwari and Chandrasekar [8] for neutrosophic gb-closed sets and neutrosophic gb-continuity. Narmatha et al. [9] researched on the neutrosophic πgβ-closed sets and neutrosophic πgβ-mappings and continued by Puvaneshwari and Bageerathi [10] who presented neutrosophic mappings concerning Feebly open sets and Feebly closed sets. Rajesh and Chandrasekar [11] investigated neutrosophic Pre-α, Semi-α and Pre-β irresolute, open and closed mappings. In addition, Salama et al. [12] presented neutrosophic closed sets and neutrosophic continuous functions whereaa Vadivel and Sundar [13] investigated neutrosophic e-continuous maps and neutrosophic e-irresolute maps in neutrosophic topological spaces. In 2017, Imran et al. [6] presented the concept of neutrosophic open sets called neutrosophic semi-α-open sets and studied their fundamental properties in neutrosophic topological spaces.

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They also presented neutrosophic semi-α-interior and neutrosophic semi-α-closure and studied some of their fundamental properties. In this paper, we introduce the concepts of neutrosophic semi–α–continuous mappings, neutrosophic contra semi–α–continous, and contra semi–α–irresolute mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.

2 Preliminaries

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set P is an object having the form $P = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$, where $\mu_P(x)$ − represents the degree of membership, $\sigma_P(x)$ − represents the degree of indeterminacy, and $\gamma_P(x)$ − represents the degree of non-membership. The class of all neutrosophic sets in X will be denoted by $\mathrm{N}(X)$.

Definition 2.2. Let X be a non-empty set and let $P = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$ and $Q = \{\langle x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x)\rangle : x \in X\}$ be two neutrosophic sets, *Then* 1. (*Empty set*) $0_N = \langle x, 0, 0, 1\rangle$ is called the neutrosophic empty set, 2. (*Universal set*) $1_N = \langle x, 1, 1, 0\rangle$ is called the neutrosophic universal set. 3. (Inclusion): $P \subseteq Q$ if and only if $\mu_P(x) \le \mu_Q(x), \sigma_P(x) \le \sigma_Q(x)$ and $\gamma_P(x) \ge \gamma_Q(x)$:$x \in X$, 4. (Equality): $P = Q$ if and only if $P \subseteq Q$ and $Q \subseteq P$, 5. (*Complement*) $P^C = 1_N - P = \{\langle x, \gamma_P(x), 1-\sigma_P(x), \mu_P(x)\rangle : x \in X\}$, 6. (*Union*) $P \cup Q = \{\langle x, \max(\mu_P(x), \mu_Q(x)), \max(\sigma_P(x), \sigma_Q(x)), \min(\gamma_P(x), \gamma_Q(x))\rangle : x \in X\}$. 7. (*Intersection*) $P \cap Q = \{\langle x, \min(\mu_P(x), \mu_Q(x)), \min(\sigma_P(x), \sigma_Q(x)), \max(\gamma_P(x), \gamma_Q(x))\rangle : x \in X\}$.

Definition 2.3. A neutrosophic point $x_{(\alpha,\beta,\gamma)}$ is said to be in the neutrosophic set A- in symbols $x_{(\alpha,\beta,\gamma)} \in A$ if and only if $\alpha < \mu_A(x)$, $\beta < \sigma_A(x)$ and $\gamma > \gamma_A(x)$.

Definition 2.4. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying $(i)\, 0_N, 1_N \in \mathrm{T}_N$. (ii) $G \cap H \in \mathrm{T}_N$ for every G, $H \in \mathrm{T}_N$, (iii) $\bigcup_{j \in J} G_j \in \mathrm{T}_N$ for every $\{G_j : j \in J\} \subseteq \tau_N$. Then the pair (X, T_N) is called a neutrosophic topological space. The elements of T_N are called neutrosophic open sets in X. A neutrosophic set A is called a neutrosophic closed set iff its complement A^C is neutrosophic open set.

