Neutrosophic Delta Beta Normal Topological Space

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Abstract. Real-life structures always include indeterminacy. The Mathematical tool which is well known in dealing with indeterminacy is neutrosophic. Smarandache proposed the approach of neutrosophic sets. Neutrosophic sets deal with uncertain data. The notion of neutrosophic sets is generally referred to as the generalization of intuitionistic fuzzy sets. In this paper, we introduce neutrosophic $\delta\beta-$normal space and strongly neutrosophic $\delta\beta-$normal space by using neutrosophic $\delta\beta-$normal sets and neutrosophic $\delta\beta-$closed sets. We investigate several fundamental properties and characterizations of these spaces as well as their relations among themselves and with already existing spaces.

Keywords: Neutrosophic topological space · Neutrosophic $\delta\beta-$open set

1 Introduction

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. There are difficulties in solving the uncertainties in data by traditional mathematical models. There are approaches such as fuzzy sets, intuitionistic fuzzy sets, vague sets, and rough sets which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminant and inconsistent data due to inadequacy of parameterization tools. Zadeh [20] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. In 2012, Salama and Alblowi [15] introduced the concept of Neutrosophic topological spaces. Arar and Jafari [3], Arokiarani et al. [4], Bageerathi and Puvaneswari [5], Das and Pramanik [6], Imran et al. [7], Iswarya and Bageerathi [8], Javyparthasarathy et al. [9], Mary and Trinita [10], Narmatha et al. [13], and Shanthi et al. [16] investigated properties and characterizations of Neutrosophic closed and open sets as well as applications of these Neutrosophic closed and open sets in different notions in Neutrosophic topological spaces. Acikgoz and Esenbel [1], Babu and Aswini [2], Mehmood et al. [11], Al-Nafee [12] and Puvaneswari et al. [14] investigated properties and characterizations of separation axioms by using different types of Neutrosophic closed sets in Neutrosophic topological spaces. Moldstov introduced soft set theory. Smarandache [17] studies neutrosophic set as an approach for solving issues that cover unreliable,

N. Wahi et al. (Eds.): ICMSS 2022, ACSR 98, pp. 47–56, 2023.
https://doi.org/10.2991/978-94-6463-014-5_6

indeterminacy and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic open sets, neutrosophic interior operator, and neutrosophic closure operator. In 2020 Vadivel, Seenivasan and Sundar [19] introduced a new type of $\delta-$open sets and $\delta-$Closed sets in Neutrosophic topological spaces. They also introduced and studied some properties and characterizations of *Neu*–$\delta\beta-$Closed in Neutrosophic topological spaces. The objective of this paper is to define neutrosophic $\delta\beta-$normal space and strongly neutrosophic $\delta\beta-$normal space in neutrosophic topological spaces and study their several fundamental properties and characterizations as well as their relations among themselves.

2 Preliminaries

We recall basic definitions and operations of neutrosophic sets and neutrosophic topological space.

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set P is an object having the form $P = \{\langle x,\ \mu_P(x),\ \sigma_P(x),\ \gamma_P(x)\rangle : x \in X\}$, *where* $\mu_P(x)-$ represents the degree of membership, $\sigma_P(x)-$ represents the degree of indeterminacy, and $\gamma_P(x)-$ represents the degree of non-membership. The class of all neutrosophic sets of X will be denoted by N (X).

Definition 2.2. Let X be a non-empty set and let $P = \{\langle x,\ \mu_P(x),\ \sigma_P(x),\ \gamma_P(x)\rangle : x \in X\}$ and $Q = \left\{\langle x,\ \mu_Q(x),\ \sigma_Q(x),\ \gamma_Q(x)\rangle : x \in X\right\}$ be two neutrosophic sets, *Then*:

