Analysis on the Solute Dispersion in Blood Flow Through an Inclined Artery with the Presence of Chemical Reaction

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Abstract. The present study discusses on the solute dispersion in a blood flow through an inclined artery with the presence of chemical reaction. The blood flow is considered to be laminar, incompressible and steady flow of Bingham model. The continuity and momentum equations are solved in cylindrical coordinate for the velocity solution using direct integration method. The steady convective-diffusion equation with the presence of chemical reaction is in the form of non-homogeneous Bessel differential equation and is solved analytically for the solute concentration. The obtained solutions are then utilized for the Taylor-Aris method for obtaining the solution for effective axial diffusion. The solutions of velocity, solute concentration and effective axial diffusion are plotted graphically to analyse the effect of angle of arterial inclination, gravitational force and chemical reaction rate on the blood flow and solute dispersion. Result shows an increase in velocity profile of blood flow as the angle of inclination increases until 90° inclination which has the highest velocity profile. As the artery inclined more, the velocity profile decreases until it reaches the lowest velocity at 270° inclination. Consequently, increase in velocity decreases the solute concentration inside the artery. Nevertheless, solute concentration increases as the angle of inclination increase. Additionally, the increase in chemical reaction rate decreases the solute concentration and leads to decrease in effective axial diffusion.

Keywords: Inclined artery · Bingham model · Solute dispersion · Chemical reaction · Taylor-Aris method

1 Introduction

The study on solute dispersion and blood flow through an inclined artery with the presence of chemical reaction is an important contribution to the medical field. Certain diseases such as cardiovascular disease and cancer rely on the treatment involving the injection of drug solutes through the artery known as intra-arterial injection. Accessing the artery for drug injection carries a high risk of complications due to the nature of the artery having a high pressure inside the artery that could cause backflow into the needle and thicker wall that could cause heavy bleeding when punctured. Therefore,

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various controllable variables should be considered and calculated to lessen the risk of complications.

There is various type of artery that have different angle of inclination inside the human body. For instance, the carotid artery which carries the blood to the head is flowing upward against the gravity. Therefore, injection through the carotid artery should consider the angle of the artery inclination for optimal procedure performance. Many researchers have done studies on blood flow through an inclined artery using Newtonian and non-Newtonian models. Bakheet et al. [1] investigated the blood flow through an inclined flexible stenosed artery using the Bingham model. Zaman et al. studied the hydro-magnetic flow of unsteady non-Newtonian blood under slip effects through an inclined artery with presence of catheter and overlapping stenosis. Ahmed and Nadeem [2] studied the blood flow through an inclined artery with presence of catheter and overlapping stenosis using the Carreau nanofluid. Abbas et al. [3] analysed the magnetohydrodynamic pulsatile flow of Sutterby fluid with the presence of periodic body acceleration through an inclined overlapping stenosis. Kumari et al. [4] considered the unsteady peristaltic transport through an inclined stenosed artery with the presence of slip effect using the MHD fluid. Thus, the effect of arterial inclination should be investigated as it represents the real-life condition of human artery.

Injection of drug solutes into the blood stream causes a chemical reaction between the drug solutes and blood cells. The rate of the chemical reaction depends on the type of drug that reacts with the blood cells. Nevertheless, the chemical reaction rate effects on the solute concentration and effective axial diffusion should be investigated. Abidin et al. [5] studied the solute dispersion through a stenosed artery with the presence of chemical reaction using the Herschel-Bulkley model. Ratchagar and Vijayakumar [6] investigated the effect of chemical reaction on the solute dispersion and concluded that the increase in chemical reaction decreases the solute concentration. Shukla et al. [7] studied the chemical reaction effects a few non-Newtonian fluids such as Power Law, Bingham and Casson fluid models adopting Taylor's dispersion theory. Chandra and Agarwal [8] investigated the effect of chemical reaction on the solute dispersion through pipe and channel in micro-fluid flow using Taylor's dispersion theory. They noted a decrease in dispersion coefficient with the increase in chemical reaction rate parameter. Thus, the effect of chemical reaction should be observed to analyse the degree of its effect on the solute dispersion.

