The Properties of Bounded-Addition Fuzzy Semi-simple Splicing Systems

Mohd Pawiro Santono[1], Mathuri Selvarajoo[1](✉), Wan Heng Fong[2], and Nor Haniza Sarmin[2]

[1] Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia
mathuri@tmsk.uitm.edu.my
[2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, UTM, 81310 Johor Bahru, Johor, Malaysia

Abstract. One of the first theoretical models for DNA computing is known as splicing system. In a splicing system, two strings of DNA molecules are cut at certain recognition sites, and the prefix of the first string is connected to the suffix of the second, resulting in new strings. For a specific form of splicing system, namely semi-simple splicing systems, the recognition sites for both strings of DNA molecules are the same. Only regular languages are known to be produced by splicing systems with finite sets of axioms and splicing rules. As a result, a variety of splicing system restrictions have been considered in order to increase their generating power. Fuzzy splicing systems have been introduced, in which truth values (i.e., fuzzy membership values) from the closed interval [0, 1] are assigned to splicing system axioms. The truth values of each generated string z from strings x and y are obtained by applying a fuzzy bounded- addition operation to their truth values. This study focuses on the characteristics of bounded-addition fuzzy semi-simple splicing systems. It has been demonstrated that fuzzy semi-simple splicing systems with bounded-addition operation increases the generative power of the splicing languages generated.

Keywords: Formal language theory · Restriction · Fuzzy splicing systems · Semi-simple splicing system

1 Introduction

In 1987, Head introduced the mathematical model of splicing systems. This splicing system model was introduced to observe the recombinant behavior of DNA molecules when restriction enzymes and ligases are present [1]. DNA is a chain of nucleotides that contains the genetic material of organisms. Adenine (A), guanine (G), cytosine (C), and thymine (T) are the chemical bases of nucleotides. A double helix is formed when nucleotides are bonded together in two long strands to form a spiral in which A pairs with T, C pairs with G, and vice versa. Restriction enzymes, which are found naturally in bacteria, cut DNA fragments at certain sequences called restriction sites, while ligase re-join DNA fragments with complementary ends [2].

N. Wahi et al. (Eds.): ICMSS 2022, ACSR 98, pp. 354–363, 2023.
https://doi.org/10.2991/978-94-6463-014-5_31

Many restrictions on splicing systems have been proposed to increase the generative power of splicing systems. This is important in the context of DNA computing, as Adleman points out in [2], where splicing systems with restriction can be considered as theoretical models for universally programmable DNA-based computers. A few operations are performed on strings, which can be combined to generate a language expression. The string pattern will then be classified into the appropriate language family using grammar, according to Chomsky's hierarchy. Recursively enumerable, context-sensitive, context-free, linear, regular, and finite languages are just a few of the classes established by Noam [3]. Paun [4] discussed the link between several variants of splicing systems, including simple splicing, semi-simple splicing, and one-sided splicing. All these variants are differentiated by the form of their splicing rules.

By assigning the truth values (i.e., fuzzy membership values) to the axioms of splicing systems from the closed interval [0, 1], the idea of bounded-addition fuzzy semi-simple splicing system is introduced in this study. The truth value of each produced string z is then determined using a bounded-addition fuzzy operation over the truth values of strings x and y. This research looks into a bounded-addition fuzzy semi-simple splicing system.

This study is organized as follows: first, some basic definitions and findings from formal language theory, splicing systems, and a brief overview of fuzzy splicing systems are explained. Next, certain bounded-addition fuzzy semi-simple splicing system definitions are presented. Furthermore, an example and the main findings on the generative power of languages generated by a bounded-addition fuzzy semi-simple splicing system are shown. The result of this research is then discussed at the conclusion of the paper.

2 Preliminaries

Some prerequisites were covered in this section by introducing the basic concepts and notation of formal language, as well as the fuzzy splicing system theories that would be applied later. More details can be referred to [1, 5–8].

Throughout the study, the following general notations are used. The term $\in$ denotes an element in a member of a set, whereas $\notin$ denotes not an element in a member of a set, while $\subseteq$ stands for (proper) inclusion, $\subset$ specifies the strictness of the inclusion. The term |X| denotes the cardinality of a set X, while the symbol $\varnothing$ represents an empty set.