1. (*Empty set*) $0_N = \langle x,\ 0,\ 0,\ 1\rangle$ is called the neutrosophic empty set,
2. (*Universal set*) $1_N = \langle x,\ 1,\ 1,\ 0\rangle$ is called the neutrosophic universal set.
3. (Inclusion): $P \subseteq Q$ if and only if $\mu_P(x) \le \mu_Q(x), \sigma_P(x) \le \sigma_Q(x)$ and $\gamma_P(x) \ge \gamma_Q(x) : \forall\ x \in X$,
4. (Equality): $P = Q$ if and only if $P \subseteq Q$ and $Q \subseteq P$,
5. (*Complement*) $P^C = 1_N - P = \{\langle x,\ \gamma_P(x),\ 1-\sigma_P(x),\ \mu_P(x)\rangle : x \in X\}$,
6. (*Union*) $P \cup Q = \left\{\langle x,\ \max\left(\mu_P(x),\ \mu_Q(x)\right),\ \max\left(\sigma_P(x),\ \sigma_Q(x)\right),\ \min\left(\gamma_P(x),\ \gamma_Q(x)\right)\rangle : x \in X\right\}$.
7. (*Intersection*) $P \cap Q = \left\{\langle x,\ \min\left(\mu_P(x),\ \mu_Q(x)\right),\ \min\left(\sigma_P(x),\ \sigma_Q(x)\right),\ \max\left(\gamma_P(x),\ \gamma_Q(x)\right)\rangle : x \in X\right\}$.

Definition 2.3. A neutrosophic point $x_{(\alpha,\beta,\gamma)}$ is said to be in the neutrosophic set A- in symbols $x_{(\alpha,\beta,\gamma)} \in A$ if and only if $\alpha < \mu_A(x),\ \beta < \sigma_A(x)$ and $\gamma > \gamma_A(x)$.

Definition 2.4. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying $(i) 0_N,\ 1_N \in \mathrm{T}_N$. (ii) $G \cap H \in \mathrm{T}_N$ for every G, $H \in \mathrm{T}_N$, (iii) $\bigcup_{j \in J} G_j \in \mathrm{T}_N$ for every $\left\{G_j : j \in J\right\} \subseteq \tau_N$. Then the pair (X, T_N) is called a neutrosophic topological space. The elements of T_N are called neutrosophic open sets in X. A neutrosophic set A in X is called a neutrosophic closed set if and only if its complememnt A^C is a neutrosophic open set.

Definition 2.5. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set. Then:

(i) The neutrosophic interior of A, denoted by $\mathrm{N}\,Int(A)$ is the union of all neutorophic open subsets of A. Clearly $\mathrm{N}\,Int(A)$ is the biggest neutrosophic open subset of X contained in A.
(ii) The neutrosophic closure of A denoted by $\mathrm{N}\,Cl(A)$ is the intersection of all neutrosophic closed sets containing A. Clearly $\mathrm{N}\,Cl(A)$ is the smallest neutrosophic closed set which contains A.

Definition 2.6. A neutrosophic subset A of a neutrosophic topological space (X, T_N) is said to be a neutrosophic regular open set if $A \subseteq \mathrm{N}\,Int[\mathrm{N}\,Cl(A)]$. The complement of a neutrosophic regular open set is called a neutrosophic regular closed set in X.

Definition 2.7. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set. Then neutrosophic $\delta-$interior of A, denoted by (*briefly* $\mathrm{N}\,\delta\,Int(A)$) is defined as the union of all neutrosophic regular open subsets of A. Equivalently, it could be as given below: $\mathrm{N}\,\delta\,Int(A) = \cup\{B : B \subseteq A \;\&\; B \textit{ is a neutrosophic regular open set in } X\}$.

Definition 2.8. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set. Then neutrosophic $\delta-$closure of A, denoted by (*briefly* $\mathrm{N}\,\delta Cl(A)$) is defined as the intersection of all neutrosophic regular closed sets containing A. Equivalently, it could be as given below: $\mathrm{N}\,\delta Cl(A) = \cap\{B : A \subseteq B \;\&\; B \textit{ is a neutrosophic regular closed set in } X\}$.

Definition 2.9. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set on X. Then A is said to be a neutrosophic $\delta-$open (resp. $\delta-$closed) set if $A = \mathrm{N}\delta\,Int(A)$ $(resp.\ A = \mathrm{N}\delta Cl(A))$.

Definition 2.10. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set of X. Then A is said to be a neutrosophic $\delta\beta-$open (briefly $\mathrm{N}\,\delta\beta-$OS) set if $A \subseteq \mathrm{N}\,Cl[\mathrm{N}\,Int(\mathrm{N}\,\delta Cl(A))]$.

Definition 2.11. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set of X. Then A is called a neutrosophic $\delta\beta-$closed(briefly $\mathrm{N}\,\delta\beta-$CS) set if its complement A^C is a neutrosophic $\delta\beta-$open set in X.