In representing the blood rheology for this present study, there are Newtonian and various non-Newtonian models to depict the characteristic of a blood flow. Nevertheless, no one model is universally accepted to perfectly depicts the rheology of blood flow. Various models have been used by researchers in the study of blood flow and dispersion such as Newtonian, Casson, Herschel-Bulkley, Power Law and many more including Bingham. Some study focuses on the blood flow in an artery under normal physiological condition. Meanwhile, other study focuses on blood flowing through an abnormal or diseased artery. Since this study also focuses on the chemical reaction between drug solutes and blood, the blood flow is under the premise of blood flowing through an artery with certain pathological condition that can be appropriately described by Bingham model. For example, people with thick blood condition that requires medication. At low stress, the blood behaves like rigid body. When the heart pumps out pressure, the blood behaves viscously. This means that the viscosity decreases with increasing shear rate which exhibits a shear-thinning property of a Bingham model. This fundamental aspect of similarity enhances the determination to consider the Bingham model.

Nevertheless, the study of chemical reaction effect on the solute dispersion through an inclined artery using the Bingham model has not yet been done. Thus, the objective of the present study is to investigate the effect of arterial inclination and chemical reaction on the blood flow and solute dispersion modelled by the Bingham model. The result of this present study could potentially benefit the medical field such as improving the intra-arterial injection procedure, making better medical device and refining drug prescribing.

2 Mathematical Formulation

A steady, axisymmetric, laminar and fully-developed unidirectional flow of Bingham model through an inclined circular artery is considered. As shown in Fig. 1, the geometry of the blood flow in an inclined circular artery where $\overline{r}$ is radius, $\overline{w}(\overline{z})$ is the velocity in axial direction and $\overline{g}$ is the gravitational force in the downward direction. The artery is inclined to a degree $\overline{\theta}$. Thus, the gravitational force in the $\overline{z}$ direction is $\overline{\rho g}\sin\theta$. The simplified continuity, momentum and constitutive equations in cylindrical coordinate are

$$\frac{\partial \overline{w}}{\partial \overline{z}} = 0, \tag{1}$$

where $\overline{w}$ is the axial velocity. The direction of flow in the artery is parallel to the horizontal axis. Thus, the momentum equation will be considered in the $\overline{z}$ direction. Moreover, the artery is inclined with an angle of $\overline{\theta}$ degree. The governing momentum equation is simplified to

$$-\frac{d\overline{p}}{d\overline{z}} - \frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}(\overline{r\tau}) + \overline{\rho g}\sin\overline{\theta} = 0, \tag{2}$$

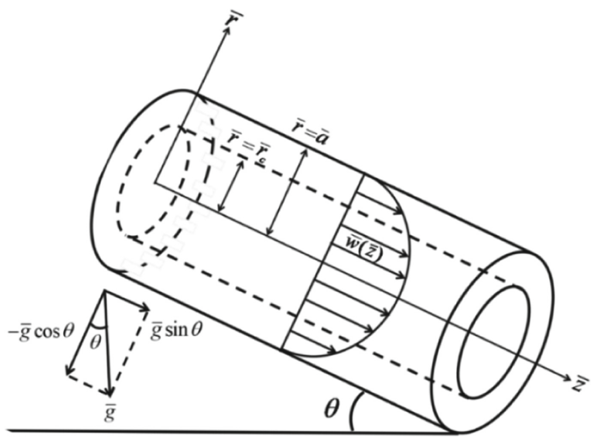

Fig. 1. Geometry of blood flow through an inclined artery.

where $\overline{p}$ is the pressure, $\overline{\tau}$ is the shear stress, $\overline{g}\sin\overline{\theta}$ is the gravitational force in the $\overline{z}$ direction and $\overline{\rho}$ is the density of the fluid. The pressure gradient $d\overline{p}/d\overline{z}$ is considered as a constant. The boundary condition for Eq. (2) is given as

$$\overline{\tau} \text{ is finite at } \overline{r} = 0. \tag{3}$$

According to Pincombe and Mazumdar [9], the constitutive equation of the Bingham model is described by

$$\frac{d\overline{w}}{d\overline{r}} = \begin{cases} -\frac{(\overline{\tau}-\overline{\tau}_y)}{\overline{\mu}_B}, & \text{if } \overline{\tau} \geq \overline{\tau}_y, \\ 0, & \text{if } \overline{\tau} < \overline{\tau}_y, \end{cases} \tag{4}$$

where $\overline{\tau}_y$ is the yield stress and $\overline{\mu}_B$ viscosity coefficient of Bingham model. The boundary condition for Eq. (4) is

$$\overline{w} = 0 \text{ at } \overline{r} = \overline{a}. \tag{5}$$

According to Sharp [10], the steady convective-diffusion equation at the core and outer flow region are consecutively given as

$$\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{C}_1}{\partial \overline{r}}\right) = \frac{\hat{w}_c}{\overline{D}_m}\frac{\partial \overline{C}}{\partial \overline{z}^*}, \tag{6}$$