Definition 2.5. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set. Then (i) The neutrosophic closure of A (*briefly* $\mathrm{N}\,Cl(A)$) is the intersection of all neutrosophic closed sets containing A. (ii) The neutrosophic interior of A, denoted by $\mathrm{N}\,Int(A)$ is the union of all neutrosophic open subsets of A.

Definition 2.6. A neutrosophic subset A of a neutrosophic topological space (X, T_N) is said to be a neutrosophic semi−open set (briefly NS−OS) if $A \subseteq \mathrm{N}\,Cl[\mathrm{N}\,Int(A)]$. The complement of a NS−OS is called a neutrosophic semi−closed set (briefly NS−CS) in (X, T_N). The family of all NS−OS (resp. NS−CS) of X is denoted by $NSO(X)$ (*resp.* $NSC(X)$).

Definition 2.7. A neutrosophic subset A of a neutrosophic topological space (X, T_N) is said to be a neutrosophic α−open set (briefly Nα−OS) if $A \subseteq \mathrm{N}\,Int[\mathrm{N}\,Cl(\mathrm{N}\,Int(A))]$. The complement of a Nα−OS is called a neutrosophic α − closed set (briefly Nα−CS) in (X, T_N). The family of all Nα−OS(resp. Nα − CS) of X is denoted by $N\alpha O(X)$ $(resp.\ N\alpha C(X))$.

Definition 2.8. The neutrosophic α−interior of a neutrosophic set A of a neutrosophic topological space (X, T_N) is the union of all Nα−OS contained in A and is denoted by $\mathrm{N}\,\alpha Int(A)$.

Definition 2.9. The neutrosophic α−closure of a neutrosophic set A of neutrosophic topological space (X, T_N) is the intersection of all Nα−CS that contains A and is denoted by $\mathrm{N}\,\alpha Cl(A)$.

Definition 2.10. A neutrosophic subset A of a neutrosophic topological space (X, T_N) is said to be a neutrosophic semi−α−open set (briefly NSα−OS) if there exists a Nα−OSH in X such that $H \subseteq A \subseteq \mathrm{N}\,Cl(H)$ or equivalently if $A \subseteq \mathrm{N}\,Cl[\mathrm{N}\,\alpha Int(A)]$. The family of all NSα−OS of X is denoted by $NS\alpha O(X)$.

Definition 2.11. The complement of a NSα−OS is called a neutrosophic semi−α−closed set (briefly NSα−CS) in (X, T_N). The family of all NSα−CS of X is denoted by $NS\alpha C(X)$.

Proposition 2.12. Let (X, T_N) be a neutrosophic topological space. Then (i) Every N−OS (resp. N−CS) is a NSα−OS (resp. NSα−CS). (ii) Every Nα−OS (resp. Nα−CS) is a NSα−OS (resp. NSα−CS).

Theorem 2.13. Let (X, T_N) be a neutrosophic topological space. Then (i) The union of any family of NSα−OS is a NSα−OS (ii) The intersection of any family of NSα−CS is a NSα−CS.

Definition 2.14. The union of all NSα−OS in a neutrosophic topological space (X, T_N) contained in a neutrosophic set $A \in \mathrm{N}(X)$, is called neutrosophic semi−α−interior of A and is denoted by $\mathrm{N}\,S\alpha Int(A)$, $\mathrm{N}\,S\alpha Int(A) = \cup\{B : B \subseteq A,\ \ B\ is\ a\ NS\alpha{-}OS\}$.

Definition 2.15. The intersection of all NSα−CS in a neutrosophic topological space (X, T_N) containing a neutrosophic set $A \in \mathrm{N}(X)$, is called neutrosophic semi−α−closure of A and is denoted by $\mathrm{N}\,S\alpha Cl(A)$, $\mathrm{N}\,S\alpha Cl(A) = \cap\{B : A \subseteq B,$ $B\ is\ a\ NS\alpha{-}CS\}$.

