Short-Term Performance of the Probability-Based Universal Portfolio

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Abstract. Universal portfolio is an investment strategy that produced an efficient performance in the stock market. Two universal portfolios generated by probability distribution, namely multinomial distribution universal portfolio and multivariate normal distribution universal portfolio are selected in this study. The above two universal portfolios are run on selected stock-price data sets from the local stock exchange. Empirical results show that the above two universal portfolios perform well for short term period.

Keywords: Universal portfolio · Multinomial distribution · Multivariate Normal distribution · short term

1 Introduction

The Investment has been a popular subject lately. Hence, a study on the investment in the stock market is required to maximize the return of capital and minimize the risk of investment. There are different investment strategies that can be used to maximize the return. The universal portfolio is used as an investment strategy to study the efficiency of the performance in the stock market. The main goal for this study is to develop a better understanding of the universal portfolio and study the performance of the universal portfolio to increase the wealth return in a short-term period.

Universal portfolios generated by different methods are of recent interest. One of early methods of generating a universal portfolio is that due to Cover and Ordentlich [1] using the moments of the Dirichlet. Tan [2] introduces the memory cum-time saving finite order universal portfolio generated by probability distribution. In Pang [3], the performance of the finite order distributed generated universal portfolio was shown to be better than Constant Rebalanced portfolio (CRP) with some selected parameters of proposed universal portfolio. Matrix-generated divergences can also be applied to generate a universal portfolio [4] and partially Convex functions have been used to generate universal portfolios in [5]. Universal portfolios generated by inequality ratio methods have been discussed in [6]. The latest research [7] in universal portfolios involves a portfolio generated from the zero-gradient set of an objective involving the estimated daily rate of wealth increase and the extend f- divergence.

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2 Methodology

The scope of the study is to investigate the performance of the finite-order universal portfolios generated by the probability distribution in the short-term period. Twelve Malaysia stocks are collected from Kuala Lumpur Stock Exchange (KLSE) from 1 January 2019 to 31 December 2019. Two order-1 universal portfolios, namely multivariate normal distribution universal portfolio (MNUP) and multinomial distribution universal portfolio (MUP) are run over the selected data sets for short term period. The performance of above two universal portfolios in the short-term period are studied and compared.

In Pang, Liew and Chang [3], let $\boldsymbol{x}_n = (x_{n1}, x_{n2}, \cdots, x_{ni})$ be the price relative vector for i^{th} stocks on trading day n. The price-relative factor, x_{ni} is the ratio of the closing price to the opening price of i^{th} stock on n^{th} trading day. Let x^n be the price-relative sequence $x_1, x_2, \ldots, x_n$.

Let $\boldsymbol{b}_n = (b_{n1}, b_{n2}, \cdots, b_{ni})$ be the portfolio vector such that $b_n \geq 0$ and $\sum_{n=1}^{i} b_n = 1$,b_n is defined as the proportion b_{ni} of wealth invested in the i^{th} stock on the trading day n.

Let $S_0(b) = 1$ be the initial wealth and let $S_n(b)$ be the accumulated wealth in the end of the trading day n.

$$S_n(b) = \prod\nolimits_{i=1}^{n} b_i^T x_i \tag{1}$$

where T is the transpose of a vector.

2.1 Order-v Universal Portfolio

In Pang, Liew and Chang [3], let $Y = (Y_1, \ldots, Y_m)$ be m independent random vector with probability distribution of $f(Y)$ such that $0 \leq Y_i \leq 1$, $\sum Y_i = 1$. The proportion of wealth for stock k^{th} on the $(n+1)^{th}$ trading day for the order v universal is

$$b_{n+1,k} = \frac{E(\mathrm{Y}_k \prod_{i=1}^{n} \mathrm{Y}^t \mathrm{x}_j)}{E(\prod_{i=1}^{n} \mathrm{Y}^t \mathrm{x}_j)} = \frac{E\big[\mathrm{Y}_k \big(\mathrm{Y}^t \mathrm{x}_n\big)\big(\mathrm{Y}^t \mathrm{x}_{n-1}\big) \ldots \big(\mathrm{Y}^t \mathrm{x}_1\big)\big]}{E\big[(\mathrm{Y}^t \mathrm{x}_n)(\mathrm{Y}^t \mathrm{x}_{n-1}) \ldots (\mathrm{Y}^t \mathrm{x}_1)\big]} \tag{2}$$

for $k = 1, 2, \ldots, m.\xi_{n+1,v}$ is denotes as the normalizing constant, whereas it is the denominator of $b_{n+1,k}$ as

$$\xi_{n+1,v} = \Big[\sum\nolimits_{k=1}^{m} \Big(\sum\nolimits_{i_1=1}^{m} \cdots \sum\nolimits_{i_v=1}^{m} x_{n,i_1} \cdots x_{n-v+1,i_v} E\big[Y_k Y_{i_1} \cdots Y_{i_v}\big]\Big)\Big]^{-1} \tag{3}$$