where $\overline{C}_1$ is the solute concentration at the core flow region, $\hat{w}_c$ is the relative velocity at the core flow region, $\overline{D}_m$ is the constant molecular diffusion coefficient and $\overline{z}^*$ is the axial coordinate and

$$\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{C}_2}{\partial \overline{r}}\right) = \frac{\hat{w}_o}{\overline{D}_m}\frac{\partial \overline{C}}{\partial \overline{z}^*}, \tag{7}$$

where $\overline{C}_2$ is the solute concentration at the outer flow region and $\hat{w}_o$ is the relative velocity at the outer flow region. The relative velocity at the core and outer flow region are defined by $\hat{w}_c = \overline{w}_c - \overline{w}_m$ and $\hat{w}_o = \overline{w}_o - \overline{w}_m$ respectively where $\overline{w}_m$ is the mean velocity defined as

$$\overline{w}_m = \frac{\int_0^{2\pi}\int_0^{\overline{a}} \overline{w}(\overline{r})\overline{r}d\overline{r}d\overline{\psi}}{\int_0^{2\pi}\int_0^{\overline{a}} \overline{r}d\overline{r}d\overline{\psi}}. \tag{8}$$

The boundary conditions are given as

$$\frac{\partial \overline{C}}{\partial \overline{r}} = 0 \quad \text{at } \overline{r} = 0, \tag{9}$$

$$\overline{C} = 0 \text{ at } \overline{r} = 0, \tag{10}$$

for Equation Eq. (6) and

$$\frac{\partial \overline{C}_2}{\partial \overline{r}} = 0 \text{ at } \overline{r} = \overline{a}, \tag{11}$$

$$\overline{C}_2 = \overline{C}_1 \text{ at } \overline{r} = \overline{r}_c. \tag{12}$$

for equation Eq. (7).

3 Method of Solution

Integrating Eq. (2) with respect to $\overline{r}$ subject to boundary Eq. (3), the shear stress $\overline{\tau}$ is obtained as

$$\overline{\tau} = -\frac{\overline{r}}{2}\left(\frac{d\overline{p}}{d\overline{z}} - \overline{\rho g}\sin\theta\right). \tag{13}$$

Substituting Eq. (13) into constitutive equation in Eq. (4) and integrate it with respect to $\overline{r}$ subject to boundary condition in Eq. (5), the resulting velocity at the outer flow region $\overline{w}_o$ is

$$\begin{aligned}\overline{w}_o =& \frac{1}{4\overline{\mu}_B}\left(\frac{d\overline{p}}{d\overline{z}} - \overline{\rho g}\sin\overline{\theta}\right)\\ &\left(\overline{r}^2 - \overline{a}^2 - 2\overline{r}_c(\overline{r} - \overline{a})\right),\end{aligned} \tag{14}$$

where $\overline{r}_c$ is radius at the core flow region and $\overline{r}_c = -2\overline{\tau}_y \big/ ((d\overline{p}/d\overline{z}) - \overline{\rho g}\sin\theta)$. The velocity at the core flow region is obtained by evaluating the radius in Eq. (14) at $\overline{r} = \overline{r}_c$ as

$$\overline{w}_c = -\frac{1}{4\overline{\mu}_B}\left(\frac{d\overline{p}}{d\overline{z}} - \overline{\rho g}\sin\overline{\theta}\right)(\overline{a} - \overline{r}_c)^2. \tag{15}$$

Equation (6) is solved using the Bessel function for core region concentration $\overline{C}_1$ subject to boundary equations in Eqs. (9) and (10). The resulting core region concentration yields

$$\begin{aligned}\overline{C}_1 = -\frac{\hat{w}_c}{\overline{R}}\left(\frac{d\overline{p}}{d\overline{z}} - \overline{\rho g}\sin\overline{\theta}\right)\left(\frac{\partial\overline{C}}{\partial\overline{z}^*}\right)\\ \left(J_0\left(i\overline{r}\sqrt{\overline{R}/\overline{D}_m}\right) - 1\right),\end{aligned} \tag{16}$$

where J_0 is Bessel function of the first kind of zeroth order. The radius in Eq. (16) is evaluated at $\overline{r} = \overline{r}_c$ to obtain the core region concentration with evaluated radius $\overline{C}_c$ as

$$\begin{aligned}\overline{C}_c = -\frac{\hat{w}_c}{\overline{R}}\left(\frac{d\overline{p}}{d\overline{z}} - \overline{\rho g}\sin\overline{\theta}\right)\left(\frac{\partial\overline{C}}{\partial\overline{z}^*}\right)\\ \left(J_0\left(i\overline{r}_c\sqrt{\overline{R}/\overline{D}_m}\right) - 1\right).\end{aligned} \tag{17}$$