Definition 2.16. The neutrosophic semi−α−frontier of a neutrosophic subset A of a neutrosophic topological space (X, T_N) is denoted by $\mathrm{N}\,S\alpha Fr(A)$ and $\mathrm{N}\,S\alpha Fr(A) = \mathrm{N}\,S\alpha Cl(A) \cap \mathrm{N}\,S\alpha Cl\left(A^C\right)$.

3 Neutrosophic Semi-α-Continous Mappings

In this section, we introduce the concept of neutrosophic semi−α−continuous mappings in neutrosophic topological spaces. Also, we study some of the main results depending on neutrosophic semi−α−open sets.

Definition 3.1. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a mapping. Then f is called a neutrosophic semi−α−continuous mapping if $f^{-1}(V)$ is a neutrosophic semi−α−open set in X for every neutrosophic open set V in Y.

Theorem 3.2. Every neutrosophic continuous mapping is neutrosophic semi−α−continuous mapping.

Proof. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be neutrosophic continuous mapping. Let V be a neutrosophic open set in (Y, σ_N). Then $f^{-1}(V)$ is neutrosophic open set in (X, T_N). Since every neutrosophic open set is neutrosophic semi−α−open $f^{-1}(V)$ is neutrosophic semi−α−open set in (X, T_N). Hence f is neutrosophic semi−α−continuous mapping.

Theorem 3.3. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ and $g : (Y, \sigma_N) \to (Z, \eta_N)$ be neutrosophic semi−α−continuous mappings. Then $gof : (X, \mathrm{T}_N) \to (Z, \eta_N)$ is a neutrosophic semi−α−continuous mapping.

Proof. Let G be a neutrosophic open set in Z. Since $g : (Y, \sigma_N) \to (Z, \eta_N)$ is neutrosophic continuous, $f^{-1}(G)$ is neutrosophic open set in Y. Since f is a neutrosophic semi−α−continuous mapping, $f^{-1}\left[f^{-1}(G)\right]$ is neutrosophic semi−α−open in X. But $f^{-1}\left[g^{-1}(G)\right] = (gof)^{-1}(G)$. Then $(gof)^{-1}(G)$ is neutrosophic semi−α−open set in X. Hence, gof is a neutrosophic semi−α−continuous mapping.

Theorem 3.4. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. Then prove that a mapping $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is neutrosophic semi−α−continuous if and only if $f^{-1}(B)$ is neutrosophic semi−α−closed set in X for every neutrosophic closed set B in Y.

Proof. Let B be a neutrosophic closed set in Y. Then B^C is neutrosophic open set in Y. Since f is neutrosophic semi−α−continuous. Therefore $f^{-1}\left(B^C\right)$ is a neutrosophic semi−α−open set in X. Since $f^{-1}\left(B^C\right) = \left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is neutrosophic semi−α−closed set in X.

Conversely, Let B be a neutrosophic open set in Y. Then B^C is neutrosophic closed set in Y. By assumption $f^{-1}\left(B^C\right)$ is neutrosophic semi−α−closed set in X. Since $f^{-1}\left(B^C\right) = \left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is neutrosophic semi−α−open set in X. Hence f is neutrosophic semi−α−continuous.

Theorem 3.5. Suppose that $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is a mapping. Then f is a neutrosophic semi−α−continuous mapping if and only if $f(\mathrm{N}S\alpha Cl(A)) \subseteq \mathrm{N}S\alpha Cl(f(A))$ for every neutrosophic set A in X.