The families of finite, regular, linear, context-free, context-sensitive and recursively enumerable languages are denoted by FIN, REG, LIN, CF, CS and RE respectively. For these family languages, the next strict inclusions, namely Chomsky hierarchy (see [7]), holds:

$$FIN \subset REG \subset LIN \subset CF \subset CS \subset RE.$$

Further, a basic definition of splicing system and a theorem on the family of languages generated by a splicing language are recalled.

Definition 1 [1] **:** A splicing system (EH) is a 4-tuple $\gamma = (V, T, A, R)$ where V is an alphabet, $T \subseteq V$ is a terminal alphabet. A is a finite subset of V^+ and $R \subseteq V{*}\#V{*}\$V{*}\#V{*}$ is the set of splicing rules.

Table 1. The family of languages generated by splicing systems.

F_1/F_2	**FIN**	**REG**	**LIN**	**CF**	**CS**	**RE**
FIN	**REG**	**RE**	**RE**	**RE**	**RE**	**RE**
REG	**REG**	**RE**	**RE**	**RE**	**RE**	**RE**
LIN	**LIN, CF**	**RE**	**RE**	**RE**	**RE**	**RE**
CF	**CF**	**RE**	**RE**	**RE**	**RE**	**RE**
CS	**RE**	**RE**	**RE**	**RE**	**RE**	**RE**
RE	**RE**	**RE**	**RE**	**RE**	**RE**	**RE**

Theorem 1 [4] **:** The relations in the following table hold, where at the intersection of the row marked with F_1 with the column marked with F_2, there appear either the family $EH(F_1, F_2)$ or two families F_3, F_4 such that $F_3 \subset EH(F_1, F_2) \subseteq F_4$ (Table 1).

Next, the definition of a fuzzy splicing system is presented.

Definition 2 [6] **:** A fuzzy splicing system (a fuzzy H system) is a 6-tuple $\gamma = (V, T, A, R, \mu, \odot)$ where *V, T, R* are defined as for a usual extended H system, $\mu : V^* \to [0, 1]$ is a fuzzy membership function, A is a subset of $V^* \times [0, 1]$ and $\odot$ is a fuzzy operation over [0, 1].

Strings x, y and z are written as $(x, \mu(x))$, $(y, \mu(y))$, $(z, \mu(z))$ and a fuzzy splicing operation is defined as follows.

Definition 3 [6] **:** For $(x, \mu(x)), (y, \mu(y)), (z, \mu(z)) \in V^* \times [0, 1]$ and $r \in R$,

$$[(x, \mu(x)), (y, \mu(y))] \mapsto_r z(z, \mu(z))$$

if and only if $(x, y) \mapsto_r z$ and $\mu(z) = \mu(x) \odot \mu(y)$.

Thus, the fuzzy of the string $z \in V^*$ obtained by splicing operation on two strings $x, y \in V^*$ is computed by multiplying their fuzzy membership values. The language generated by the fuzzy splicing system is defined next.

Definition 4 [6] **:** The language generated by fuzzy splicing system $\gamma = (V, T, A, R, \mu, \odot)$ is defined as

$$L_f(\gamma) = \{(z, \mu(z)) \in \sigma_f^*(A^{\oplus}) : z \in T^*\}.$$

Further, the definition of semi-simple splicing system is recalled.

Definition 5 [9] **:** A semi-simple splicing system (SSEH) is a triple $\gamma_{ss} = (V, A, R)$ where V is an alphabet, A is a finite subset of V^+, R is the rule in the form $(a, 1; b, 1)$ for $a, b \in A$, and $R \subseteq V$.

3 Main Results

In this section, the concept of bounded-addition fuzzy semi-simple splicing system is introduced. First, the truth values (i. e., fuzzy membership values) are assigned to the axioms of splicing systems from the closed interval [0, 1]. The truth value of each produced string z is then determined using a bounded-addition fuzzy operation over the truth values of strings x and y.

Definition 6: A bounded-addition fuzzy semi-simple splicing system is a 5-tuple $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ where V is defined as the usual extended splicing systems, R is the rule in the form $(a, 1; b, 1)$, $\mu : V^* \to [0, 1]$ is a fuzzy membership function, $A^{\oplus}$ is a subset of $V^* \times [0, 1]$ such that

$$\sum_{i=1}^{n} \mu(x_i) \leq 1$$

and $\oplus$ is a bounded-addition fuzzy operation on [0, 1] defined by

$$\mu_{A+B} = \mu_A + \mu_B - \mu_A \mu_B \text{ where } \mu_A \mu_B \in \mu(x_i).$$

A bounded-addition fuzzy semi-simple splicing operation is defined next.