To obtain the outer region concentration $\overline{C}_2$, Eq. (7) is solved using the non-homogeneous Bessel function subject to boundary condition in Eqs. (11) and (12). Due to the complexity of the solution $\overline{C}_2$, it is omitted here. The relative velocities and concentrations of $\hat{w}_o$, $\hat{w}_c$, $\overline{C}_c$ and $\overline{C}_2$ are then used to find the volumetric flow rate $\overline{q}$ which is defined as

$$\overline{q} = \frac{1}{\pi\overline{a}^2}\left\{\int_0^{\overline{r}_c}\left(\hat{w}_c\overline{C}_c - \overline{D}_m\frac{\partial\overline{C}}{\partial\overline{z}^*}\right)2\pi\overline{r}d\overline{r}\right.$$

$$+ \int_{\overline{r}_c}^{\overline{a}} \left(\hat{w}_o \overline{C}_2 - \overline{D}_m \frac{\partial \overline{C}}{\partial \overline{z}^*} \right) 2\pi \overline{r} d\overline{r} \Bigg\}. \tag{18}$$

The integrals within the $\overline{q}$ are evaluated using the numerical method of Simpson's 3/8 due to the complexity of the expressions. The effective axial diffusion is obtained by using the Taylor-Aris definition of

$$\overline{D}_{eff} = -\frac{\overline{q}}{\left(\partial \overline{C} / \partial \overline{z}^*\right)}. \tag{19}$$

Non-dimensional effective axial diffusion is obtained in the form of $\overline{D}_{eff} \big/ \overline{D}_m$. For the solution of velocities in Eqs. (14) and (15), it is non-dimensionalized using the non-dimensional variable as follows

$$\begin{aligned} &r = \frac{\overline{r}}{\overline{a}},\; w_c = \frac{\overline{w}_c}{\overline{U}},\; w_o = \frac{\overline{w}_o}{\overline{U}},\; p = \frac{\overline{pa}}{\overline{\mu U}}, \\ &z = \frac{\overline{z}}{\overline{a}},\; g = \frac{\overline{ga}}{\overline{U}^2},\; \rho = \frac{\overline{\rho a U}}{\overline{\mu}}, \end{aligned} \tag{20}$$

for the purpose of data plotting.

4 Discussion

The present study analyses the solute dispersion and blood flow behaviour under the influence of arterial inclination and chemical reaction. In the real-life situation, the human arteries have various inclination depending on the location in the human body. Thus, it is significant to consider the orientation of the arterial inclination to observe the effects on the blood flow and solute dispersion. Additionally, the introduction of drug solutes into the blood stream initiates a chemical reaction between the blood cells and drug solutes. The presence of chemical reaction in the blood stream also affects the blood flow and solute dispersion as shown by the graphical plotting of the data. The solutions for the blood velocity and solute concentration are solved analytically using a direct integration method. The solute concentration method is then utilized in solving for the effective axial diffusion using the Taylor-Aris method. In solving for the effective axial diffusion, the integrals within the problem solving are handled numerically using the Simpson's 3/8 method due to the complex expressions. In plotting the solutions, certain variables such as angle of artery inclination, chemical reaction rate and gravitational force are varied to observe their effects. The artery inclination is taken within $0° \leq \overline{\theta} \leq 360°$. The value of chemical reaction rate parameter depends on the type of drug that is reacting with the blood and Bird et al. [11] stated that the chemical reaction rate value can be experimentally determined. However, for a theoretical study, if the appropriate chemical reaction rate is unknown, a small value of chemical reaction rate such as $\overline{R} = 10^{-5}$ can be used to start the computation and increasing the reaction rate until the results are constant. Since this present study focuses on the effect of chemical reaction rate of a non-specific drug, an arbitrary range of number is chosen as long as the range of number is in an increasing manner, can be plotted and does not give a constant result. Hence, the chemical reaction rate used in this study is $\overline{R} = 0.1, 0.2, 05$ to observe the impact of increasing chemical reaction rate in different interval value. Mandin et al. [12] used the value of $\overline{g} = 9.81$ to represent the earth gravity constant which is rounded up to $\overline{g} = 10$ in this study. Nevertheless, increment in gravitational force of $\overline{g} = 15$ and $\overline{g} = 20$ are also used to observe the impact on the blood velocity. Other variables are constant throughout the plotting of the solution such as $\overline{\mu}_B = 1$, $\overline{a} = 1$, $d\overline{p}\big/ d\overline{z} = -15$, $\overline{\rho} = 1$, $\overline{D}_m = 1$, $\partial\overline{C}\big/ \partial\overline{z} = 1$ and $\overline{r}_c = 0.04$. All the solutions are plotted graphically.