Proof. Let A be a neutrosophic set in X and f be a neutrosophic semi−α−continuous mapping. Then evidently $f(A) \subseteq \mathrm{N}S\alpha Cl[f(A)]$. Now, $A \subseteq f^{=1}[f(A)] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(f(A))]$ and $\mathrm{N}S\alpha Cl(A) \subseteq \mathrm{N}S\alpha Cl[f^{-1}(\mathrm{N}S\alpha Cl(f(A)))]$. Since f is a neutrosophic semi−α−continuous mapping and $\mathrm{N}S\alpha Cl[f(A)]$ is a neutrosophic semi−α−closed set. Thus $\mathrm{N}S\alpha Cl[f^{-1}(\mathrm{N}S\alpha Cl(f(A)))] = f^{-1}[\mathrm{N}S\alpha Cl(f(A))]$. Hence, $f[\mathrm{N}S\alpha Cl(A)] \subseteq \mathrm{N}S\alpha Cl[f(A)]$.

Conversely, let $f[\mathrm{N}S\alpha Cl(A)] \subseteq \mathrm{N}S\alpha Cl[f(A)]$, for each neutrosophic set A in X. Let F be a neutrosophic closed set in Y. Then $\mathrm{N}S\alpha Cl[f(f^{-1}(F))] \subseteq \mathrm{N}S\alpha Cl(F) = F$. By assumption, $f[\mathrm{N}S\alpha Cl(f^{-1}(F))] \subseteq \mathrm{N}S\alpha Cl[f(f^{-1}(F))] \subseteq F$ and hence $\mathrm{N}S\alpha Cl[f^{-1}(F)] \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq \mathrm{N}S\alpha Cl[f^{-1}(F)]$, $\mathrm{N}S\alpha Cl[f^{-1}(F)] = f^{-1}(F)$. This implies that $f^{-1}(F)$ is a neutrosophic semi−α−closed set in X. Thus by Theorem 3.4, f is a neutrosophic semi−α−continuous mapping.

Theorem 3.6. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a mapping. Then f is a neutrosophic semi−α−continuous mapping if and only if $\mathrm{N}S\alpha Cl[f^{-1}(B)] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(B)]$ for every neutrosophic set B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic semi−α−continuous mapping. Clearly $f^{-1}(B) \subseteq f^{-1}[\mathrm{N}S\alpha Cl(B)]$. Then, $\mathrm{N}S\alpha Cl[f^{-1}(B)] \subseteq \mathrm{N}S\alpha Cl[f^{-1}(\mathrm{N}S\alpha Cl(B))]$. Since $\mathrm{N}S\alpha Cl(B)$ is neutrosophic semi−α−closed set in Y. So by Theorem 3.4, $f^{-1}[\mathrm{N}S\alpha Cl(B)]$ is a neutrosophic semi−α−closed set in X. Thus, $\mathrm{N}S\alpha Cl[f^{-1}(B)] \subseteq \mathrm{N}S\alpha Cl[f^{-1}(\mathrm{N}S\alpha Cl(B))] = f^{-1}[\mathrm{N}S\alpha Cl(B)]$.

Conversely, $\mathrm{N}S\alpha Cl[f^{-1}(B)] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(B)]$ for all neutrosophic sets B in Y. Let F be a neutrosophic closed set in Y. Since every neutrosophic closed set is neutrosophic semi−α−closed set, $\mathrm{N}S\alpha Cl[f^{-1}(F)] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(F)] = f^{-1}(F)$. This implies that $f^{-1}(F)$ is a neutrosophic semi−α−closed set in X. Thus by Theorem 3.4, f is a neutrosophic semi−α−continuous mapping.

Theorem 3.7. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a bijective mapping. Then f is neutrosophic semi−α−continuous if and only if $\mathrm{N}S\alpha Int[f(A)] \subseteq f[\mathrm{N}S\alpha Int(A)]$ for every neutrosophic set A in X.

