Alternative Interval Estimation for the Generalized Exponential Distribution with Interval-Censored and Fixed Covariate

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Abstract. This paper investigates several alternative methods of constructing confidence interval estimates based on the bootstrap and jackknife techniques for the interval censored generalized exponential distribution with fixed covariates. Simulation studies were conducted to compare the bootstrap techniques, which includes bootstrap-t, bootstrap-percentile, and bootstrap-normal interval estimation methods, with the jackknife confidence interval estimation methods using coverage probability. The results indicate that the bootstrap techniques works better than the jackknife techniques when dealing with interval-censored generalized exponential with fixed covariates.

Keywords: Bootstrap · Jackknife · Interval-censoring · Fixed covariate · Coverage probability study

1 Introduction

In inferential studies, building confidence intervals play vital roles in making decisions in hypothesis testing. The smaller the intervals, the better the model. Several researches have been conducted in which confidence intervals have been built, depending on some normality assumptions. These confidence intervals include Wald intervals, Likelihood ratio intervals, score intervals, Jeffery intervals, etc. Some of these intervals have been extensively studied and applied, such as works by [1–5], etc. Recently, alternative techniques for constructing confidence intervals without having to depend solely on asymptotic normality theory has become rather popular due to the deficiency of the asymptotic confidence intervals which suffers from a serious systematic negative bias in its coverage probability [6]. Examples of these alternative confidence intervals are the bootstrap confidence intervals, jackknife confidence intervals, etc. Several researches have been done on these alternative confidence intervals. The bootstrap percentile interval was proposed by Efron [7] where he found out that the bootstrap confidence interval reduces most

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of the error in standard approximation. Other works on bootstrap confidence interval includes [8–11] and so on.

In this paper, our goal is to study the inferential procedures by evaluating the performance of some alternative confidence interval techniques on the parameters of the interval censored generalized exponential distribution with fixed covariate through a coverage probability study. The coverage probability studies are conducted on the parameter estimates obtained through simulations which deals with the interval censoring. These studies were conducted to assess how close the estimated probability errors for all parameters are to the significance levels under different sample sizes with various censoring proportions and interval lengths. This study was conducted at significance level $\zeta = 0.10$ and 0.05. Four alternative confidence intervals are considered. They are the Jackknife, Bootstrap-t, Bootstrap-percentile and the Bootstrap-Normal confidence intervals.

The generalized exponential distribution as obtained by [12] is an extended form of the exponential distribution, obtained by introduction of the shape parameter in order to achieve better flexibility of the distribution to real life data. Given that $X_1, X_2, ..., X_n$ is a set of random samples assumed to be the distributed to the generalized exponential distribution, then the random variable X has the following cumulative distribution function (CDF),

$$F(t;\alpha,\lambda,\mu) = (1 - e^{-\frac{t-\mu}{\lambda}})^{\alpha}, t > \mu, \alpha > 0, \lambda > 0 \tag{1}$$

corresponding probability density function (pdf) as,

$$f(t;\alpha,\lambda,\mu) = \frac{\alpha}{\lambda}(1 - e^{-\frac{t-\mu}{\lambda}})^{\alpha-1} e^{-\frac{t-\mu}{\lambda}}, t > \mu, \alpha > 0, \lambda > 0 \tag{2}$$

and the survival function as:

$$S(t;\alpha,\lambda,\mu) = 1 - (1 - e^{-\frac{t-\mu}{\lambda}})^{\alpha}, t > \mu, \alpha > 0, \lambda > 0, \tag{3}$$

where α is a shape parameter; λ is a scale parameter, and μ is a location parameter. Some works have assessed some properties of this distribution, such as [13–15], etc. None of these work had considered studying the generalized exponential distribution under interval censoring mechanisms.

In order to incorporate the interval-censored event time and likelihood function of the generalized exponential distribution, the definition of some indicator variables for non-censored event times, left, right and interval-censored data is necessary. The indicator variables for i^{th} observation are defined as follows:

$$\delta_{E_i} = \begin{cases} 1, \text{ if the data is uncensored at } \mathrm{t_i} \\ 0, \text{ otherwise;} \end{cases}$$

$$\delta_{R_i} = \begin{cases} 1, \text{ if the data is right censored at } \mathrm{t_i} \\ 0, \text{ otherwise;} \end{cases}$$

$$\delta_{L_i} = \begin{cases} 1, \text{ if the event is left censored at } \mathrm{t_i} \\ 0, \text{ otherwise;} \end{cases}$$

$$\delta_{I_i} = \begin{cases} 1, \text{ if the event is interval} - \text{censored at } (\mathrm{L_i}, \mathrm{R_i}) \\ 0, \text{ otherwise.} \end{cases}$$

[16], among others, represent the likelihood construction for right, left, and interval-censored data for complete sample with $i = i, .., n$ is then:

$$L = \prod_{i=1}^{n} \left\{ f_i(t_i)^{\delta_{E_i}} F_i(t_{L_i})^{\delta_{L_i}} \left[S_i(t_{R_i}) \right]^{\delta_{R_i}} \left[S_i(t_{L_i}) - S_i(t_{R_i}) \right]^{\delta_{I_i}} \right\}.$$

where $L_i < t_i < R_i$, and the log–likelihood function is given by the following:

$$l = \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln[f_i(t_i)] + \delta_{L_i} \ln[F_i(t_{L_i})] + \delta_{R_i} \ln[S_i(t_{R_i})] + \delta_{I_i} \ln[S_i(t_{L_i}) - S(t_{R_i})] \right\} \tag{4}$$

Substituting the probability distribution and survivorship function of the model into the log–likelihood function, we obtain the likelihood function of the model without covariate as:

$$l = \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln\left[\alpha\beta (1 - e^{-\beta(t_i-\mu)})^{\alpha-1} e^{-\beta(t_i-\mu)} \right] + \delta_{L_i} \ln\left[(1 - e^{-\beta(t_{L_i}-\mu)})^{\alpha} \right] \right.$$
$$\left. + \delta_{R_i} \ln\left[1 - (1 - e^{-\beta(t_{R_i}-\mu)})^{\alpha} \right] + \delta_{I_i} \ln\left[1 - (1 - e^{-\beta(tL_i-\mu)})^{\alpha} - (1 - \left(1 - e^{-\beta(t_{R_i}-\mu)})^{\alpha} \right) \right] \right\}. \tag{5}$$

If we incorporate a single fixed covariate into the model (5) by setting $\beta = e^{(b_0+b_1x_i)}$, then the likelihood function of the interval censored generalized exponential int he presence of covariates will give,

$$l = \sum_{i=1}^{n} \left\{ \delta_{E_i} \ln\alpha + \delta_{E_i} \ln e^{(b_0+b_1x_i)} + \delta_{E_i}(\alpha - 1) \ln\left(1 - e^{-e^{(b_0+b_1x_i)}(t_i-\mu)} \right) \right.$$
$$- \delta_{E_i} e^{(b_0+b_1x_i)}(t_i - \mu) + \delta_{L_i} \alpha \ln\left[\left(1 - e^{-e^{(b_0+b_1x_i)}(t_{L_i}-\mu)} \right) \right]$$
$$- \delta_{R_i} \alpha \ln\left[\left(1 - e^{-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)} \right) \right] + \delta_{I_i} \alpha \ln\left(1 - e^{-exp^{(b_0+b_1x_i)}(t_{R_i}-\mu)} \right)$$
$$\left. - \delta_{I_i} \alpha \ln\left(1 - e^{-exp^{(b_0+b_1x_i)}(t_{L_i}-\mu)} \right) \right\}. \tag{6}$$

2 Confidence Interval Estimates

In this section, we assessed possible methods for creating confidence intervals for the model's parameters of the interval censored generalized exponential distribution with fixed covariates. We first considered the Bootstrap methods, which include the bootstrap-t, bootstrap normal and the bootstrap percentile. Then, we further considered the jack-knife method of confidence interval estimation. To create all of the bootstrap samples, we'll be using bootstrapping resampling using nonparametric samples. This method requires the resampling of a large number of B bootstrap samples with replacement from the original dataset with each observation having equal probability of being chosen. Then we estimate the bootstrap estimate $\widehat{\phi^b}$, where $b = 1, 2, ..., B$. Therefore, using this method with the generalized exponential distribution under interval censoring mechanism with fixed covariates, for all bootstrap methods considered, we need to estimate, say $\hat{\phi}_0^1, \hat{\phi}_0^2, \hat{\phi}_0^3, \ldots, \hat{\phi}_0^B$ which are obtained as the maximum likelihood estimates of ϕ_0 based on each of the B bootstrap samples. The bootstrap methods are discussed as follows:

2.1 Bootstrap-T Confidence Interval

If M Bootstrap samples of size n for $m = 1, 2, \ldots, M$, the bootstrap estimates $\hat{\gamma}_m$ is calculated from each of the bootstrap samples. The mean of the bootstrap estimates, $\hat{\gamma}$ is given as,

$$\hat{\gamma} = \frac{\sum_{m=1}^{M} \hat{\gamma}_m}{M} \tag{7}$$

Furthermore, the estimated bootstrap SE, se($\hat{\gamma}$) is obtained as,

$$se(\hat{\gamma}) = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M} \left(\hat{\gamma}_m - \overline{\hat{\gamma}}\right)}, \tag{8}$$