Definition 7: For strings $(x, \mu(x))$, $(y, \mu(y))$, $(z, \mu(z)) \in V^* \times [0, 1]$ and $r \in R$, the bounded-addition fuzzy semi-simple splicing operation is defined as

$$[(x, \mu(x)), (y, \mu(y))] \mapsto_r z(z, \mu(z))$$

if and only if $(x, y) \mapsto_r z$ and $\mu(z) = \mu(x) \oplus \mu(y)$ is defined by

$$\mu_{x+y} = \mu_x + \mu_y - \mu_x \mu_y \text{ where } \mu_x \mu_y \in \mu(x_i)$$

and $r = (a, 1; b, 1) \in R$.

The value of string z, μ(z), is computed using a semi-simple splicing operation on two strings $x, y \in V^*$ using the $\oplus$ operation on these two strings.

Definition 8: The language generated by bounded-addition fuzzy semi-simple splicing system $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is defined as

$$L_f(\gamma_{ss}^{\oplus}) = \{z \in T | (z, \mu(z)) \in \sigma^*(A^{\oplus})\}.$$

From the definition of bounded-addition fuzzy semi-simple splicing system, an example is given to illustrate the application of bounded-addition fuzzy semi-simple splicing systems. Here, the symbol $\mapsto$ denotes the splicing operations on the strings.

Example 1: Consider the bounded-addition fuzzy semi-simple splicing system

$$\gamma_{ss}^{\oplus} = (\{a, b, c, d\}, \{(ab, \frac{1}{2}), (ac, \frac{1}{3}), (bac, \frac{1}{4}), (dca, \frac{1}{5}),$$

$$(cba, \frac{1}{6})\}, r_1, r_2, r_3, r_4, r_5, \mu, \oplus)$$

where $r_1 = a\#1\$b\#1$, $r_2 = b\#1\$a\#1$, $r_3 = c\#1\$a\#1$, $r_4 = a\#1\$c\#1$ and $r_5 = b\#1\$d\#1$.

When the first rule r_1 is applied in strings ab and bac, the string obtained is

$$[(ab, \frac{1}{2}), (bac, \frac{1}{4})] \mapsto_{r_1} (aac, \frac{5}{8}).$$

By iterative splicing operation between the same string using the rule r_1, the string

$$(a^{k+1}c, 1 - \frac{3^k}{2.4^k}), k \geq 1$$

is obtained. By applying the rule r_2 to the strings cba and ab, the string obtained is

$$[(cba, \frac{1}{6}), (ab, \frac{1}{2})] \mapsto_{r_2} (cbb, \frac{7}{12}).$$

By iterative splicing operation between the same string using the rule r_2, the string

$$(cb^m, 1 - \frac{5^m}{3.2^m}), m > 1$$

is obtained. By applying the rule r_3 to the strings dca and ac, the string obtained is

$$[(dca, \frac{1}{5}), (ac, \frac{1}{3})] \mapsto_{r_3} (dcc, \frac{7}{15}).$$

By iterative splicing operation between the same string using the rule r_3, the string

$$(dc^{n+1}, 1 - \frac{4.2^n}{5.3^n}), n \geq 1$$

is obtained.

The non-terminals c and d from these strings are eliminated by rules r_4 and r_5. Hence, the following results are obtained:

$$(a^{k+1}c, 1 - \frac{3^k}{2.4^k}), (cb^m, 1 - \frac{5}{3.2^m}) \mapsto_{r_4} (a^{k+1}b^m, 1 - \frac{5.3^k}{6.4^k 2^m})$$

and

$$(a^{k+1}b^m, 1 - \frac{5.3^k}{6.4^k 2^m}), (dc^{n+1}, 1 - \frac{4.2^n}{5.3^n}) \mapsto_{r_5}.$$

$$(a^{k+1}b^m c^{n+1}, 1 - \frac{3^k 2^{n+1}}{4^k 2^m 3^{n+1}}), k, n \geq 1, m > 1.$$