Proof. Let A be any neutrosophic set in X and f be a bijective and neutrosophic semi−α−continuous mapping. Let $f(A) = B$. Clearly $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq f^{-1}(B)$. Since f is an injective mapping, $f^{-1}(B) = A$, so that $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq A$. Therefore, $\mathrm{N}S\alpha Int[f^{-1}(\mathrm{N}S\alpha Int(B))] \subseteq \mathrm{N}S\alpha Int(A)$. Since f is neutrosophic semi−α−continuous $f^{-1}[\mathrm{N}S\alpha Int(B)]$ is neutrosophic semi−α−open set in X and $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq \mathrm{N}S\alpha Int(A)$, $f[f^{-1}(\mathrm{N}S\alpha Int(B))] \subseteq f[\mathrm{N}S\alpha Int(A)]$. Hence, $\mathrm{N}S\alpha Int[f(A)] \subseteq f[\mathrm{N}S\alpha Int(A)]$.

Conversely, $\mathrm{N}S\alpha Int[f(A)] \subseteq f[\mathrm{N}S\alpha Int(A)]$ for every neutrosophic set A in X. Let V be a neutrosophic open set in Y. Then V is neutrosophic semi−α−open set in Y. Since f is surjective and so $V = \mathrm{N}S\alpha Int(V) = \mathrm{N}S\alpha Int[f(f^{-1}(V))] \subseteq f[\mathrm{N}S\alpha Int(f^{-1}(V))]$. It follows that $f^{-1}(V) \subseteq \mathrm{N}S\alpha Int[f^{-1}(V)]$. Hence $f^{-1}(V)$ is neutrosophic semi−α−open set in X. Thus f is a neutrosophic semi−α−continuous mapping.

Theorem 3.8. Let $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ be a mapping. Then f is a neutrosophic semi−α−continuous mapping if and only if $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq \mathrm{N}S\alpha Int\left[f^{-1}(B)\right]$ for every neutrosophic set B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic semi−α−continuous mapping. Clearly $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq f^{-1}(B)$ implies $\mathrm{N}S\alpha Int\left[f^{-1}(\mathrm{N}S\alpha Int(B))\right] \subseteq \mathrm{N}S\alpha Int\left[f^{-1}(B)\right]$. Since $\mathrm{N}S\alpha Int(B)$ is neutrosophic semi−α−open set in Y and f is neutrosophic semi−α−continuous, $f^{-1}[\mathrm{N}S\alpha Int(B)]$ is neutrosophic *semi−α−open* set in X. Hence $\mathrm{N}S\alpha Int\left[f^{-1}(\mathrm{N}S\alpha Int(B))\right] \subseteq f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq \mathrm{N}S\alpha Int\left[f^{-1}(B)\right]$. Conversely, $f^{-1}[\mathrm{N}S\alpha Int(B)] \subseteq \mathrm{N}S\alpha Int\left[f^{-1}(B)\right]$ for every neutrosophic set B in Y. Let G be any neutrosophic open set in Y. Then $f^{-1}(G) = f^{-1}[\mathrm{N}S\alpha Int(G)] \subseteq \mathrm{N}S\alpha Int\left[f^{-1}(G)\right]$ and therefore $f^{-1}(G) = \mathrm{N}S\alpha Int\left[f^{-1}(G)\right]$. Thus $f^{-1}(G)$ is neutrosophic semi−α−open set in X. Hence f is a neutrosophic semi−α−continuous mapping.

Theorem 3.9. Let $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ be a bijective mapping. Then f is a neutrosophic semi−α−continuous mapping if and only if $f[\mathrm{N}S\alpha Fr(A)] \subseteq \mathrm{N}S\alpha Fr\left[f(A)\right]$ for every neutrosophic set A in X.