4.1 Velocity Profile

In Fig. 2(a) to (c), the velocity profiles of blood flow through an inclined artery are investigated. The velocity profiles are observed under the influence of arterial inclination of 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315° and 360°. At 0° inclination, the artery is in a horizontal position and considered as not under any influence of inclination. As the arterial inclination increases to 45°, the velocity of the blood flow increases. It can be said that increase in arterial inclination increases the velocity profile. Nevertheless, this is only true until a certain angle of inclination. At 90° inclination, in which the artery is in a vertical position, the velocity profile is the highest compared to all other angle of inclination. It can be logically assumed that the blood is flowing in a downward direction alongside the direction of the gravitational force. The gravitational force is fully acting on the blood flow and accelerates the blood flow velocity. In comparison with 45° inclination, the gravitational force is only partially acting on the blood flow which explains the smaller increase in the velocity profile. Inclining the artery above 90°, the velocity starts to decrease until it approaches the lowest velocity profile at 270° inclination. At 270° inclination, the blood is flowing against the gravitational force fully acting on the flow. Thus, the gravitational force slows down the velocity of the blood flow significantly. It can also be noted at 180° and 360° inclinations, the velocity profile is similar to the velocity profile at 0° inclination.

The blood flow velocity is also affected by the gravitational force depending on the angle of arterial inclination as shown in Fig. 3. For instance, at 90° inclination where the blood is flowing in the direction of the gravitational force and fully aided by the gravitational force, the higher the gravitational force, the higher the velocity profile. On the contrary, at 270° inclination where the blood is flowing against the gravitational force, the increase in gravitational force decreases the velocity profile to a point of causing a backflow in the artery. Note that at $g = 15$, the velocity is constant due to the pressure gradient inside the artery have a similar value to the gravitational force. Meanwhile at $g = 20$, the velocity is in a negative value. This indicates that the flow is in the opposite direction inside the artery due to the higher gravitational force than the pressure gradient. Furthermore, the velocity of artery that is not inclined is constant and unaffected by the gravitational force as shown at 0° inclination.

4.2 Solute Concentration

The solute concentration through an inclined artery with the presence of chemical reaction is analysed in Fig. 4(a) to (c). The behaviour of solute concentration exhibits a