The total wealth in the end of $(n+1)^{th}$ trading day, S_{n+1} is

$$S_{n+1}(b) = \Big(b_{n+1}^T x_{n+1}\Big)\Big(b_n^T x_n\Big) \cdots (b_{n-v+2}^T x_{n-v+2}) \cdot S_{n-v+1}(b) \tag{4}$$

When $\boldsymbol{v} = 1$**,** the order 1 universal portfolio becomes

$$b_{n+1,k} = \xi_{n,1} \Big(\sum\nolimits_{i=1}^{m} x_{n,i} E[Y_k Y_i]\Big) \tag{5}$$

for $k = 1, 2, \ldots, m$ where the normalizing constant, $\xi_{n,1}$ is

$$\xi_{n,1} = \left[\sum_{k=1}^{m}\left(\sum_{i=1}^{m} x_{n,i} E[Y_k Y_i]\right)\right]^{-1} \tag{6}$$

and the total wealth, S_n is

$$S_{n+1}(b) = \left(b_{n+1}^{T} x_{n+1}\right) \cdot S_n(b) \tag{7}$$

When $m = 3$,

$$b_{n+1,k} = \frac{\left(x_{n,1}E[Y_k Y_1] + x_{n,2}E[Y_k Y_2] + x_{n,3}E[Y_k Y_3]\right)}{\xi_{n,1}} \tag{8}$$

for $k = 1, 2, 3$ where the denominator,$\xi_{n,1}$ is

$$\begin{aligned}\xi_{n,1} &= x_{n,1}E[Y_1Y_1] + x_{n,1}E[Y_2Y_1] + x_{n,1}E[Y_3Y_1] \\ &+ x_{n,2}E[Y_1Y_2] + x_{n,2}E[Y_2Y_2] + x_{n,2}E[Y_3Y_2] \\ &+ x_{n,3}E[Y_1Y_3] + x_{n,3}E[Y_2Y_3] + x_{n,3}E[Y_3Y_3]\end{aligned} \tag{9}$$

Multinomial Distribution Universal Portfolio (MUP)

In Tan and Pang [8], let $Y = (Y_1, \ldots, Y_m)$ be a vector randomly selected, whereas it has a joint multinomial distribution of parameters $N, p_1, p_2, \ldots, p_{m-1}$ where $0 < p_i < 1$ for $i = 1, 2, \ldots, m-1$, then $0 < p_m < 1$, where $p_m = 1 - p_1 - p_2 - \cdots - p_{m-1}$. N is larger than the number of market stocks. The joint distribution $f(Y) = f\left(y_1, y_2, \ldots, y_m\right)$ is

$$f\left(y_1, y_2, \ldots, y_m\right) = \binom{N}{y_1 y_2 \ldots y_m} p_1^{y_1} p_2^{y_1} \ldots p_m^{y_m} = \frac{N}{y_1! y_2! \ldots y_m!} p_1^{y_1} p_2^{y_1} \ldots p_m^{y_m} \tag{10}$$

where $y_i = 0, 1, 2, \ldots, N$ for $i = 1, 2, \ldots, m$ and $\sum y_i = N$.

The joint moment generating function for $(Y_1, \ldots, Y_m)$ is

$$M(\tau_1, \tau_2, \ldots, \tau_m) = \left(p_1 e^{\tau_1} + p_2 e^{\tau_2} + \cdots + p_m e^{\tau_m}\right)^N \tag{11}$$

Let partial derivative of the moment generating functions be

$$M^{n_1, n_2, \ldots, n_m}(\tau_1, \tau_2, \ldots, \tau_m) = \frac{\partial^{n_1+n_2+\cdots+n_m}}{\partial \tau_1^{n_1} \partial \tau_2^{n_2} \ldots \partial \tau_m^{n_m}} M(\tau_1, \tau_2, \ldots, \tau_m) \tag{12}$$

The moments of Y are

$$E\left[Y_1^{n_1} Y_2^{n_2} \ldots Y_m^{n_m}\right] = M^{n_1, n_2, \ldots, n_m}(0, 0, \ldots, 0) \tag{13}$$

Multivariate Normal Distribution Universal Portfolio (MNUP)