Then, the language generated by the bounded-addition fuzzy semi-simple splicing system $\gamma_{ss}^{\oplus}$,

$$L_f(\gamma_{ss}^{\oplus}) = \{(a^{k+1}b^m c^{n+1}, 1 - \frac{3^k 2^{n+1}}{4^k 2^m 3^{n+1}}) | k, n \geq 1, m > 1\}.$$

When bounded-addition fuzzy semi-simple splicing systems with different thresholds and modes are considered, the threshold languages generated are

1. $L_f(\gamma_{ss}^{\oplus} = 0) = \varnothing \in FIN$,
2. $L_f(\gamma_{ss}^{\oplus} = 1 - \frac{3^k 2^{n+1}}{4^k 2^m 3^{n+1}}) = \{(a^{k+1} b^m c^{n+1} | k, n \geq 1, m > 1\} \in \textbf{REG}$,
3. $L_f(\gamma_{ss}^{\oplus} = \frac{11}{12}) = \{a^2 b^2 c^2\} \in FIN$,
4. $L_f(\gamma_{ss}^{\oplus} = 1 - \frac{1}{3}(\frac{1}{4})^n) = \{a^{n+1} b^{n+1} c^{n+1} | n \geq 1\} \in CS - CF$.

The example above demonstrated that with this restriction, the generative power of bounded-addition fuzzy semi-simple splicing systems can be increased up to the context-sensitive languages. The next lemma follows immediately.

Lemma 1: For all families $F \in \{FIN, REG, CF, LIN, CS, RE\}$,

$$\mathrm{SSEH}(FIN, F) \subseteq f^{\oplus}\mathrm{SSEH}(F).$$

Proof: Let $\gamma_{ss} = (V, A, R)$ be a simple splicing system generating the language $L(\gamma_{ss}) \in$ SSEH $(FIN,$ F) where $F \in \{FIN, REG, CF, CS, RE\}$. Let $A = \{x_1, x_2, \ldots, x_n\}, n \geq 1$. A bounded-addition fuzzy semi-simple splicing system is defined by $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ where the set of axioms is defined by

$$A^{\oplus} = \{(x_i, \mu(x_i)) : x_i \in A^{\oplus}, 1 \leq n\}$$

where $\mu(x_i) = 1/n$ for all $1 \leq n$, then

$$\sum_{i=1}^{n} \mu(x_i) \leq 1$$

and $\oplus$ is a bounded-addition fuzzy operation on [0, 1] defined by

$$\mu_{A+B} = \mu_A + \mu_B - \mu_A \mu_B \text{ where } \mu_A \mu_B \in \mu(x_i).$$

The threshold language generated by $\gamma_{ss}^{\oplus}$ is defined as $L_f(\gamma_{ss}^{\oplus} > 0)$, then it is clear that $L(\gamma_{ss}) = L_f(\gamma_{ss}^{\oplus} > 0)$. Hence, $\mathrm{SSEH}(FIN, F) \subseteq f^{\oplus}\mathrm{SSEH}(F)$ for all families $F \in \{FIN, REG, CF, LIN, CS, RE\}$. ■

From Theorem 1, Lemma 1 and Example 1, the following two theorems are obtained.

Theorem 2: Let $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded- addition fuzzy semi-simple splicing system, where $0 < \mu(x) < 1$ for all $x \in A^{\oplus}$ and $\alpha \in [0, 1]$. Then,

1. $L_f(\gamma_{ss}^{\oplus} > \alpha)$ is a finite language,
2. $L_f(\gamma_{ss}^{\oplus} \leq \alpha)$ is a regular language,
3. $L_f(\gamma_{ss}^{\oplus} \in I)$ is a regular language where I is a subsegment of [0, 1].

Proof:
Case 1: Let $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded-addition fuzzy semi-simple splicing system where

$$A^{\oplus} = \{(x_1, \mu(x_1)), (x_2, \mu(x_2)), \ldots, (x_n, \mu(x_n))\}$$

and $\mu(x_i) = \mu_i$, $1 \le i \le n$. Since $0 < \mu(x_i) < 1$,

$$\sum_{j=1}^{k+1} \mu(x_{ij}) > \sum_{j=1}^{k} \mu(x_{ij}), \mu_{ij} \in \{\mu_1, ..., \mu_n\}.$$

Then, there exists a positive integer $m = k + 1 \in \mathbb{N}$ such that

$$\sum_{j=1}^{m} \mu(x_{ij}) < \alpha, \mu_{ij} \in \{\mu_1, ..., \mu_n\}$$

where $1 \le j \le m$.