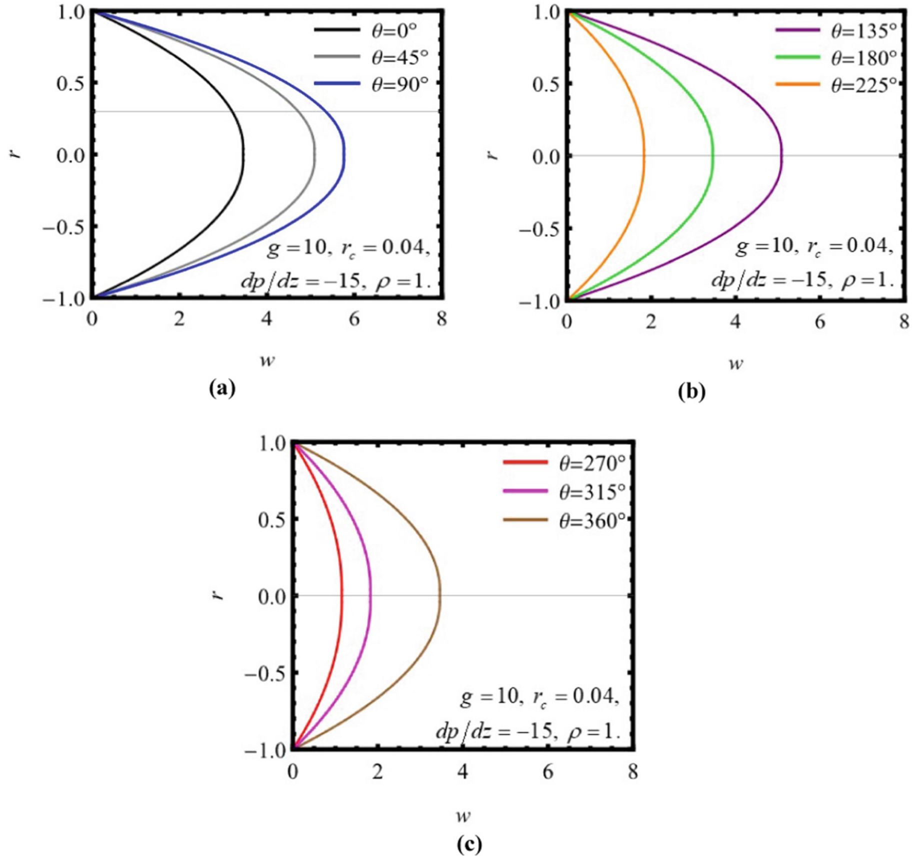

Fig. 2. Velocity profiles at various arterial inclination with $dp/dz = -15$, $\rho = 1$, $g = 10$ and $r_c = 0.04$.

similar pattern to the blood flow velocity when it is affected by the arterial inclination by means of increased solute concentration as the inclination increases approaching 90° inclination. Not to mention, the highest and lowest solute concentration are also at 90° and 270° inclinations. It can also be observed that the solute concentration is higher at the outer flow region compared to the core flow region. This is due to the higher velocity at the core flow region compared to the outer flow region as seen in Fig. 3(a) to (c). Higher velocity at the core flow region causes the solute to diffuse along the artery efficiently with the aid of blood flow movement. Thus, less solute is concentrated at the core region. Similarly, low velocity at the outer flow region increases the solute concentration at the outer region due to lack of blood movement to aid the solute dispersion at the region. Nonetheless, arterial inclination affects the solute concentration at the outer region significantly compared to the core region.

Figures 5(a) and (b) show the graphical plotting of solute concentration are affected by the chemical reaction rate parameter of $\overline{R} = 0.1, 0.2$ and 0.5 at arterial inclination of 90° and 270°. It can be seen that the solute concentration decreases as the chemical

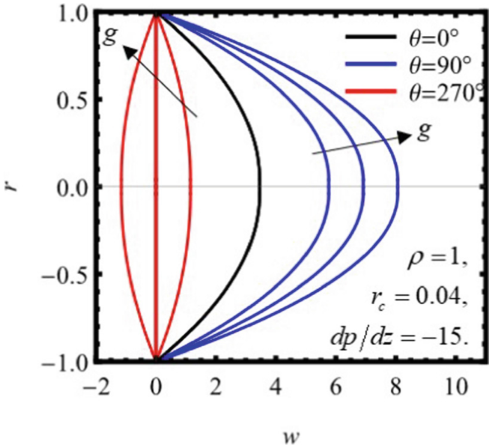

Fig. 3. Velocity profiles at $g = 10, 15, 20$ with $dp/dz = -15$, $\rho = 1$, $r_c = 0.04$ and $\theta = 0°, 90°, 270°$.

reaction rate increases from 0.1 to 0.5. This is because increase in chemical reaction rate decreases the solute molecules in blood stream. The higher the chemical reaction between solutes and blood cells, the faster the decreases of solute concentration and less solutes are left inside the artery. Thus, reducing the solute molecules to disperse along the artery. It is also interesting to note that the decrease in solute concentration is higher at 90° inclination compared to the 270° inclination. This is due to the higher gravitational force aiding the flow velocity at 90° inclination which in return, aiding the decreases in solute concentration. Another significant observation is that the increase in chemical reaction rate affects the solute concentration at the outer region more than the core region. This is due to the higher solutes at the outer region. Therefore, it is affected more compared to the core region which has less or no solute molecules. Not to mention, both graphical plotting exhibits a slight concentration peak at the core region due to the applied shear stress is less than the yield stress. Thus, the blood represented by the Bingham model behaves like a rigid body. The blood flow behaves like a solid medium in the core layer and the solid plug moves within the flow. The dispersion at the core region is unaffected by the increase in chemical reaction rate. Hence, the slight concentration peak at the core flow region. This is because to the dispersion in the core region is very small (or none) due to the rigid behaviour of the blood. Any drug solutes or blood cells does not disperse well at the plug region.