Therefore, the 100(1 − ϵ)% bootstrap-t CI for parameter γ can be derived as in,

$$\hat{\gamma} - t_{1-\frac{\epsilon}{2}} se(\hat{\gamma}) < \gamma < \hat{\gamma} - t_{\frac{\epsilon}{2}} se(\hat{\gamma}). \tag{9}$$

2.2 Percentile Bootstrap Confidence Interval

The Percentile bootstrap confidence interval is the interval that exist between $\frac{100\epsilon}{2}$ and $100\left(\frac{1-\epsilon}{2}\right)$ percentiles of the model parameters τ. To obtain the bootstrap percentile of τ, M random bootstrap samples is a first generated, then the parameter estimate is derived from each bootstrap samples. All the bootstrap model parameter estimates are placed in order of magnitude, starting from the lowest to the highest. The confidence interval is obtained to be,

$$\left\{\tau_{lowerlimit}, \tau_{upperlimit}\right\}. \tag{10}$$

2.3 Bootstrap-Normal Confidence Interval

If M Bootstrap samples of size n for $m = 1, 2, \ldots, M$, the bootstrap estimates $\hat{\tau}_m$ is calculated from each of the bootstrap samples. The mean of the bootstrap estimates, $\overline{\hat{\tau}}$ is given as,

$$\hat{\tau} = \frac{\sum_{m=1}^{M} \hat{\tau}_m}{M}, \tag{11}$$

and the estimated Bias is:

$$Bias(\tau) = \hat{\tau} - \hat{\tau}, \tag{12}$$

Furthermore, the estimated bootstrap SE, se($\hat{\tau}$) is obtained as,

$$se(\hat{\tau}) = \sqrt{\frac{1}{M-1}\sum_{m=1}^{M}(\hat{\tau}_m - \hat{\tau})}, \tag{13}$$

Therefore, the 100(1 − ϵ)% Normal bootstrap CI for parameter τ can be derived as,

$$\hat{\tau} - m_\tau + z_{\frac{\epsilon}{2}} se(\hat{\tau}) < \tau < \hat{\tau} - m_\tau + z_{1-\frac{\epsilon}{2}} se(\hat{\tau}). \tag{14}$$

2.4 Jackknife Interval Estimates

Another alternative confidence interval estimation method considered in this study is the Jackknife confidence interval method. The Jackknife was proposed by Quenouille [17] and later refined and given its current name by Tukey [18]. Quenouille [17] originally developed the method as a procedure for correcting bias. Later, Tukey [18] described its use in constructing confidence limits for a large class of estimators. It is similar to the bootstrap in that it involves resampling, but instead of sampling with replacement, the method samples without replacement. According to Arasan and Lunn [9] for a data set consisting of n observation, the i th jackknife sample is defined to be y with the i th observations removed. So, the i th jackknife sample would consist of $(n-1)$ observations, all except the i th observation.

$$y_{(i)} = (y_1, y_2, y_3, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n) \tag{15}$$

If $\hat{\tau}$ is the MLE of the parameter τ. Then, the new estimate $\widehat{\tau_{jack}}$ is defined by,

$$\widehat{\tau_{jack}} = \hat{\tau} - (n-1)\left(\widehat{\tau_{(.)}} - \hat{\tau}\right), \tag{16}$$

where, $\widehat{\tau_{(.)}} = \sum_{i=n}^{n}\frac{\widehat{\tau_{(i)}}}{n}$.

The jackknife estimate of the standard error (SE) is,

$$\widehat{SE}_{\tau_{jack}}(\hat{\tau}) = \sqrt{\frac{n-1}{n}\sum_{i=n}^{n}(\widehat{\tau_{(i)}} - \widehat{\tau_{(.)}})^2}.$$

the 100(1 − ϵ)% CI for τ is given as,

$$\widehat{\tau_{jack}} - t_{(1-\frac{\epsilon}{2}, n-1)}\widehat{SE}_{jack}(\hat{\tau}) < \tau < \widehat{\tau_{jack}} + t_{(1-\frac{\epsilon}{2}, n-1)}\widehat{SE}_{\tau_{jack}}(\hat{\tau}).$$

3 Coverage Probability Study

The coverage probability is the proportion of time a confidence interval includes the true population parameter value. A simulation study with $N = 1000$ samples for sample size, $n = 50$, 100, 200 and 500 were conducted to compare the performances of the confidence interval estimates at two nominal error probabilities, $\zeta = 0.05$ and 0.10 The approach of Manoharan et al. [19] was adopted to calculate the error probabilities from the left (lep) and right. (rep) and the corresponding total error probability (tep) which is the sum of (lep) and (rep). Upon computing tep, the confidence interval is termed anticonservative (AC) if $tep > \alpha + 2.58 s.e.(\hat{\alpha})$, conservative (C) if $tep < \alpha + 2.58 s.e.(\hat{\alpha})$ with $s.e.(\hat{\alpha}) = \sqrt{\alpha(1-\alpha)/N}$. For the asymmetric (AS), the larger error probability will be 1.5 times the smaller one. The confidence interval is optimal if AC, C and AS are close to zero and lep, rep and tep are close 0.025(0.05) and 0.05(0.10) respectively.

3.1 Coverage Probability Results of the Interval Censored Generalized Exponential Model with Fixed Covariates Using Bootstrap CI

In this section, a coverage probability study was conducted for the interval censored generalized exponential model with covariates using the jackknife CI. This study was conducted for the parameters of the interval censored generalized exponential under the settings of combination of different interval lengths and censoring percentages of (2.5, 0% (No censoring)), (2.5, 30% cp), (2.5, 50%), (2.5, 70%), (3.5, 0% (No censoring)), (3.5, 30% cp), (3.5, 50%), (3.5, 70%), (6.5, 0% (No censoring)), (6.5, 30% cp), (6.5, 50%), (6.5, 70%). Tables 1, 2, 3, 4, 5 and 6 show the estimated lep and rep generated by the alternative confidence interval using bootstrap procedures for each of the parameter estimates of the generalized exponential model with fixed covariates under the interval censoring mechanism. The table is reported for sample sizes $n = 50$, 100, 200 and 500.

Table 1 and 2 reveals the estimated error probabilities of the interval censored generalized exponential distribution using the bootstrap-t confidence interval. The figures for all the model parameters in both tables show values that are very close to their respective confidence interval, i.e. 0.10 and 0.05 respectively. This shows a good performance of the confidence interval method and its suitability to the generalized exponential model. Furthermore, the result shows that the lower the interval length, the closer the values of the error probabilities to the significant levels.

Tables 3 and 4 reveals the estimated error probabilities of the interval censored generalized exponential distribution using the bootstrap-percentile confidence interval. The figures for all the model parameters in both tables show values the values are all zeros, indicating that error probabilities for the model parameter diverges as the values were either close to 0 or 1.

Tables 5 and 6 reveals the estimated error probabilities of the interval censored generalized exponential distribution using the bootstrap-normal confidence interval. The figures for all the model parameters in both tables show values that are very close to their respective confidence interval, i.e. 0.10 and 0.05 respectively. This shows a good performance of the confidence interval method and its suitability to the generalized exponential model.

Tables 7, 8, 9, 10, 11 and 12 reveals the total number of AC, C and AS intervals for all the bootstrap methods examined. Considering at 10% significance level, the Bootstrap Normal confidence interval produced the lowest anticonservative and conservative intervals, when compared to the other bootstrap methods. This indicates that the Bootstrap normal performs best among the method. This also applies to 5% significance level.