For any string $x \in \sigma_f^i(A^\oplus)$, $i \ge m$ that was obtained from some strings of $\sigma_f^{i-1}(A^\oplus)$ using more than or equal to m splicing operations, $\mu(x) < \alpha$ is produced. Thus, $L_f(\gamma_{ss}^\oplus > \alpha)$ contains a finite number of strings.

Case 2: Let $L_f(\gamma_{ss}^\oplus) = L_f(\gamma_{ss}^\oplus > \alpha) \cup L_f(\gamma_{ss}^\oplus \le \alpha)$. Since $L_f(\gamma_{ss}^\oplus)$ is regular and $L_f(\gamma_{ss}^\oplus > \alpha)$ is finite, then $L_f(\gamma_{ss}^\oplus \le \alpha)$ is regular.

Case 3: IF $I = (\alpha_1, \alpha_2)$, then $L_f(\gamma_{ss}^\oplus \in I) = L_f(\gamma_{ss}^\oplus > \alpha) \cap L_f(\gamma_{ss}^\oplus < \alpha)$. Hence, according to **Case 1** and **Case 2**, $L_f(\gamma_{ss}^\oplus \in I)$ is regular. ■

Theorem 3: Let $\gamma_{ss}^\oplus = (V, A^\oplus, R, \mu, \oplus)$ be a bounded-addition fuzzy semi-simple splicing system and $L_f(\gamma_{ss}^\oplus * \alpha)$ be a threshold language where $\oplus \in \{min, max\}$, $* \in \{>, <, =\}$ and $\alpha \in [0, 1]$. Then,

1. $L_f(\gamma_{ss}^\oplus * \alpha)$ is a regular language,
2. If α is *max*, then $L_f(\gamma_{ss}^\oplus > \alpha) = \varnothing$ and $L_f(\gamma_{ss}^\oplus \le \alpha) = L_f(\gamma_{ss}^\oplus)$,
3. If α is *min*, then $L_f(\gamma_{ss}^\oplus \le \alpha) = \varnothing$ and $L_f(\gamma_{ss}^\oplus > \alpha) = L_f(\gamma_{ss}^\oplus)$,
4. If I is a subsegment of [0, 1], then $L_f(\gamma_{ss}^\oplus \in I)$ is a regular language.

Proof:
Case 1: Let $\gamma_{ss}^\oplus = (V, A^\oplus, R, \mu, \oplus)$ be a bounded-addition fuzzy semi-simple splicing system with

$$A^\oplus = \{(x_1, \mu(x_1)), (x_2, \mu(x_2)), ..., (x_n, \mu(x_n))\}.$$

Consider *max* as a bounded-addition fuzzy operation and $>$ as a threshold mode. Then, the set of $\sigma_f^*(A^\oplus)$ can be represented as $\sigma_f^*(A^\oplus) = \sigma_{f1}^*(A^\oplus) \cup \sigma_{f2}^*(A^\oplus)$ where

$$\sigma_{f1}^*(A^\oplus) = \{(x, \mu(x)) \in \sigma_{f1}^*(A^\oplus) : \mu(x) > \alpha\},$$

and

$$\sigma_{f2}^*(A^\oplus) = \{(x, \mu(x)) \in \sigma_{f1}^*(A^\oplus) : \mu(x) \le \alpha\}.$$

Let $\sigma^*_{f1}(A^\oplus) = A_i^\oplus$, $i = 1, 2$. Then, $A^\oplus = A_1^\oplus \cup A_2^\oplus$ where $A_1^\oplus = \{x \in A^\oplus : \mu(x) > \alpha\}$, and $A_2^\oplus = \{x \in A^\oplus : \mu(x) \leq \alpha\}$.