Proof. Let f be a bijective and neutrosophic semi−α−continuous mapping. Let A be a neutrosophic set in X. By definition, $\mathrm{N}S\alpha Fr(A) = \mathrm{N}S\alpha Cl(A) \cap \mathrm{N}S\alpha Cl\left(A^C\right)$. By Theorem 3.7, $\mathrm{N}S\alpha Int\left[f(A)\right] \subseteq f[\mathrm{N}S\alpha Int(A)]$ and by Theorem 3.5, $f[\mathrm{N}S\alpha Cl(A)] \subseteq \mathrm{N}S\alpha Cl\left[f(A)\right]$, $f[\mathrm{N}S\alpha Fr(A)] = f[\mathrm{N}S\alpha Cl(A)] \cap f\left[\mathrm{N}S\alpha Cl\left(A^C\right)\right] \subseteq \mathrm{N}S\alpha Cl\left[f(A)\right] \cap \mathrm{N}S\alpha Cl\left[f(A)\right]^C = \mathrm{N}S\alpha Fr\left[f(A)\right]$. Conversely, $f[\mathrm{N}S\alpha Fr(A)] \subseteq \mathrm{N}S\alpha Fr\left[f(A)\right]$ for every neutrosophic set A in X. Then $f[\mathrm{N}S\alpha Cl(A)] = f[\mathrm{N}S\alpha Int(A)] \cup f[\mathrm{N}S\alpha Fr(A)] \subseteq f(A) \cup \mathrm{N}S\alpha Fr\left[f(A)\right] \subseteq \mathrm{N}S\alpha Cl\left[f(A)\right]$. By Theorem 3.5, f is a neutrosophic semi−α−continuous mapping.

Theorem 3.10. Let $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ be a bijective mapping. Then f is a neutrosophic semi−α−continuous mapping if and only if $\mathrm{N}S\alpha Fr\left[f^{-1}(B)\right] \subseteq f^{-1}[\mathrm{N}S\alpha Fr(B)]$ for every neutrosophic set B in Y.

Proof. Let f be a bijective and neutrosophic semi−α−continuous mapping. Let B be a neutrosophic set in Y. By Theorem 3.6, $\mathrm{N}S\alpha Cl\left[f^{-1}(B)\right] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(B)]$. So $f^{-1}[\mathrm{N}S\alpha Fr(B)] = f^{-1}\left[(\mathrm{N}S\alpha Cl(B)) \cap \mathrm{N}S\alpha Cl\left(B^C\right)\right] = f^{-1}[\mathrm{N}S\alpha Cl(B)] \cap f^{-1}\left[\mathrm{N}S\alpha Cl\left(B^C\right)\right] \supseteq \mathrm{N}S\alpha Cl\left[f^{-1}(B)\right] \cap \mathrm{N}S\alpha Cl\left[f^{-1}\left(B^C\right)\right] = \mathrm{N}S\alpha Cl\left[f^{-1}(B)\right] \cap \mathrm{N}S\alpha Cl\left[\left(f^{-1}(B)\right)^C\right] = \mathrm{N}S\alpha Fr\left[f^{-1}(B)\right]$. Therefore $\mathrm{N}S\alpha Fr\left[f^{-1}(B)\right] \subseteq f^{-1}[\mathrm{N}S\alpha Fr(B)]$.

Conversely since $\mathrm{N}S\alpha Fr\left[f^{-1}(B)\right] \subseteq f^{-1}[\mathrm{N}S\alpha Fr(B)]$ for every neutrosophic set B in Y. This implies that $\mathrm{N}S\alpha Cl\left[f^{-1}(B)\right] \subseteq f^{-1}[\mathrm{N}S\alpha Cl(B)]$. By Theorem 3.6, f is a neutrosophic semi−α−continuous mapping.