The results from these tables shows that error probabilities from the Bootstrap Normal confidence intervals are close to the significance probability, that is ζ at both 0.10 and

Table 1. Estimated error probabilities for model using the Bootstrap-t Confidence Interval for Interval Censoring at 10% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*Tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	0%	50	0.006	0.049	0.055	0.018	0	0.018	0.065	0.046	0.111	0.022	0.088	0.11
		100	0.088	0.025	0.113	0.022	0	0.022	0.067	0.045	0.112	0.036	0.084	0.12
		200	0.058	0.027	0.085	0.058	0	0.058	0.049	0.043	0.092	0.028	0.076	0.104
		500	0.053	0.048	0.101	0.063	0.006	0.069	0.053	0.048	0.101	0.034	0.063	0.097
	30%	50	0.012	0.042	0.054	0.024	0	0.024	0.079	0.044	0.123	0.022	0.098	0.12
		100	0.073	0.033	0.106	0.028	0	0.028	0.065	0.048	0.113	0.032	0.084	0.116
		200	0.048	0.035	0.083	0.065	0	0.065	0.055	0.042	0.097	0.041	0.075	0.116
		500	0.064	0.039	0.103	0.066	0.015	0.081	0.056	0.049	0.105	0.042	0.06	0.102
	50%	50	0.004	0.04	0.044	0.02	0	0.02	0.071	0.036	0.107	0.036	0.094	0.13
		100	0.063	0.029	0.092	0.015	0	0.015	0.059	0.046	0.105	0.034	0.067	0.101
		200	0.052	0.033	0.085	0.062	0	0.062	0.049	0.045	0.094	0.029	0.075	0.104
		500	0.057	0.045	0.102	0.067	0.011	0.078	0.051	0.047	0.098	0.034	0.062	0.096
	70%	50	0.009	0.038	0.047	0.017	0	0.017	0.078	0.038	0.116	0.031	0.082	0.113
		100	0.085	0.031	0.116	0.029	0	0.029	0.063	0.048	0.111	0.035	0.084	0.119
		200	0.065	0.028	0.093	0.066	0	0.066	0.054	0.04	0.094	0.031	0.071	0.102
		500	0.063	0.048	0.111	0.06	0.01	0.07	0.052	0.051	0.103	0.04	0.065	0.105
3.5	0%	50	0.006	0.045	0.051	0.024	0	0.024	0.07	0.051	0.121	0.031	0.069	0.1
		100	0.069	0.041	0.11	0.033	0	0.033	0.078	0.043	0.121	0.032	0.083	0.115
		200	0.053	0.034	0.087	0.059	0	0.059	0.051	0.034	0.085	0.032	0.074	0.106
		500	0.069	0.038	0.107	0.071	0.019	0.09	0.056	0.048	0.104	0.034	0.067	0.101
	30%	50	0.004	0.043	0.047	0.024	0	0.024	0.076	0.05	0.126	0.028	0.085	0.113
		100	0.071	0.034	0.105	0.018	0	0.018	0.055	0.042	0.097	0.028	0.084	0.112
		200	0.052	0.034	0.086	0.056	0	0.056	0.049	0.047	0.096	0.034	0.083	0.117
		500	0.063	0.035	0.098	0.057	0.015	0.072	0.054	0.046	0.1	0.039	0.074	0.113
	50%	50	0.004	0.05	0.054	0.03	0	0.03	0.074	0.045	0.119	0.03	0.08	0.11
		100	0.075	0.028	0.103	0.047	0	0.047	0.077	0.036	0.113	0.039	0.07	0.109
		200	0.07	0.028	0.098	0.065	0.001	0.066	0.066	0.039	0.105	0.036	0.067	0.103
		500	0.063	0.047	0.11	0.065	0.007	0.072	0.047	0.056	0.103	0.038	0.065	0.103
	70%	50	0.051	0.037	0.088	0.018	0	0.018	0.063	0.05	0.113	0.028	0.099	0.127
		100	0.069	0.036	0.105	0.03	0	0.03	0.065	0.034	0.099	0.032	0.088	0.12
		200	0.054	0.039	0.093	0.068	0	0.068	0.058	0.045	0.103	0.036	0.076	0.112
		500	0.047	0.04	0.087	0.074	0.012	0.086	0.059	0.041	0.1	0.041	0.06	0.101
6.5	0%	50	0.011	0.042	0.053	0.03	0	0.03	0.064	0.051	0.115	0.024	0.095	0.119
		100	0.067	0.028	0.095	0.015	0	0.015	0.067	0.049	0.116	0.039	0.076	0.115
		200	0.057	0.039	0.096	0.071	0	0.071	0.063	0.047	0.11	0.033	0.068	0.101
		500	0.051	0.04	0.091	0.069	0.007	0.076	0.061	0.037	0.098	0.046	0.06	0.106

(*continued*)

Table 1. (*continued*)

	cp	n	μ			α			b_0			b_1		
			lep	rep	tep	lep	rep	Tep	lep	rep	tep	lep	rep	tep
	30%	50	0.02	0.037	0.057	0.045	0	0.045	0.072	0.037	0.109	0.018	0.104	0.122
		100	0.089	0.033	0.122	0.05	0	0.05	0.064	0.045	0.109	0.024	0.084	0.108
		200	0.054	0.029	0.083	0.059	0	0.059	0.061	0.042	0.103	0.041	0.065	0.106
		500	0.035	0.041	0.076	0.067	0.002	0.069	0.054	0.046	0.1	0.048	0.056	0.104
	50%	50	0.009	0.045	0.054	0.024	0	0.024	0.074	0.041	0.115	0.029	0.092	0.121
		100	0.066	0.032	0.098	0.012	0	0.012	0.078	0.039	0.117	0.034	0.071	0.105
		200	0.045	0.034	0.079	0.054	0	0.054	0.063	0.046	0.109	0.033	0.074	0.107
		500	0.042	0.051	0.093	0.075	0.004	0.079	0.047	0.049	0.096	0.042	0.063	0.105
	70%	50	0.003	0.053	0.056	0.021	0	0.021	0.075	0.039	0.114	0.032	0.083	0.115
		100	0.06	0.033	0.093	0.022	0	0.022	0.064	0.04	0.104	0.026	0.084	0.11
		200	0.055	0.025	0.08	0.056	0	0.056	0.068	0.043	0.111	0.038	0.068	0.106
		500	0.059	0.042	0.101	0.071	0.012	0.083	0.054	0.039	0.093	0.046	0.061	0.107

Table 2. Estimated error probabilities using the Bootstrap-t Confidence Interval for Interval Censoring at 5% significance level for various *n* and *cp*.

	cp	n	μ			α			b_0			b_1		
			lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
2.5	0%	50	0.001	0.039	0.04	0.016	0	0.016	0.026	0.029	0.055	0.007	0.051	0.058
		100	0.067	0.019	0.086	0.015	0	0.015	0.037	0.024	0.061	0.02	0.047	0.067
		200	0.031	0.018	0.049	0.04	0	0.04	0.028	0.029	0.057	0.015	0.042	0.057
		500	0.022	0.029	0.051	0.051	0.003	0.054	0.027	0.032	0.059	0.021	0.032	0.053
	30%	50	0.002	0.032	0.034	0.022	0	0.022	0.035	0.025	0.06	0.009	0.059	0.068
		100	0.051	0.026	0.077	0.025	0	0.025	0.031	0.026	0.057	0.018	0.048	0.066
		200	0.022	0.023	0.045	0.046	0	0.046	0.033	0.022	0.055	0.022	0.04	0.062
		500	0.029	0.023	0.052	0.049	0.003	0.052	0.026	0.023	0.049	0.019	0.033	0.052
	50%	50	0.002	0.034	0.036	0.018	0	0.018	0.043	0.02	0.063	0.017	0.051	0.068
		100	0.049	0.024	0.073	0.012	0	0.012	0.027	0.022	0.049	0.021	0.036	0.057
		200	0.026	0.024	0.05	0.041	0	0.041	0.021	0.026	0.047	0.014	0.038	0.052
		500	0.026	0.026	0.052	0.036	0.004	0.04	0.023	0.028	0.051	0.017	0.037	0.054
	70%	50	0	0.031	0.031	0.013	0	0.013	0.039	0.024	0.063	0.018	0.054	0.072
		100	0.063	0.021	0.084	0.015	0	0.015	0.037	0.028	0.065	0.01	0.056	0.066
		200	0.031	0.022	0.053	0.046	0	0.046	0.027	0.022	0.049	0.016	0.047	0.063
		500	0.028	0.027	0.055	0.047	0	0.047	0.024	0.023	0.047	0.012	0.033	0.045

(*continued*)