4.3 Effective Axial Diffusion

The effect of chemical reaction rate on the effective axial diffusion is analysed in Fig. 6(a) and (b). The increase in chemical reaction rate from 0.1 to 0.5 reduces the effective axial diffusion for all 0°, 90° and 270° inclinations. This is because of the lesser solute concentration due to the high chemical reaction rate that leads to less axial solute diffusion. Thus, the effective axial diffusion decreases. The decrease in solute concentration is more significant at 0° and 90° inclinations compared to 270° inclination. Albeit very small,

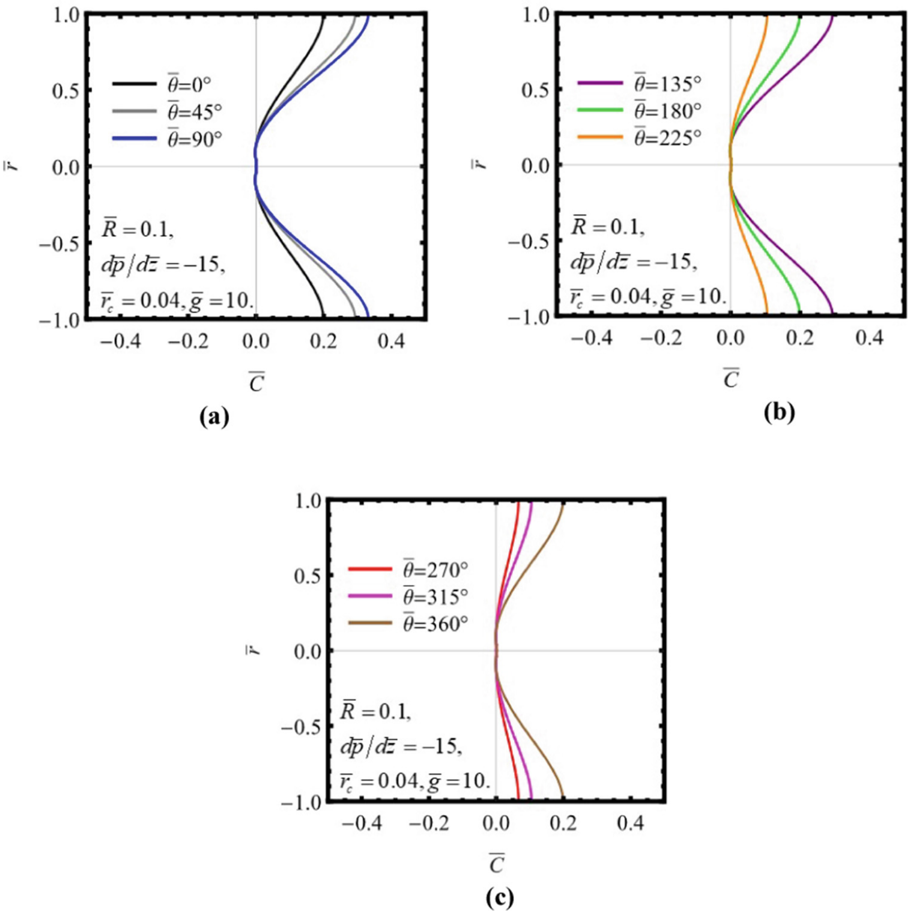

Fig. 4. Solute concentration at various arterial inclination with $\overline{R} = 0.1, \overline{\mu}_B = 1, \overline{a} = 1, \frac{d\overline{p}}{dz} = -15, \overline{\rho} = 1, \overline{r}_c = 0.04, \overline{g} = 10, \overline{D}_m = 1$ and $\partial\overline{C}/\partial\overline{z} = 1$.

the decrease in effective axial diffusion is still present at 270° inclination as shown in Fig. 6(b). From this observation, one can conclude that the blood flow should be against the gravitational force to lessen the decrease in diffusion for chemically reactive drug. For all angle of inclination, as the core radius increases, the effective axial diffusion decreases. Large core radius increases the yield stress that leads to increased blood viscosity due to more blood cells presence. Highly viscous blood makes it hard for solutes to diffuse smoothly. Hence, the decrease in effective axial diffusion. Not to mention, the effective axial diffusion decreases as the core radius increases until it reaches a certain core radius and start diffuses at a constant rate as shown in Fig. 6(a).