In Tan and Pang [9] and Pang, Liew and Chang [3], let $Y = (Y_1, \ldots, Y_m)$ be a randomly selected, whereas it has a joint multivariate normal distribution with the density function of $f(Y)$ which is defined over the domain, $D = \{(Y_1, Y_2, \ldots, Y_m) : -\infty < Y_i < \infty, i = 1, \ldots, m, f(Y_1, Y_2, \ldots, Y_m) > 0\}$.

$$f(Y) = \frac{1}{\left(\sqrt{2\pi}\right)^n |K|^{\frac{1}{2}}} e^{-\frac{1}{2}(Y-\mu)^T K^{-1}(Y-\mu)} \quad (14)$$

where $|K|$ is the determinant of the covariance matrix of Y.

$$K = \begin{pmatrix} {\sigma_{11}}^2 & {\sigma_{12}}^2 & {\sigma_{13}}^2 \\ {\sigma_{21}}^2 & {\sigma_{22}}^2 & {\sigma_{23}}^2 \\ {\sigma_{31}}^2 & {\sigma_{32}}^2 & {\sigma_{33}}^2 \end{pmatrix} = \begin{pmatrix} {\sigma_{11}}^2 & 0 & 0 \\ 0 & {\sigma_{22}}^2 & 0 \\ 0 & 0 & {\sigma_{33}}^2 \end{pmatrix} \quad (15)$$

$$\mu = (E(Y_1), E(Y_2), \ldots, E(Y_m)) \quad (16)$$

is the Y's mean vector. Then, Y has a multivariate normal distribution of N (μ, K).

3 Result and Discussion

Table 1 gives the list of Malaysian Companies selected form Kuala Lumpur Exchange for the empirical study. The data is collected for the period 1st January 2019 to 31st December 2019, which consists of 246 trading days. The stock-price data consists of four sets A, B, C and D.

3.1 Result of Order-1 Multinomial Distribution Generated Universal Portfolio

In this study, short term period used are 7 trading days continuously, 14 trading days continuously and 30 trading days continuously selected randomly from the period of 1st January 2019 to 31st December 2019. The parameter, refer to Eq. (10),$(p_1, p_2, N) = (0.001, 0.8, 600)$ is selected for MUP as this parameters outperformed other parameters in Pang, Liew and Chang [3]. The order-1 MUP with the above selected parameter is run

Table 1. List of Malaysian Companies in data set A, B, C, and D

Set A	Set B	Set C	Set D
Parkson Holdings Bhd	Poh Kong Holdings Bhd	Ajinomoto Malaysia Bhd	Ynh Property Bhd
Genting Bhd	Sime Darby Bhd	British American Tobacco Bhd	Muar Ban Lee Group Bhd
Nestle (Malaysia) Bhd	Spritzer Bhd	LCTITAN	Globaltec Formation Bhd

over data set A, B, C and D for 7 trading days, 14 trading days and 30 trading days. The wealth S_n achieved after 7 trading days, 14 trading days and 30 trading days are listed in Table 2, 3 and 4.

Table 2 shows the best wealth is obtained for data sets B and D while the lowest wealth for data set A. Table 2 also shows that data sets A and C are poor portfolios achieving wealth of 0.9965 and 0.9971. The empirical result from Table 3 shows that data sets A, B, C and D are good performing portfolios achieving wealth of 1.0242. 1.1326 1.0092 and 1.2310. Equivalently, Table 3 also shows that data sets A, B and D are good performing portfolios with wealth of 1.0051 and 1.0805 and 1.3831.

Table 2. The wealth obtained over data Set A, B, C and D for 7 trading days together with the final portfolio *b*

	(b_1, b_2, b_3)	S7
Set A	(0.00100000, 0.80000000, 0.19900000)	0.9965
Set B	(0.00100000, 0.80000000, 0.19900000)	1.0758
Set C	(0.00100000, 0.80000000, 0.19900000)	0.9971
Set D	(0.00100000, 0.80000000, 0.19900000)	1.0877

Table 3. The wealth obtained over data Set A, B, C and D for 14 trading days together with the final portfolio *b*

	(b_1, b_2, b_3)	S14
Set A	(0.00099996, 0.79999737, 0.19900267)	1.0242
Set B	(0.00099993, 0.80000129, 0.19899878)	1.1326
Set C	(0.00099996, 0.80000807, 0.19899197)	1.0092
Set D	(0.00099978, 0.80002511, 0.19897511)	1.2310

Table 4. The wealth obtained over data Set A, B, C and D for 30 trading days together with the final portfolio *b*

	(b_1,b_2, b_3)	S30
Set A	(0.00099996, 0.80000114, 0.19899899)	1.0051
Set B	(0.00100003, 0.79999391, 0.19900606)	1.0805
Set C	(0.00099999, 0.80000346, 0.19899655)	0.8787
Set D	(0.00100002, 0.79999999, 0.19899999)	1.3831