Table 2. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
3.5	0%	50	0	0.038	0.038	0.023	0	0.023	0.032	0.031	0.063	0.009	0.051	0.06
		100	0.046	0.024	0.07	0.024	0	0.024	0.038	0.023	0.061	0.014	0.048	0.062
		200	0.019	0.025	0.044	0.039	0	0.039	0.026	0.019	0.045	0.016	0.039	0.055
		500	0.037	0.027	0.064	0.043	0.004	0.047	0.025	0.027	0.052	0.016	0.041	0.057
	30%	50	0	0.039	0.039	0.023	0	0.023	0.039	0.024	0.063	0.016	0.05	0.066
		100	0.055	0.022	0.077	0.014	0	0.014	0.021	0.026	0.047	0.016	0.049	0.065
		200	0.032	0.022	0.054	0.036	0	0.036	0.026	0.02	0.046	0.012	0.043	0.055
		500	0.032	0.027	0.059	0.039	0.003	0.042	0.027	0.021	0.048	0.024	0.031	0.055
	50%	50	0	0.04	0.04	0.027	0	0.027	0.045	0.027	0.072	0.011	0.047	0.058
		100	0.064	0.021	0.085	0.032	0	0.032	0.045	0.021	0.066	0.019	0.04	0.059
		200	0.041	0.015	0.056	0.046	0	0.046	0.035	0.017	0.052	0.014	0.036	0.05
		500	0.022	0.028	0.05	0.045	0.002	0.047	0.027	0.025	0.052	0.019	0.03	0.049
	70%	50	0.004	0.028	0.032	0.013	0	0.013	0.038	0.028	0.066	0.011	0.055	0.066
		100	0.06	0.023	0.083	0.019	0	0.019	0.028	0.019	0.047	0.015	0.05	0.065
		200	0.018	0.021	0.039	0.056	0	0.056	0.028	0.026	0.054	0.015	0.047	0.062
		500	0.019	0.027	0.046	0.049	0.003	0.052	0.025	0.025	0.05	0.022	0.033	0.055
6.5	0%	50	0.003	0.032	0.035	0.025	0	0.025	0.037	0.024	0.061	0.008	0.049	0.057
		100	0.041	0.022	0.063	0.012	0	0.012	0.033	0.021	0.054	0.021	0.042	0.063
		200	0.029	0.024	0.053	0.052	0	0.052	0.033	0.024	0.057	0.013	0.039	0.052
		500	0.02	0.022	0.042	0.046	0.001	0.047	0.028	0.025	0.053	0.023	0.031	0.054
	30%	50	0.004	0.034	0.038	0.038	0	0.038	0.031	0.027	0.058	0.007	0.065	0.072
		100	0.063	0.019	0.082	0.037	0	0.037	0.038	0.028	0.066	0.011	0.053	0.064
		200	0.034	0.019	0.053	0.042	0	0.042	0.031	0.023	0.054	0.023	0.038	0.061
		500	0.013	0.021	0.034	0.047	0	0.047	0.028	0.023	0.051	0.023	0.033	0.056
	50%	50	0.001	0.036	0.037	0.018	0	0.018	0.041	0.031	0.072	0.012	0.045	0.057
		100	0.043	0.024	0.067	0.008	0	0.008	0.041	0.02	0.061	0.021	0.043	0.064
		200	0.021	0.024	0.045	0.042	0	0.042	0.029	0.017	0.046	0.016	0.047	0.063
		500	0.015	0.03	0.045	0.051	0	0.051	0.027	0.023	0.05	0.022	0.032	0.054
	70%	50	0	0.044	0.044	0.019	0	0.019	0.041	0.027	0.068	0.016	0.046	0.062
		100	0.046	0.024	0.07	0.019	0	0.019	0.029	0.025	0.054	0.013	0.047	0.06
		200	0.025	0.019	0.044	0.039	0	0.039	0.031	0.029	0.06	0.019	0.04	0.059
		500	0.026	0.022	0.048	0.047	0.003	0.05	0.031	0.022	0.053	0.022	0.036	0.058

Table 3. Estimated error probabilities for model using the Bootstrap-percentile Confidence Interval for Interval Censoring at 10% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	*0%*	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	30%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
3.5	*0%*	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	30%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

(*continued*)

Table 3. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0
6.5	0%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0
	30%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0
	50%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0
	70%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0

Table 4. Estimated error probabilities for model using the Bootstrap-percentile Confidence Interval for Interval Censoring at 5% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*let*	*rep*	*tep*	*lep*	*rep*	*rep*
2.5	0%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0
	30%	50	0	0	0	0	0	0	0	0	0	0	0	0
		100	0	0	0	0	0	0	0	0	0	0	0	0
		200	0	0	0	0	0	0	0	0	0	0	0	0
		500	0	0	0	0	0	0	0	0	0	0	0	0

(*continued*)

Table 4. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*let*	*rep*	*tep*	*lep*	*rep*	*rep*
	50%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
3.5	*0%*	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	30%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
6.5	*0%*	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

(*continued*)

Table 4. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*let*	*rep*	*tep*	*lep*	*rep*	*rep*
	30%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Table 5. Estimated error probabilities for model using the Bootstrap Normal Confidence Interval for Interval Censoring at 10% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	*0%*	*50*	*0.001*	*0.057*	*0.058*	*0.018*	*0*	*0.018*	*0.044*	*0.055*	*0.099*	*0.038*	*0.059*	*0.097*
		100	*0.056*	*0.041*	*0.097*	*0.019*	*0*	*0.019*	*0.056*	*0.058*	*0.114*	*0.058*	*0.054*	*0.112*
		200	*0.031*	*0.041*	*0.072*	*0.046*	*0*	*0.046*	*0.04*	*0.05*	*0.09*	*0.05*	*0.053*	*0.103*
		500	*0.027*	*0.066*	*0.093*	*0.061*	*0.01*	*0.071*	*0.052*	*0.05*	*0.102*	*0.045*	*0.05*	*0.095*
	30%	*50*	*0.002*	*0.054*	*0.056*	*0.022*	*0*	*0.022*	*0.046*	*0.059*	*0.105*	*0.036*	*0.065*	*0.101*
		100	*0.058*	*0.045*	*0.103*	*0.027*	*0*	*0.027*	*0.041*	*0.064*	*0.105*	*0.045*	*0.055*	*0.1*
		200	*0.026*	*0.052*	*0.078*	*0.061*	*0*	*0.061*	*0.046*	*0.053*	*0.099*	*0.059*	*0.055*	*0.114*
		500	*0.048*	*0.052*	*0.1*	*0.064*	*0.021*	*0.085*	*0.048*	*0.054*	*0.102*	*0.052*	*0.046*	*0.098*
	50%	*50*	*0.002*	*0.052*	*0.054*	*0.018*	*0*	*0.018*	*0.046*	*0.053*	*0.099*	*0.051*	*0.057*	*0.108*
		100	*0.049*	*0.039*	*0.088*	*0.015*	*0*	*0.015*	*0.047*	*0.049*	*0.096*	*0.048*	*0.047*	*0.095*
		200	*0.029*	*0.051*	*0.08*	*0.057*	*0.001*	*0.058*	*0.037*	*0.054*	*0.091*	*0.042*	*0.056*	*0.098*
		500	*0.049*	*0.053*	*0.102*	*0.059*	*0.016*	*0.075*	*0.049*	*0.049*	*0.098*	*0.041*	*0.054*	*0.095*
	70%	*50*	*0*	*0.048*	*0.048*	*0.015*	*0*	*0.015*	*0.046*	*0.065*	*0.111*	*0.044*	*0.059*	*0.103*
		100	*0.064*	*0.05*	*0.114*	*0.018*	*0*	*0.018*	*0.053*	*0.062*	*0.115*	*0.051*	*0.07*	*0.121*

(*continued*)

Table 5. (*continued*)

	cp	n	μ			α			b_0			b_1		
			lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
		200	0.036	0.044	0.08	0.059	0.001	0.06	0.046	0.057	0.103	0.048	0.054	0.102
		500	0.042	0.058	0.1	0.058	0.021	0.079	0.043	0.057	0.1	0.055	0.05	0.105
3.5	0%	50	0	0.055	0.055	0.023	0	0.023	0.049	0.063	0.112	0.043	0.061	0.104
		100	0.046	0.051	0.097	0.028	0	0.028	0.059	0.055	0.114	0.052	0.055	0.107
		200	0.023	0.047	0.07	0.05	0	0.05	0.046	0.049	0.095	0.049	0.048	0.097
		500	0.044	0.057	0.101	0.068	0.02	0.088	0.053	0.049	0.102	0.042	0.055	0.097
	30%	50	0.001	0.052	0.053	0.023	0	0.023	0.051	0.061	0.112	0.049	0.053	0.102
		100	0.055	0.053	0.108	0.016	0	0.016	0.037	0.053	0.09	0.054	0.051	0.105
		200	0.036	0.046	0.082	0.046	0	0.046	0.046	0.057	0.103	0.048	0.057	0.105
		500	0.043	0.046	0.089	0.056	0.015	0.071	0.05	0.052	0.102	0.051	0.063	0.114
	50%	50	0	0.057	0.057	0.028	0	0.028	0.047	0.059	0.106	0.044	0.063	0.107
		100	0.064	0.042	0.106	0.034	0	0.034	0.05	0.051	0.101	0.05	0.05	0.1
		200	0.041	0.05	0.091	0.055	0.004	0.059	0.051	0.044	0.095	0.057	0.047	0.104
		500	0.041	0.053	0.094	0.063	0.008	0.071	0.045	0.057	0.102	0.045	0.048	0.093
	70%	50	0.003	0.06	0.063	0.016	0	0.016	0.048	0.063	0.111	0.05	0.057	0.107
		100	0.06	0.047	0.107	0.025	0	0.025	0.042	0.054	0.096	0.053	0.053	0.106
		200	0.025	0.055	0.08	0.063	0	0.063	0.046	0.055	0.101	0.047	0.059	0.106
		500	0.031	0.056	0.087	0.071	0.016	0.087	0.053	0.051	0.104	0.051	0.046	0.097
6.5	0%	50	0.002	0.064	0.066	0.028	0	0.028	0.049	0.069	0.118	0.047	0.053	0.1
		100	0.049	0.041	0.09	0.014	0	0.014	0.053	0.057	0.11	0.05	0.052	0.102
		200	0.035	0.045	0.08	0.065	0.001	0.066	0.053	0.06	0.113	0.046	0.051	0.097
		500	0.036	0.048	0.084	0.062	0.011	0.073	0.046	0.049	0.095	0.054	0.055	0.109
	30%	50	0.003	0.046	0.049	0.04	0	0.04	0.05	0.045	0.095	0.039	0.065	0.104
		100	0.06	0.055	0.115	0.043	0	0.043	0.053	0.059	0.112	0.048	0.056	0.104
		200	0.036	0.039	0.075	0.051	0	0.051	0.048	0.053	0.101	0.049	0.051	0.1
		500	0.021	0.052	0.073	0.062	0.007	0.069	0.049	0.053	0.102	0.056	0.042	0.098
	50%	50	0.001	0.055	0.056	0.021	0	0.021	0.053	0.057	0.11	0.049	0.054	0.103
		100	0.051	0.042	0.093	0.012	0	0.012	0.052	0.045	0.097	0.053	0.053	0.106
		200	0.022	0.044	0.066	0.049	0	0.049	0.048	0.056	0.104	0.049	0.059	0.108
		500	0.03	0.056	0.086	0.068	0.015	0.083	0.046	0.053	0.099	0.049	0.051	0.1
	70%	50	0	0.057	0.057	0.02	0	0.02	0.055	0.054	0.109	0.05	0.058	0.108
		100	0.057	0.045	0.102	0.021	0	0.021	0.041	0.057	0.098	0.048	0.053	0.101
		200	0.028	0.045	0.073	0.047	0	0.047	0.045	0.053	0.098	0.047	0.054	0.101
		500	0.042	0.054	0.096	0.063	0.016	0.079	0.05	0.044	0.094	0.055	0.051	0.106