The semi-simple splicing system $\gamma_{ss} = (V, A_2, R)$ is constructed where $L(\gamma_{ss}) = \alpha^*(A_2) \cap T^*$ is regular. Moreover, it is shown that $\sigma^*_{f2}(A^\oplus) = \sigma^*_f(A_2^\oplus)$. First, $\sigma^*_f(A_2^\oplus) \subseteq \sigma^*_{f2}(A^\oplus)$ since $A_2^\oplus \subseteq A^\oplus$. On the other hand, $\sigma^*_{f2}(A^\oplus) \subseteq \sigma^*_f(A_2^\oplus)$. Let $x \notin \sigma^*_f(A_2^\oplus)$. Then, there is an axiom $(x, \mu(x)) \in A_1^\oplus$ such that

$$((x_1, \mu(x_1)), (x_2, \mu(x_2))) \mapsto (z_1, \mu(z_1)),\ ((z_1, \mu(z_1)), (z_2, \mu(z_2))) \mapsto (z_3, \mu(z_3)),$$
$$\vdots$$
$$((z_k, \mu(z_k)), (z_{k+1}, \mu(z_{k+1}))) \mapsto (x, \mu(x)),$$

where $(x_2, \mu(x_2)) \in A^\oplus$ and $(z_1, \mu(z_1)) \in \sigma^*_f(A^\oplus)$. Then,

$$max\{\mu(x_1), \mu(x_2)\} = \mu(z_1) > \alpha,$$
$$\vdots$$
$$max\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) > \alpha.$$

Consequently, $(x, \mu(x)) \notin \sigma^*_{f2}(A^\oplus)$. Thus, $\sigma^*_{f2}(A^\oplus) = \sigma^*_f(A_2^\oplus)$. It follows that the language $L_f(\gamma_{ss}^\oplus \leq \alpha) = \sigma^*_{f2}(A^\oplus) \cap T^*$ is regular. Hence, $\sigma^*_{f1}(A^\oplus) = \sigma^*_f(A^\oplus) \backslash \sigma^*_{f2}(A^\oplus)$ and the language $L_f(\gamma_{ss}^\oplus > \alpha) = L_f(\gamma_{ss}^\oplus) \backslash L_f(\gamma_{ss}^\oplus \leq \alpha)$ is also regular. Similarly, if the bounded-addition fuzzy operation is *min*, it can be proven that $L_f(\gamma_{ss}^\oplus > \alpha)$ and $L_f(\gamma_{ss}^\oplus \leq \alpha)$ are regular.

Case 2: Let $\alpha > max\{\mu_1, \mu_2, ..., \mu_n\}$. Based on **Case 1**, *max* as bounded-addition fuzzy operation was considered. Then, it produces an axiom such that

$$max\{\mu(x_1), \mu(x_2)\} = \mu(z_1) < \alpha,$$
$$\vdots$$
$$max\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) < \alpha.$$

Thus, the language $L_f(\gamma_{ss}^\oplus \leq \alpha)$ is regular and $L_f(\gamma_{ss}^\oplus > \alpha)$ is an empty set produced. The language $L_f(\gamma_{ss}^\oplus) = L_f(\gamma_{ss}^\oplus > \alpha) \cup L_f(\gamma_{ss}^\oplus \leq \alpha)$. Since $L_f(\gamma_{ss}^\oplus > \alpha)$ is empty and $L_f(\gamma_{ss}^\oplus \leq \alpha)$ is regular, then $L_f(\gamma_{ss}^\oplus)$ is regular. Hence, $L_f(\gamma_{ss}^\oplus) = L_f(\gamma_{ss}^\oplus \leq \alpha)$.

Case 3: Let $\alpha > min\{\mu_1, \mu_2, ..., \mu_n\}$. Based on **Case 1**, *min* as bounded-addition fuzzy operation was considered. Then, it produces an axiom such that

$$min\{\mu(x_1), \mu(x_2)\} = \mu(z_1) > \alpha,$$
$$\vdots$$
$$min\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) > \alpha.$$

Thus, the language $L_f(\gamma_{ss}^\oplus > \alpha)$ is regular and $L_f(\gamma_{ss}^\oplus \leq \alpha)$ is an empty set produced. The language $L_f(\gamma_{ss}^\oplus) = L_f(\gamma_{ss}^\oplus > \alpha) \cup L_f(\gamma_{ss}^\oplus \leq \alpha)$. Since $L_f(\gamma_{ss}^\oplus \leq \alpha)$ is empty and $L_f(\gamma_{ss}^\oplus > \alpha)$ is regular, then $L_f(\gamma_{ss}^\oplus)$ is regular. Hence, $L_f(\gamma_{ss}^\oplus) = L_f(\gamma_{ss}^\oplus > \alpha)$.