Definition 3.11. Let $x_{(r,t,s)}$ be a neutrosophic point of a neutrosophic topological space (X, T_N). A neutrosophic set A of X is called neutrosophic neighborhood of $x_{(r,t,s)}$ if there exists a neutrosophic open set B such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.12. Let f be a mapping from a neutrosophic topological space (X, T_N) to a neutrosophic topological space (Y, σ_N). Then the following assertions are equivalent.

i. f is neutrosophic semi−α−continuous.
ii. For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighborhood A of $f\left(x_{(r,t,s)}\right)$, there exists a neutrosophic semi−α−open set B such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$.
iii. For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighborhood A of $f\left(x_{(r,t,s)}\right)$, there exists a neutrosophic semi−α−open set B in X such that $x_{(r,t,s)} \in B$ and $f(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii) : Let $x_{(r,t,s)} \in X$ be a neutrosophic point in X and let A be a neutrosophic neighborhood of $f\left(x_{(r,t,s)}\right)$. Then there exists a neutrosophic open set B in Y such that $f\left(x_{(r,t,s)}\right) \in B \subseteq A$. Since f is neutrosophic semi−α−continuous, therefore it implies that $f^{-1}(B)$ is a neutrosophic semi−α−open set in X and $x_{(r,t,s)} \in f^{-1}\left(f_{(r,t,s)}\right) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii) : Let $x_{(r,t,s)}$ be a neutrosophic point in X and let A be a neutrosophic neighborhood of $f\left(x_{(r,t,s)}\right)$. The condition (ii) implies that there exists a neutrosophic semi−α−open set B in X such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $f(B) \subseteq f\left[f^{-1}(A)\right] \subseteq A$. Hence (iii) is true.

(iii) $\Rightarrow$ (i) : Let B be a neutrosophic open set in Y and let $x_{(r,t,s)} \in f^{-1}(B)$. Since B is neutrosophic open set, $f\left(x_{(r,t,s)}\right) \in B$, and so B is neutrosophic neighborhood of $f\left(x_{(r,t,s)}\right)$. It follows from (iii) that there exists a neutrosophic semi−α−open set A in X such that $x_{(r,t,s)} \in A$ and $f(A) \subseteq B$ so that $x_{(r,t,s)} \in A \subseteq f^{-1}\left[f(A)\right] \subseteq f^{-1}(B)$. This implies by definition that $f^{-1}(B)$ is a neutrosophic semi−α−open set in X. Therefore, f is a neutrosophic semi−α−continuous mapping.

4 Neurosophic Contra Semi-α-Continuous and Contra Semi-α-Irresolute Mappings

In this section, we introduce the concepts of neutrosophic contra semi−α−continuous mappings and neutrosophic contra semi−α−irresolute mappings and investigate their fundamental properties and characterizations.

Definition 4.1. A mapping $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is said to be neutrosophic contra−continuous if the inverse image of every neutrosophic open set in Y is neutrosophic *closed* set in X.

Definition 4.2. A mapping $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is called neutrosophic contra semi−α−continuous if the inverse image of every neutrosophic open set in Y is neutrosophic semi−α−closed set in X.

Theorem 4.3. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a neutrosophic contra−continuous mapping. Then f is neutrosophic contra semi−α−continuous.

Proof. Let V be any neutrosophic open set in Y. Since f is neutrosophic contra continuous, $f^{-1}(V)$ is neutrosophic closed set in X. As every neutrosophic closed set is neutrosophic semi−α−closed, we have $f^{-1}(V)$ is neutrosophic semi−α−closed set in X. Therefore f is neutrosophic contra semi−α−continuous.

Theorem 4.4. A mapping $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ is neutrosophic contra semi−α−continuous if and only if the inverse image of every neutrosophic closed set in Y is neutrosophic *semi−α−open* set in X.

Proof. Let V ba a neutrosophic closed set in Y. Then V^C is neutrosophic open set in Y. Since f is neutrosophic contra semi−α−continuous, $f^{-1}(V^C)$ is neutrosophic semi−α−closed set in X. But $f^{-1}(V^C) = 1 – f^{-1}(V)$ and so $f^{-1}(V)$ is neutrosophic semi−α−open set in X.

Conversely, assume that the inverse image of every neutrosophic closed set in Y is neutrosophic semi−α−open in X. Let W be a neutrosophic open set in Y. Then W^C is neutrosophic closed in Y. By hypothesis $f^{-1}(W^C) = 1 – f^{-1}(W)$ is neutrosophic semi−α−open in X, and so $f^{-1}(W)$ is neutrosophic semi−α−closed set in X. Thus f is neutrosophic contra *semi−α−continuous*.