Table 6. Estimated error probabilities for model using the Bootstrap Normal Confidence Interval for Interval Censoring at 5% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	0%	50	0	0.047	0.047	0.014	0	0.014	0.014	0.04	0.054	0.018	0.034	0.052
		100	0	0.027	0.027	0.015	0	0.015	0.027	0.034	0.061	0.032	0.03	0.062
		200	0.025	0.026	0.051	0.037	0	0.037	0.022	0.032	0.054	0.024	0.028	0.052
		500	0.009	0.042	0.051	0.049	0.003	0.052	0.025	0.033	0.058	0.027	0.025	0.052
	30%	50	0	0.043	0.043	0.022	0	0.022	0.021	0.035	0.056	0.021	0.037	0.058
		100	0.012	0.032	0.044	0.019	0	0.019	0.019	0.036	0.055	0.029	0.027	0.056
		200	0.017	0.033	0.05	0.043	0	0.043	0.025	0.028	0.053	0.034	0.027	0.061
		500	0.018	0.03	0.048	0.048	0.006	0.054	0.023	0.028	0.051	0.025	0.024	0.049
	50%	50	0	0.038	0.038	0.017	0	0.017	0.027	0.032	0.059	0.033	0.035	0.068
		100	0	0.027	0.027	0.012	0	0.012	0.02	0.031	0.051	0.028	0.028	0.056
		200	0.016	0.031	0.047	0.039	0	0.039	0.019	0.031	0.05	0.018	0.029	0.047
		500	0.021	0.034	0.055	0.036	0.004	0.04	0.021	0.029	0.05	0.018	0.031	0.049
	70%	50	0	0.039	0.039	0.013	0	0.013	0.025	0.033	0.058	0.03	0.037	0.067
		100	0.038	0.031	0.069	0.013	0	0.013	0.025	0.034	0.059	0.023	0.035	0.058
		200	0.023	0.026	0.049	0.042	0	0.042	0.02	0.029	0.049	0.025	0.032	0.057
		500	0.017	0.034	0.051	0.046	0.002	0.048	0.021	0.03	0.051	0.024	0.024	0.048
3.5	0%	50	0	0.045	0.045	0.022	0	0.022	0.025	0.038	0.063	0.023	0.041	0.064
		100	0.013	0.039	0.052	0.02	0	0.02	0.024	0.032	0.056	0.024	0.03	0.054
		200	0.016	0.032	0.048	0.03	0	0.03	0.02	0.021	0.041	0.027	0.029	0.056
		500	0.022	0.034	0.056	0.043	0.004	0.047	0.024	0.027	0.051	0.022	0.026	0.048
	30%	50	0	0.043	0.043	0.018	0	0.018	0.023	0.039	0.062	0.024	0.028	0.052
		100	0.029	0.035	0.064	0.014	0	0.014	0.017	0.03	0.047	0.024	0.035	0.059
		200	0.023	0.029	0.052	0.034	0	0.034	0.024	0.026	0.05	0.027	0.027	0.054
		500	0.017	0.032	0.049	0.038	0.003	0.041	0.026	0.025	0.051	0.027	0.026	0.053
	50%	50	0	0.047	0.047	0.025	0	0.025	0.025	0.038	0.063	0.02	0.034	0.054
		100	0.044	0.028	0.072	0.03	0	0.03	0.026	0.03	0.056	0.032	0.032	0.064
		200	0.021	0.028	0.049	0.043	0	0.043	0.028	0.021	0.049	0.028	0.03	0.058
		500	0.012	0.035	0.047	0.045	0.002	0.047	0.026	0.028	0.054	0.022	0.024	0.046
	70%	50	0	0.041	0.041	0.013	0	0.013	0.021	0.037	0.058	0.026	0.04	0.066
		100	0.031	0.036	0.067	0.018	0	0.018	0.023	0.026	0.049	0.029	0.03	0.059
		200	0.014	0.035	0.049	0.051	0	0.051	0.024	0.03	0.054	0.025	0.032	0.057
		500	0.014	0.035	0.049	0.047	0.005	0.052	0.024	0.026	0.05	0.028	0.026	0.054
6.5	0%	50	0.001	0.043	0.044	0.023	0	0.023	0.024	0.036	0.06	0.02	0.034	0.054
		100	0	0.028	0.028	0.012	0	0.012	0.02	0.029	0.049	0.032	0.028	0.06
		200	0.022	0.035	0.057	0.045	0	0.045	0.028	0.035	0.063	0.02	0.029	0.049
		500	0.014	0.03	0.044	0.036	0.004	0.04	0.022	0.03	0.052	0.033	0.026	0.059
	30%	50	0	0.04	0.04	0.036	0	0.036	0.021	0.032	0.053	0.019	0.038	0.057
		100	0.041	0.034	0.075	0.033	0	0.033	0.026	0.038	0.064	0.018	0.035	0.053

(*continued*)

Table 6. (*continued*)

	cp	n	μ			α			b0			b1		
			lep	rep	tep	lep	rep	tep	lep	rep	tep	lep	rep	tep
		200	0.028	0.025	0.053	0.038	0	0.038	0.028	0.025	0.053	0.033	0.022	0.055
		500	0.009	0.025	0.034	0.04	0.002	0.042	0.021	0.027	0.048	0.028	0.025	0.053
	50%	50	0	0.045	0.045	0.016	0	0.016	0.028	0.037	0.065	0.029	0.024	0.053
		100	0	0.031	0.031	0.007	0	0.007	0.024	0.029	0.053	0.029	0.032	0.061
		200	0.018	0.032	0.05	0.037	0	0.037	0.022	0.024	0.046	0.025	0.028	0.053
		500	0.015	0.034	0.049	0.045	0.002	0.047	0.027	0.025	0.052	0.026	0.026	0.052
	70%	50	0	0.05	0.05	0.017	0	0.017	0.031	0.033	0.064	0.024	0.032	0.056
		100	0	0.029	0.029	0.018	0	0.018	0.024	0.03	0.054	0.022	0.03	0.052
		200	0.023	0.024	0.047	0.032	0	0.032	0.022	0.034	0.056	0.026	0.031	0.057
		500	0.017	0.029	0.046	0.042	0.003	0.045	0.029	0.024	0.053	0.026	0.028	0.054

Table 7. Total number of AC, C, and AS intervals for model parameters using the Bootstrap-t Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 10%.

	cp	n	μ			α			b_0			b_1		
			AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
2.5	0%	50	0	1	1	0	1	1	0	0	0	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	1	1	0	0	0	0	0	1
		500	0	0	0	0	1	1	0	0	0	0	0	1
	30%	50	0	1	1	0	1	1	0	0	1	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	0	0	1	1	0	0	0	0	0	1
		500	0	0	1	0	0	1	0	0	0	0	0	0
	50%	50	0	1	1	0	1	1	0	0	1	1	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	1	1	0	0	0	0	0	1
		500	0	0	0	0	0	1	0	0	0	0	0	1
	70%	50	0	1	1	0	1	1	0	0	1	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	1	1	0	0	0	0	0	1
		500	0	0	0	0	1	1	0	0	0	0	0	1

(*continued*)

Table 7. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
3.5	*0%*	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	30%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*1*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	50%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	70%	*50*	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*1*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
6.5	*0%*	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*
	30%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*