3.2 Result of Order-1 Multivariate Normal Distribution Generated Universal Portfolio

The parameter refers to Eq. (15) and (16),$\left(\mu_1, \mu_2, \mu_3, \sigma^2\right) = (0.2, 0.8, 7.1, 1.0)$ is selected as it showed well performed in Pang, Liew and Chang [3]. The order-1 universal portfolio generated by multivariate normal distribution with the above selected parameter is run over data set A, B, C and D for the same short-term period used in multinomial generated universal portfolio. The wealth S_n achieved after 7 trading days, 14 trading days and 30 trading days are listed in Table 5, 6 and 7.

According to Table 5, the best wealth is obtained for data set D. Table 5 also shows that all four data sets are good portfolios achieving wealth of more than 1. The empirical result from Table 6 shows that data sets A, B and D are good performing portfolios achieving wealth of 1.0263, 1.1917 and 1.1994. Equivalently, Table 7 also shows that

Table 5. The wealth obtained over data Set A, B, C and D for 7 trading days together with the final portfolio *b*

	(b_1, b_2, b_3)	S7
Set A	(0.03885257, 0.10891835, 0.85222908)	1.0111
Set B	(0.03836789, 0.10875311, 0.85287900)	1.0612
Set C	(0.03813354, 0.10908018, 0.85278628)	1.0014
Set D	(0.03805688, 0.10904378, .85289934))	1.1161

Table 6. The wealth obtained over data Set A, B, C and D for 14 trading days together with the final portfolio *b*

	(b_1, b_2, b_3)	S14
Set A	(0.03790639, 0.10929233, .85280128)	1.0263
Set B	(0.03820265, 0.10891831, 0.85287903)	1.1917
SetC	(0.03851909, 0.10894227, 0.85253863)	0.9838
Set D	(0.03837900, 0.10946258, 0.85215842)	1.1994

Table 7. The wealth obtained over data Set A, B, C and D for 30 trading days together with the final portfolio *b*

	(b_1,b_2,b_3)	S30
Set A	(0.03792418, 0.10896603, .85310980)	1.0206
Set B	(0.03846623, 0.10898467, 0.85254911)	1.2091
Set C	(0.03836882, 0.10921364, 0.85241754)	0.9178
Set D	(0.03849318, 0.10874289, 0.85276393)	1.3870

data sets A, B and D are good performing portfolios with wealth of 1.0206 1.2091 and 1.3870.

4 Conclusion

The results in Table 2 to Table 7 show that data set D outperformed the other three data sets for 7, 14 and 30 trading days. The second stock of data set D is performing well in the market when generated by order 1 MUP. While the third stock of data set D is performing well in the market when generated with order 1 MNUP. Hence the portfolios assign more weights on these two stocks and lead to higher wealth return for all the three trading period. The study concluded that the two universal portfolios, MUP and MNUP perform well for short term period. More sets of stocks with different set of parameters will be investigated in the future work to improve the performances on the two proposed universal portfolios.

References

1. Cover, T.M., Ordentlich, E.: EEE Trans. Inf. Theory **42**, 348–363 (1996)
2. Tan, C.P.: Performance bounds for the distribution-generated universal portfolio. In: Proceedings of the 59th World Statistics Congress of the International Statistical Hong Kong, pp. 5327 (2013)
3. Pang, S.T., Liew, H.H., Chang, Y.F.: Performance of finite order distributed generated universal portfolio. In: The 4th International Conference of Mathematical Sciences, UKM AIP Conference Proceedings, p. 1830(2017)
4. Tan, C. P., Lim, W.X.: Universal portfolios generated by the Mahalanobis squared divergence. East-WestJournal of Mathematics, pp. 225–235(2012)
5. Tan, C.P., Kuang, K.S.: Universal portfolios generated by f-disparity differences. In: AIP Conference Proceedings, vol. 2184, 050024 (2019)
6. Tan, C.P., Lee, Y.J.: Universal portfolios generated by inequality ratios. IOSR J. Math. **15**(3), 24–28 (2019)
7. Tan, C.P.: Universal portfolio generated by extended f-divergence. IQSR J. Math. 16, 25–38(2020)
8. Tan, C.P., Pang, S.T.: The finite and moving order multinomial universal portfolio. J. Phys. Conf. Ser. **435**(1), 012039 (2013)
9. Tan, C.P., Pang, S.T.: The finite order multivariate normal universal portfolio (Session 5C (IASC-ARS). In: Proceedings of the symposium of Japanese Society of Computational Statistics, vol. 26, pp. 169–172 (2012)