Case 4: Let $L_f(\gamma_{ss}^{\oplus} \in I) = L_f(\gamma_{ss}^{\oplus} > \alpha_1) \cap L_f(\gamma_{ss}^{\oplus} < \alpha_2)$ where $I = (\alpha_1, \alpha_2)$. From **Case 1**, $L_f(\gamma_{ss}^{\oplus} > \alpha_1)$ and $L_f(\gamma_{ss}^{\oplus} < \alpha_2)$ are regular. Therefore, their intersections are also regular. ■

As direct consequences of the theorems above, interesting facts of bounded-addition fuzzy semi-simple splicing systems are obtained, as stated in Corollary 1, Corollary 2 and Corollary 3.

Corollary 1: IF the fuzzy membership of each axiom $\mu(x) \in A^{\oplus}$ in a bounded-addition fuzzy semi-simple splicing system $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is nonzero, then the threshold language $L_f(\gamma_{ss}^{\oplus} = 0)$ is an empty set, i.e., $L_f(\gamma_{ss}^{\oplus} = 0) = \varnothing$.

Corollary 2: IF the fuzzy membership of each axiom $\mu(x) \in A^{\oplus}$ in a bounded-addition fuzzy semi-simple splicing system $\gamma_{ss}^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is not greater than 1, then every threshold language $L_f(\gamma_{ss}^{\oplus} = \alpha)$ with $\alpha \in [0, 1]$ is finite.

Corollary 3: Every fuzzy semi-simple splicing system with the bounded-addition operation *max* or *min*, and the cut-points of any number in [0, 1] or any subinterval of [0, 1], generates a regular language.

4 Conclusion

This paper establishes the concept of bounded-addition fuzzy semi-simple splicing systems and determines its preliminary characteristics. Each axiom is associated with truth values from the closed interval [0, 1], and the truth value of a string z formed from strings x and y is computed by performing a bounded-addition fuzzy operation on the truth values. It has been demonstrated that the generative power of the languages generated can be increased by introducing bounded-addition fuzzy semi-simple splicing systems. Furthermore, some threshold languages can generate non-regular languages if appropriate cut-points are chosen.

Acknowledgments. The first and second authors would like to thank the Ministry of Higher Education and Universiti Teknologi MARA Malaysia for the financial funding through research grant FRGS-RACER (600-IRMI/FRGS-RACER 5/3 (050/2019)). The third and fourth authors would also like to thank the Ministry of Higher Education and Research Management Center (RMC) UTM for the UTMShine Grant Vote No. Q.J130000.2454.09G89.

References

1. Head, T.: Formal language theory and DNA: An analysis of the generative capacity of specific recombinant behaviors. Bull. Math. Biol. **49**(6), 737–759 (1987). https://doi.org/10.1007/BF02481771
2. Adleman, L.: Molecular solution to computational problems. Science **266**, 1021–1024 (1994)
3. Chomsky, N.: Three models for the description of language. IRE Trans. Inf. Theory **2**, 113–124 (1956)

4. Păun, G., Rozenberg, G., Salomaa, A.: DNA computing: new computing paradigms. Comput. Math. Appl. **37**(3), 134 (1998). https://doi.org/10.1016/s0898-1221(99)90411-x
5. Mizumoto, M., Tanaka, K.: Fuzzy-fuzzy automata. Kybernetes **5**(3), 107–112 (1976)
6. Karimi, F., Turaev, S., Sarmin, N.H., Fong, W.H.: Fuzzy splicing systems. In: Hwang, D., Jung, J.J., Nguyen, N.-T. (eds.) ICCCI 2014. LNCS (LNAI and LNB), vol. 8733, pp. 20–29. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-11289-3_3
7. Moraga, C.: An approach to fuzzy context free languages. ESTYLF08, Cuencas Mineras (Mieres - Langreo), pp. 17–19 (2008)
8. Laun, T.E.G.: Constants and splicing systems. Ph.D thesis, State University of New York, Binghamton ProQuest Dissertations Publishing (1999)
9. Goode, E., Pixton, D.: Semi-simple splicing systems. Where Math. Comput. Sci. Linguist. Biol. Meet 343–352 (2001). https://doi.org/10.1007/978-94-015-9634-3_30