Table 8. Total number of AC, C, and AS intervals for model parameters using the Bootstrap-t Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 5%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	*0%*	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	30%	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*0*	*0*	*1*	*1*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*1*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*1*
	50%	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*0*	*0*	*2*	*2*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*0*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	70%	*50*	*0*	*2*	*2*	*0*	*2*	*2*	*0*	*0*	*2*	*1*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
3.5	*0%*	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*2*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*1*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	30%	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*1*	*0*	*2*	*0*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*1*
	50%	*50*	*0*	*1*	*2*	*0*	*2*	*2*	*1*	*0*	*2*	*0*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*2*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*2*	*0*	*0*	*2*
		500	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	70%	*50*	*0*	*1*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*1*	*0*	*0*	*2*

(*continued*)

Table 8. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
		200	0	0	0	0	1	2	0	0	0	0	0	2
		500	0	0	0	0	0	2	0	0	0	0	0	0
6.5	0%	50	0	1	2	0	2	2	0	0	1	0	0	2
		100	0	0	2	0	2	2	0	0	1	0	0	2
		200	0	0	0	0	1	2	0	0	0	0	0	2
		500	0	0	0	0	0	2	0	0	1	0	0	0
	30%	50	0	1	2	0	1	2	0	0	1	1	0	2
		100	1	0	2	0	1	2	0	0	0	0	0	2
		200	0	0	2	0	1	2	0	0	0	0	0	2
		500	0	0	1	0	1	2	0	0	0	0	0	0
	50%	50	0	1	2	0	2	2	1	0	1	0	0	2
		100	0	0	2	0	2	2	0	0	2	0	0	2
		200	0	0	0	0	1	2	0	0	1	0	0	2
		500	0	0	1	0	0	2	0	0	0	0	0	0
	70%	50	0	1	2	0	2	2	1	0	2	0	0	2
		100	1	0	2	0	2	2	0	0	1	0	0	2
		200	0	0	1	0	1	2	0	0	1	0	0	2
		500	0	0	0	0	0	2	0	0	0	0	0	1

Table 9. Total number of AC, C, and AS intervals for model parameters using the Bootstrap Percentile Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 10%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	0%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	30%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0

(*continued*)

Table 9. (*continued*)

	cp	n	μ			α			b_0			b_1		
			AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	50%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	70%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
3.5	0%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	30%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	50%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	70%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
6.5	0%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0
		200	0	1	0	0	1	0	0	1	0	0	1	0
		500	0	1	0	0	1	0	0	1	0	0	1	0
	30%	50	0	1	0	0	1	0	0	1	0	0	1	0
		100	0	1	0	0	1	0	0	1	0	0	1	0

(*continued*)

Table 9. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
		200	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		500	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
	50%	*50*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		100	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		200	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		500	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
	70%	*50*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		100	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		200	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*
		500	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*0*

Table 10. Total number of AC, C, and AS intervals for model parameters using the Bootstrap-Percentile Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 5%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	*0%*	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	30%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	50%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	70%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*

(*continued*)

Table 10. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
3.5	*0%*	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	30%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	50%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	70%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
6.5	*0%*	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	30%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	50%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
	70%	*50*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		100	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		200	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*
		500	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*2*	*0*

Table 11. Total number of AC, C, and AS intervals for model parameters using the Bootstrap Normal Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 10%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	*0%*	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*1*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	30%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*0*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
3.5	*0%*	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	30%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	50%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*

(*continued*)

Table 11. (*continued*)

	cp	n	μ			α			b_0			b_1		
			AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
		200	0	0	1	0	1	1	0	0	0	0	0	0
		500	0	0	1	0	0	1	0	0	0	0	0	0
6.5	0%	50	0	1	1	0	1	1	0	0	0	0	0	0
		100	0	0	0	0	1	1	0	0	0	0	0	0
		200	0	0	0	0	1	1	0	0	0	0	0	0
		500	0	0	0	0	1	1	0	0	0	0	0	0
	30%	50	0	1	1	0	1	1	0	0	0	0	0	1
		100	0	0	0	0	1	1	0	0	0	0	0	0
		200	0	1	0	0	1	1	0	0	0	0	0	0
		500	0	1	1	0	1	1	0	0	0	0	0	0
	50%	50	0	1	1	0	1	1	0	0	0	0	0	0
		100	0	0	0	0	1	1	0	0	0	0	0	0
		200	0	1	1	0	1	1	0	0	0	0	0	0
		500	0	0	1	0	0	1	0	0	0	0	0	0
	70%	50	0	1	1	0	1	1	0	0	0	0	0	0
		100	0	0	0	0	1	1	0	0	0	0	0	0
		200	0	1	1	0	1	1	0	0	0	0	0	0
		500	0	0	0	0	0	1	0	0	0	0	0	0

Table 12. Total number of AC, C, and AS intervals for model parameters using the Bootstrap Normal Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 5%.

	cp	n	μ			α			b_0			b_1		
			AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
2.5	0%	50	0	1	2	0	2	2	0	0	1	0	0	2
		100	0	1	1	0	2	2	0	0	0	0	0	0
		200	0	1	0	0	1	2	0	0	0	0	0	0
		500	0	0	2	0	1	2	0	0	0	0	0	0
	30%	50	0	1	2	0	2	2	0	0	1	0	0	2
		100	0	0	1	0	2	2	0	0	2	0	0	0

(*continued*)

Table 12. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
		200	0	0	2	0	1	2	0	0	0	0	0	0
		500	0	0	1	0	0	2	0	0	0	0	0	0
	50%	50	0	1	2	0	2	2	0	0	0	1	0	0
		100	0	1	1	0	2	2	0	0	1	0	0	0
		200	0	0	2	0	1	2	0	0	1	0	0	1
		500	0	0	1	0	1	2	0	0	0	0	0	1
	70%	50	0	1	2	0	2	2	0	0	0	0	0	0
		100	1	0	0	0	2	2	0	0	0	0	0	1
		200	0	0	0	0	1	2	0	0	0	0	0	0
		500	0	0	1	0	0	2	0	0	0	0	0	0
3.5	0%	50	0	1	2	0	2	2	0	0	1	0	0	1
		100	0	0	1	0	2	2	0	0	0	0	0	0
		200	0	1	2	0	2	2	0	0	0	0	0	0
		500	0	0	1	0	0	2	0	0	0	0	0	0
	30%	50	0	1	2	0	2	2	0	0	1	0	0	0
		100	0	0	0	0	2	2	0	0	1	0	0	0
		200	0	0	0	0	1	2	0	0	0	0	0	0
		500	0	0	1	0	1	2	0	0	0	0	0	0
	50%	50	0	1	2	0	2	2	0	0	1	0	0	1
		100	1	0	2	0	2	2	0	0	0	0	0	0
		200	0	0	0	0	1	2	0	0	0	0	0	0
		500	0	0	1	0	1	2	0	0	0	0	0	0
	70%	50	0	1	2	0	2	2	0	0	1	0	0	1
		100	0	0	0	0	2	2	0	0	0	0	0	0
		200	0	0	2	0	1	2	0	0	0	0	0	0
		500	0	0	2	0	0	2	0	0	0	0	0	0
6.5	0%	50	0	1	2	0	2	2	0	0	0	0	0	1
		100	0	1	1	0	2	2	0	0	0	0	0	0
		200	0	0	1	0	1	2	0	0	0	0	0	0
		500	0	0	1	0	1	2	0	0	0	0	0	0

(*continued*)

Table 12. (*continued*)

	cp	n	μ			α			b_0			b_1		
			AC	C	AS	AC	C	AS	AC	C	AS	AC	C	AS
	30%	50	0	1	2	0	1	2	0	0	1	0	0	2
		100	1	0	0	0	1	2	0	0	0	0	0	1
		200	0	1	0	0	1	2	0	0	0	0	0	0
		500	0	1	2	0	1	2	0	0	0	0	0	0
	50%	50	0	1	2	0	2	2	0	0	0	0	0	0
		100	0	1	1	0	2	2	0	0	0	0	0	0
		200	0	1	2	0	1	2	0	0	0	0	0	0
		500	0	0	2	0	0	2	0	0	0	0	0	0
	70%	50	0	1	2	0	2	2	0	0	0	0	0	0
		100	0	1	1	0	2	2	0	0	0	0	0	0
		200	0	1	1	0	2	2	0	0	1	0	0	0
		500	0	0	1	0	0	2	0	0	0	0	0	0

0.05. μ, α, b_0 and b_1 parameters of the model have all error probabilities close to ζ, both at 0.1 and 0.5 across all the censoring proportions. Also, for all the parameters, the error probabilities converge at lower censoring points but diverges as the censoring points increases, even for all the sample sizes. For Bootstrap-t, the error probabilities for μ and b_1 are closer to the significance values, For bootstrap-p.

Checking through the probability errors for the parameters of the model under right censoring observations, the results show that the Bootstrap-Normal confidence interval performs well for the μ, α, b_0 and b_1 due to the closeness of the probability error values for both parameters to the significance values 0.1 and 0.05, even at different censoring proportions. Furthermore, from the results obtained from the interval censoring, this was done considering different censoring proportions and different censoring lengths. Results also show that the Bootstrap Normal Confidence Interval performs well for μ and b_1. Furthermore, the probability errors gets converges to ζ at 0.10 and 0.05, no matter the interval length increases for all sample proportions. There is no effect of the increased or decreased values of the interval length.