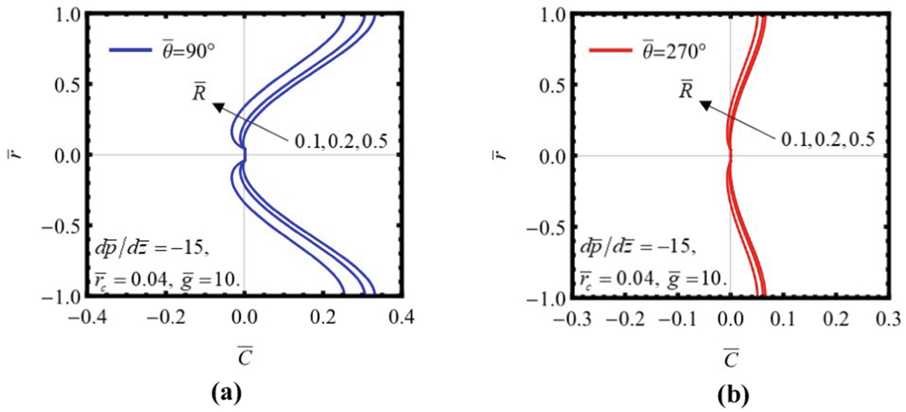

Fig. 5. Solute concentration at various chemical reaction rate with $\overline{\mu}_B = 1, \overline{a} = 1, \frac{d\overline{p}}{dz} = -15, \overline{\rho} = 1, \overline{r}_c = 0.04, \overline{D}_m = 1, \partial\overline{C}/\partial\overline{z} = 1$ for a) $\overline{\theta} = 90°$ and b) $\overline{\theta} = 270°$.

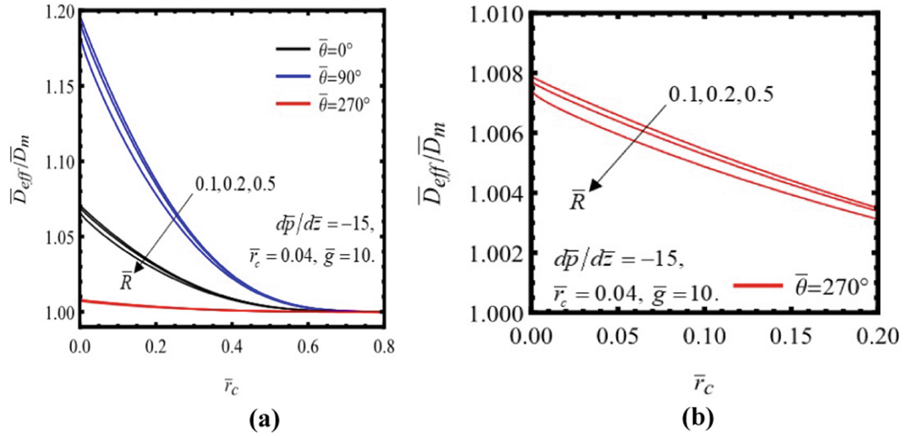

Fig. 6. Effective axial diffusion at various chemical reaction rate with $\overline{a} = 1, \frac{d\overline{p}}{dz} = -15, \overline{g} = 10, \overline{\rho} = 1, \overline{r}_c = 0.04, \overline{\mu}_B = 1, \overline{D}_m = 1, \partial\overline{C}/\partial\overline{z} = 1$ and $\overline{\theta} = 0°, 90°, 270°$.

5 Discussion

The present study investigates the steady dispersion of solute in a Bingham model through an inclined artery with the presence of chemical reaction. The blood is treated as Bingham model. The effect of arterial inclination, gravitational force and chemical reaction rate on the blood flow velocity, solute concentration and effective axial diffusion are investigated. For both blood flow and solute dispersion, the research results show an increase in blood velocity and solute concentration as the arterial inclination increases to 90° inclination. As the inclination increases more, the blood velocity decreases as it approaches 270° inclination. The highest and lowest velocity is at 90° and 270° inclinations respectively. This is due to the gravitational force fully acting in the blood flow either in the same flow direction at 90° inclination or against the flow at 270° inclination. Moreover, as the gravitational force is increased, it affects the blood velocity. At 90° inclination where the velocity is already increased, it enhances the increment significantly. Whereas at 270° inclination, increase in gravitational force enhances the decrease in blood velocity. The solute concentration solution also exhibits a similar pattern to the blood velocity solution. The highest and lowest concentration is at 90° and 270° inclinations respectively. Additionally, increase in chemical reaction rate decreases both solute concentration and effective axial diffusion. The solute concentration decreases after chemically reacting with the blood cells. The low concentration of solutes causes the dispersion along the artery to be inefficient. Hence the decreases in effective axial diffusion.

This present research considered the simple geometry of an artery. In reality, the human circulatory system is a more complex system. Not to mention, all the various factors affecting the blood flow and solute dispersion through the artery aside from arterial inclination and chemical reaction. This explores the possibilities for future studies by extending the present work. The present study can be further extended to research the solute dispersion in a blood flow through a different geometry of artery such as catheterized artery, pulsating artery, tapered artery, bifurcated artery and many more. Not to mention, applying additional condition such as body force acceleration and magnetic field to the blood flow and solute dispersion to observe their effects coupled with inclination and chemical reaction. Hopefully, this present study and the suggested research problems may inspire researchers to conduct future studies that contribute more in understanding the solute dispersion process in blood flow.

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