Theorem 4.5. If a mapping $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ is neutrosophic contra semi−α−continuous and $g : (Y, \sigma_N) \rightarrow (Z, \eta_N)$ is neutrosophic continuous, then their composition $gof : (X, \mathrm{T}_N) \rightarrow (Z, \eta_N)$ is neutrosophic contra semi−α−continuous.

Proof. Let W ba a neutrosophic open set in Z. Since g is neutrosophic continuous, g^{-1} (W) is neutrosophic open set in Y. Since f is neutrosophic contra ωα−continuous, f^{-1} $[g^{-1}(W)]$ is neutrosophic semi−α−closed set in X. But $(gof)^{-1}(W) = f^{-1}[g^{-1}(W)]$. Thus gof is neutrosophic contra semi−α−continuous.

Definition 4.6. A mapping $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ is said to be neutrosophic contra semi−α−irresolute mapping if the inverse image of every neutrosophic semi−α−open set in Y is neutrosophic semi−α−closed in X.

Theorem 4.7. If a mapping $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ is neutrosophic contra semi−α−irresolute, then it is neutrosophic contra semi−α−continuous.

Proof. Let V be a neutrosophic open set in Y. Since every neutrosophic open set is neutrosophic semi−α−open, V is neutrosophic semi−α−open set in Y. Since f is neutrosophic contra semi−α−irresolute, $f^{-1}(V)$ is neutrosophic semi−α−closed set in X. Thus f is neutrosophic contra semi−α−continuous.

Theorem 4.8. Let $(X, \mathrm{T}_N).(Y, \sigma_N)$ and (Z, η_N) be neutrosophic topological spaces. If $f : (X, \mathrm{T}_N) \rightarrow (Y, \sigma_N)$ is neutrosophic contra semi−α−irresolute and $g : (Y, \sigma_N) \rightarrow (Z, \eta_N)$ is neutrosophic semi−α−continuous, then their composition $gof : (X, \mathrm{T}_N) \rightarrow (Z, \eta_N)$ is neutrosophic contra semi−α−continuous.

Proof. Let W be any neutrosophic open set in Z. Since g is neutrosophic *semi−α−continuous*, $g^{-1}(W)$ is neutrosophic semi−α−open set in Y. Since f is neutrosophic contra semi−α−irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic *semi−α−closed* set in X. But $(gof)^{-1}(W) = f^{-1}\left[g^{-1}(W)\right]$. Thus gof is neutrosophic contra semi−α−continuous.

Theorem 4.9. Let (X, T_N), (Y, σ_N) and (Z, η_N) be neutrosophic topological spaces. If $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is neutrosophic semi−α−irresolute and $g : (Y, \sigma_N) \to (Z, \eta_N)$ is neutrosophic contra semi−α−irresolute, then their composition $gof : (X, \mathrm{T}_N) \to (Z, \eta_N)$ is neutrosophic contra semi−α−irresolute mapping.

Proof. Let W be any neutrosophic *semi−α−open* set in Z. Since g is neutrosophic contra semi−α−irresolute, $g^{-1}(W)$ is neutrosophic semi−α−closed set in Y. Since f is neutrosophic semi−α−irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic semi−α−closed set in X. But $(gof)^{-1}(W) = f^{-1}\left[g^{-1}(W)\right]$. Thus gof is neutrosophic contra semi−α−irresolute.

5 Conclusion

We introduced the concepts of neutrosophic *semi − α − continuous* mappings, neutrosophic contra *semi − α − continuous* and contra *semi − α − irresolute* mappings in neutrosophic topological spaces. We investigated and obtained several properties and characterizations concerning these mappings in neutrosophic topological spaces.

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