3.2 Coverage Probability Results of Interval Censored Generalized Exponential Model with Fixed Covariates Using Jackknife CI

In this section, a coverage probability study was conducted for the interval censored generalized exponential model with covariates using the jackknife CI. This study was conducted for the parameters of the interval censored generalized exponential under the settings of combination of different interval lengths and censoring percentages of (2.5, 0% (No censoring)), (2.5, 30% cp), (2.5, 50%), (2.5, 70%), (3.5, 0% (No censoring)), (3.5, 30% cp), (3.5, 50%), (3.5, 70%), (6.5, 0%(No censoring)), (6.5, 30% cp), (6.5, 50%), (6.5, 70%).

Results from Tables 13 and 14 show the estimated lep and rep generated by the Jackknife confidence interval for each of the parameter estimates of the generalized exponential model with fixed covariates under the interval censoring mechanism. The table is reported for sample sizes $n = 50$, 100, 200 and 500 (Tables 15 and 16).

Table 13. Estimated error probabilities for model using the Jackknife Confidence Interval for Interval Censoring at 10% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	0%	50	0.074	0.032	0.106	0.017	0	0.017	0.08	0.034	0.114	0.028	0.092	0.12
		100	0.062	0.029	0.091	0.063	0	0.063	0.064	0.047	0.111	0.029	0.085	0.114
		200	0.053	0.035	0.088	0.07	0.007	0.077	0.055	0.047	0.102	0.035	0.07	0.105
		500	0.07	0.031	0.101	0.046	0.028	0.074	0.058	0.043	0.101	0.043	0.065	0.108
	30%	50	0.097	0.029	0.126	0.014	0	0.014	0.06	0.05	0.11	0.025	0.105	0.13
		100	0.062	0.03	0.092	0.05	0	0.05	0.066	0.038	0.104	0.026	0.063	0.089
		200	0.058	0.024	0.082	0.06	0.009	0.069	0.062	0.045	0.107	0.039	0.08	0.119
		500	0.071	0.035	0.106	0.061	0.033	0.094	0.055	0.062	0.117	0.037	0.076	0.113
	50%	50	0.078	0.028	0.106	0.032	0	0.032	0.056	0.042	0.098	0.032	0.096	0.128
		100	0.063	0.027	0.09	0.073	0	0.073	0.068	0.054	0.122	0.039	0.084	0.123
		200	0.045	0.037	0.082	0.073	0.011	0.084	0.052	0.049	0.101	0.043	0.062	0.105
		500	0.069	0.033	0.102	0.064	0.027	0.091	0.046	0.048	0.094	0.044	0.057	0.101
	70%	50	0.091	0.031	0.122	0.036	0	0.036	0.066	0.044	0.11	0.023	0.087	0.11
		100	0.045	0.039	0.084	0.043	0	0.043	0.062	0.035	0.097	0.035	0.071	0.106
		200	0.062	0.028	0.09	0.055	0.003	0.058	0.054	0.044	0.098	0.028	0.076	0.104
		500	0.066	0.036	0.102	0.061	0.019	0.08	0.058	0.047	0.105	0.036	0.069	0.105
3.5	0%	50	0.076	0.027	0.103	0.007	0	0.007	0.061	0.048	0.109	0.022	0.088	0.11
		100	0.054	0.031	0.085	0.058	0	0.058	0.053	0.05	0.103	0.035	0.069	0.104
		200	0.065	0.03	0.095	0.059	0.004	0.063	0.056	0.051	0.107	0.037	0.066	0.103
		500	0.067	0.035	0.102	0.064	0.032	0.096	0.048	0.062	0.11	0.032	0.071	0.103
	30%	50	0.084	0.035	0.119	0.03	0	0.03	0.065	0.046	0.111	0.039	0.083	0.122
		100	0.057	0.028	0.085	0.061	0	0.061	0.061	0.046	0.107	0.028	0.063	0.091
		200	0.05	0.04	0.09	0.062	0.003	0.065	0.049	0.054	0.103	0.032	0.08	0.112
		500	0.066	0.035	0.101	0.059	0.031	0.09	0.053	0.044	0.097	0.041	0.065	0.106
	50%	50	0.095	0.026	0.121	0.029	0	0.029	0.068	0.038	0.106	0.025	0.074	0.099
		100	0.061	0.029	0.09	0.045	0	0.045	0.059	0.05	0.109	0.029	0.078	0.107
		200	0.04	0.042	0.082	0.06	0.002	0.062	0.055	0.033	0.088	0.032	0.066	0.098
		500	0.057	0.041	0.098	0.065	0.027	0.092	0.055	0.052	0.107	0.041	0.06	0.101
	70%	50	0.077	0.035	0.112	0.03	0	0.03	0.06	0.05	0.11	0.024	0.086	0.11
		100	0.062	0.029	0.091	0.062	0	0.062	0.052	0.045	0.097	0.037	0.08	0.117

(*continued*)

Table 13. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
		200	0.063	0.035	0.098	0.061	0.006	0.067	0.046	0.048	0.094	0.024	0.071	0.095
		500	0.074	0.034	0.108	0.065	0.029	0.094	0.051	0.055	0.106	0.044	0.063	0.107
6.5	0%	50	0.07	0.034	0.104	0.02	0	0.02	0.057	0.044	0.101	0.022	0.102	0.124
		100	0.068	0.032	0.1	0.063	0	0.063	0.055	0.045	0.1	0.028	0.086	0.114
		200	0.065	0.035	0.1	0.06	0.007	0.067	0.064	0.04	0.104	0.035	0.07	0.105
		500	0.069	0.031	0.1	0.055	0.03	0.085	0.049	0.053	0.102	0.033	0.066	0.099
	30%	50	0.082	0.028	0.11	0.019	0	0.019	0.059	0.048	0.107	0.03	0.087	0.117
		100	0.065	0.025	0.09	0.068	0	0.068	0.064	0.053	0.117	0.031	0.075	0.106
		200	0.049	0.032	0.081	0.071	0.006	0.077	0.058	0.051	0.109	0.035	0.068	0.103
		500	0.066	0.038	0.104	0.065	0.021	0.086	0.053	0.045	0.098	0.049	0.063	0.112
	50%	50	0.082	0.026	0.108	0.011	0	0.011	0.059	0.048	0.107	0.029	0.09	0.119
		100	0.07	0.03	0.1	0.057	0	0.057	0.058	0.045	0.103	0.03	0.079	0.109
		200	0.054	0.041	0.095	0.062	0.003	0.065	0.056	0.046	0.102	0.028	0.074	0.102
		500	0.058	0.037	0.095	0.058	0.03	0.088	0.056	0.052	0.108	0.042	0.056	0.098
	70%	50	0.072	0.032	0.104	0.02	0	0.02	0.049	0.052	0.101	0.033	0.087	0.12
		100	0.04	0.019	0.059	0.01	0	0.01	0.06	0.047	0.107	0.029	0.075	0.104
		200	0.049	0.032	0.081	0.062	0.006	0.068	0.05	0.032	0.082	0.033	0.067	0.1
		500	0.072	0.033	0.105	0.052	0.019	0.071	0.051	0.047	0.098	0.041	0.072	0.113

Table 14. Estimated error probabilities for model using the Jackknife Confidence Interval for Interval Censoring at 5% significance level for various *n* and *cp*.

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
2.5	0%	50	0.034	0.026	0.06	0.013	0	0.013	0.047	0.019	0.066	0.017	0.047	0.064
		100	0.029	0.019	0.048	0.046	0	0.046	0.036	0.021	0.057	0.017	0.046	0.063
		200	0.029	0.023	0.052	0.047	0.001	0.048	0.033	0.028	0.061	0.015	0.041	0.056
		500	0.038	0.017	0.055	0.03	0.006	0.036	0.035	0.02	0.055	0.015	0.036	0.051
	30%	50	0.061	0.02	0.081	0.013	0	0.013	0.023	0.024	0.047	0.015	0.059	0.074
		100	0.029	0.019	0.048	0.036	0	0.036	0.027	0.018	0.045	0.017	0.035	0.052
		200	0.03	0.015	0.045	0.048	0	0.048	0.032	0.021	0.053	0.017	0.045	0.062
		500	0.037	0.023	0.06	0.038	0.012	0.05	0.025	0.019	0.044	0.019	0.039	0.058
	50%	50	0.051	0.018	0.069	0.026	0	0.026	0.028	0.026	0.054	0.018	0.055	0.073
		100	0.039	0.019	0.058	0.05	0	0.05	0.03	0.025	0.055	0.016	0.048	0.064
		200	0.024	0.02	0.044	0.047	0	0.047	0.026	0.031	0.057	0.024	0.033	0.057
		500	0.034	0.02	0.054	0.03	0.007	0.037	0.018	0.026	0.044	0.021	0.029	0.05

(*continued*)

Table 14. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			lep	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*	*lep*	*rep*	*tep*
	70%	50	0.058	0.019	0.077	0.024	0	0.024	0.036	0.028	0.064	0.014	0.054	0.068
		100	0.023	0.026	0.049	0.033	0	0.033	0.037	0.021	0.058	0.023	0.041	0.064
		200	0.036	0.019	0.055	0.039	0.002	0.041	0.028	0.025	0.053	0.015	0.046	0.061
		500	0.036	0.017	0.053	0.035	0.005	0.04	0.032	0.025	0.057	0.021	0.04	0.061
3.5	0%	50	0.052	0.018	0.07	0.007	0	0.007	0.025	0.024	0.049	0.013	0.055	0.068
		100	0.026	0.023	0.049	0.047	0	0.047	0.025	0.026	0.051	0.017	0.035	0.052
		200	0.038	0.012	0.05	0.039	0	0.039	0.025	0.029	0.054	0.017	0.035	0.052
		500	0.039	0.017	0.056	0.039	0.013	0.052	0.024	0.031	0.055	0.02	0.035	0.055
	30%	50	0.024	0.024	0.048	0.023	0	0.023	0.033	0.028	0.061	0.023	0.047	0.07
		100	0.034	0.017	0.051	0.039	0	0.039	0.027	0.019	0.046	0.015	0.041	0.056
		200	0.024	0.025	0.049	0.047	0.001	0.048	0.025	0.031	0.056	0.019	0.034	0.053
		500	0.042	0.017	0.059	0.04	0.009	0.049	0.025	0.026	0.051	0.019	0.035	0.054
	50%	50	0.052	0.019	0.071	0.026	0	0.026	0.037	0.026	0.063	0.011	0.045	0.056
		100	0.032	0.017	0.049	0.027	0	0.027	0.029	0.024	0.053	0.017	0.041	0.058
		200	0.019	0.022	0.041	0.038	0	0.038	0.033	0.02	0.053	0.017	0.037	0.054
		500	0.036	0.024	0.06	0.04	0.008	0.048	0.03	0.023	0.053	0.027	0.03	0.057
	70%	50	0.036	0.025	0.061	0.025	0	0.025	0.026	0.028	0.054	0.015	0.05	0.065
		100	0.035	0.019	0.054	0.044	0	0.044	0.024	0.027	0.051	0.016	0.041	0.057
		200	0.034	0.02	0.054	0.042	0	0.042	0.026	0.028	0.054	0.011	0.041	0.052
		500	0.036	0.016	0.052	0.037	0.008	0.045	0.029	0.026	0.055	0.019	0.038	0.057
6.5	0%	50	0.035	0.024	0.059	0.016	0	0.016	0.029	0.027	0.056	0.01	0.058	0.068
		100	0.037	0.021	0.058	0.049	0	0.049	0.031	0.026	0.057	0.015	0.051	0.066
		200	0.044	0.018	0.062	0.047	0.001	0.048	0.028	0.026	0.054	0.009	0.041	0.05
		500	0.034	0.018	0.052	0.035	0.011	0.046	0.02	0.026	0.046	0.016	0.029	0.045
	30%	50	0.057	0.019	0.076	0.016	0	0.016	0.027	0.024	0.051	0.015	0.054	0.069
		100	0.045	0.008	0.053	0.049	0	0.049	0.028	0.029	0.057	0.017	0.04	0.057
		200	0.028	0.02	0.048	0.049	0	0.049	0.022	0.029	0.051	0.018	0.044	0.062
		500	0.034	0.017	0.051	0.041	0.009	0.05	0.028	0.018	0.046	0.022	0.036	0.058
	50%	50	0.03	0.021	0.051	0.009	0	0.009	0.026	0.029	0.055	0.017	0.048	0.065
		100	0.031	0.021	0.052	0.041	0	0.041	0.032	0.016	0.048	0.013	0.049	0.062
		200	0.028	0.026	0.054	0.048	0	0.048	0.027	0.025	0.052	0.014	0.042	0.056
		500	0.032	0.019	0.051	0.04	0.008	0.048	0.031	0.022	0.053	0.017	0.031	0.048
	70%	50	0.055	0.023	0.078	0.014	0	0.014	0.028	0.034	0.062	0.015	0.052	0.067
		100	0.022	0.016	0.038	0.008	0	0.008	0.034	0.027	0.061	0.015	0.038	0.053
		200	0.031	0.024	0.055	0.04	0	0.04	0.021	0.021	0.042	0.015	0.032	0.047
		500	0.038	0.021	0.059	0.038	0.005	0.043	0.03	0.021	0.051	0.016	0.038	0.054

Table 15. Total number of AC, C, and AS intervals for model parameters using the Jackniffe Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 10%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	*0%*	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	30%	*50*	*1*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*1*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	50%	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*1*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
3.5	*0%*	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	30%	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		500	*0*	*0*	*1*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
	50%	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		200	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
		500	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*
		100	*0*	*0*	*1*	*0*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*1*

(*continued*)

Table 15. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
		200	0	0	1	0	1	1	0	0	0	0	0	1
		500	0	0	1	0	0	1	0	0	0	0	0	0
6.5	0%	50	0	0	1	0	1	1	0	0	0	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	1	1	0	0	1	0	0	1
		500	0	0	1	0	0	1	0	0	0	0	0	1
	30%	50	0	0	1	0	1	1	0	0	0	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	0	1	0	0	0	0	0	1
		500	0	0	1	0	0	1	0	0	0	0	0	0
	50%	50	0	0	1	0	1	1	0	0	0	0	0	1
		100	0	0	1	0	1	1	0	0	0	0	0	1
		200	0	0	0	0	1	1	0	0	0	0	0	1
		500	0	0	1	0	0	1	0	0	0	0	0	0
	70%	50	0	0	1	0	1	1	0	0	0	0	0	1
		100	0	1	1	0	1	1	0	0	0	0	0	1
		200	0	0	1	0	1	1	0	0	1	0	0	1
		500	0	0	1	0	1	1	0	0	0	0	0	1

Table 16. Total number of AC, C, and AS intervals for model parameters using the Jackknife Confidence Interval for interval censoring over varying sample sizes, different interval length, censored proportions and Type I error 5%.

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
2.5	0%	50	0	0	1	0	2	2	0	0	2	0	0	2
		100	0	0	2	0	1	2	0	0	1	0	0	2
		200	0	0	1	0	0	2	0	0	0	0	0	2
		500	0	0	2	0	1	2	0	0	1	0	0	2
	30%	50	2	0	2	0	2	2	0	0	0	2	0	2
		100	0	0	2	0	1	2	0	0	2	0	0	2

(*continued*)

Table 16. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	50%	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*2*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*0*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*0*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*2*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
3.5	*0%*	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	30%	*50*	*0*	*0*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
	50%	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*2*	*0*	*0*	*2*
		500	*0*	*0*	*0*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*0*
	70%	*50*	*0*	*0*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*1*
6.5	*0%*	*50*	*0*	*0*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*

(*continued*)

Table 16. (*continued*)

	cp	*n*	μ			α			b_0			b_1		
			AC	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*	*AC*	*C*	*AS*
	30%	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*1*	*0*	*2*
		100	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*1*	*0*	*0*	*1*
	50%	*50*	*0*	*0*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		100	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		200	*0*	*0*	*0*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*0*	*2*	*0*	*0*	*0*	*0*	*0*	*1*
	70%	*50*	*1*	*0*	*2*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		100	*0*	*1*	*1*	*0*	*2*	*2*	*0*	*0*	*0*	*0*	*0*	*2*
		200	*0*	*0*	*1*	*0*	*1*	*2*	*0*	*0*	*1*	*0*	*0*	*2*
		500	*0*	*0*	*2*	*0*	*1*	*2*	*0*	*0*	*0*	*0*	*0*	*2*

Results show that the Jackknife Confidence Interval performs well for μ and b_1. Also, as the interval length increases, the probability errors deviate further away from ζ at 0.10 and 0.05. Also, as the interval length increases, the probability gets closer to zero. In addition to these, as the lower the interval length, the closer the probability errors to 0.1 and 0.05 as the case may be.

The Table 14 also reveals that as the censoring proportions increases, the probability errors either gets bigger or smaller.

4 Conclusion

In this section, we have considered four different asymptotic inferential procedures by evaluating the performances of the alternative confidence intervals for the parameters of the interval censored generalized exponential distribution with fixed covariates. These studies were conducted to assess how close the estimated probability errors for all parameters are to the significance levels under different sample sizes with various censoring proportions. This study was conducted at significance level $\zeta = 0.10$ and 0.05. Results of a simulation studies for each category show that probability estimates of the values for the parameters of the generalized exponential model with interval censored using the bootstrap-normal confidence interval performed best when compared to the other confidence intervals. Its error probabilities are close to the significance level across the censoring points and interval lengths. The values converge easily. On the part of computation, simulation runs faster and few Anti-conservatives and Conservative values, when compared with the jackknife confidence interval methods.

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