The family of all neutrosophic $\delta\beta-$open (resp. $\delta\beta-$closed) in a neutrosophic topological space (X, T_N) is denoted by $\mathrm{N}\,\delta\beta OS(X, T_N)$ or $\mathrm{N}\,\delta\beta OS(X)$ $(resp.\ \mathrm{N}\,\delta\beta CS(X, T_N))$ or $(resp.\ \mathrm{N}\,\delta\beta CS(X))$.

Preposition 2.12. Let (X, T_N) be a neutrosophic topological space. Then the following statements are true:

(i) Every neutrosophic $\delta-$open (resp. $\delta-$closed) set is neutrosophic open (*resp. closed*) set in X.
(ii) Every neutrosophic open (*resp. closed*) set is neutrosophic $\delta\beta-$open (resp. $\delta\beta-$closed) set in X.

Preposition 2.13. Let (X, T_N) be a neutrosophic topological space. Then the union (resp. Intersection) of any family of $\mathrm{N}\,\delta\beta OS(X, \mathrm{T}_N)$ $(resp.\ \mathrm{N}\,\delta\beta CS(X, \mathrm{T}_N))$ is in $\mathrm{N}\,\delta\beta OS(X, \mathrm{T}_N)$ $(resp.\ \mathrm{N}\,\delta\beta CS(X, \mathrm{T}_N))$.

Preposition 2.14. Let (X, T_N) be a neutrosophic topological space. Let A be an N δ−OS and B be an N δβ−OS. Then $A \cap B$ is an N δβ−OS.

Definition 2.15. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set of X. Then A is said to be a neutrosophic regular δβ−open set if $A = \mathrm{N}\,delta\beta Int[\mathrm{N}\,\delta\beta Cl(A)]$. The complement of a neutrosophic δβ−regular open set is called a neutrosophic δβ−regular closed set in X.

Lemma 2.16. Assume that A is a neutrosophic subset of a neutrosophic topological space (X, T_N). Then the following relations hold.

$(a)\ X - \mathrm{N}\,\delta\beta Int(U) = \mathrm{N}\,\delta\beta Cl(X - U)$. $(b)\ X - \mathrm{N}\,\delta\beta Cl(U) = \mathrm{N}\,\delta\beta Int(X - U)$.

Definition 2.17. A function $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is called a neutrosophic continuous (δβ−continuous) function if the inverse $f^{-1}(B)$ is a neutrosophic open (resp. δβ−open) set in X, for every neutrosophic open set B in Y.

Definition 2.18. A function $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is called a neutrosophic δβ−irresolute function if $f^{-1}(B)$ is a neutrosophic δβ−open set in X, for every neutrosophic δβ−open set B in Y.

Lemma 2.19. A function $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is a neutrosophic δβ−irresolute function if and only if $f^{-1}(B)$ is a neutrosophic δβ−closed set in X, for every neutrosophic δβ−closed set B in Y.

Definition 2.20. A function $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ is called a neutrosophic δβ−open (resp. δβ−closed) function if image set $f(A)$ is a neutrosophic δβ−open (resp. δβ−closed) set in Y, for every neutrosophic open (*resp. closed*) set A in X.

3 Neutrospohic $\delta\beta-$ Normal Spaces

In this section, we introduce neutrosophic δβ−normal space and study its properties and characterizations.

Definition 3.1. A neutrosophic topological space (X, T_N) is said to be neutrosophic δβ−normal if for any two disjoint neutrosophic δβ−closed sets A and B, there exist disjoint neutrosophic δβ−open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 3.2. Let (X, T_N) be a neutrosophic topological space. Then the following statements are equivalent:

(a) X is neutrosophic δβ−normal.

(b) For every neutrosophic δβ−closed set A in X and every neutrosophic δβ−open set U containing A, there exists a neutrosophic δβ−open set V containing A such that $\mathrm{N}\,\delta\beta Cl(V) \subseteq U$.
(c) For each pair of disjoint neutrosophic δβ−closed sets A and B in X, there exists a neutrosophic δβ−open set U containing A such that $\mathrm{N}\,\delta\beta Cl(U) \cap B = 0_N$.
(d) For each pair of disjoint neutrosophic δβ−closed sets A and B in X, there exist neutrosophic δβ−open sets U and V containing A and B respectively such that $\mathrm{N}\,\delta\beta Cl(U) \cap \mathrm{N}\,\delta\beta Cl(V) = 0_N$.

Proof. (a) ⇒ (b) : Let U be a neutrosophic δβ−open set containing the neutrosophic δβ−closed set A. Then $B = U^C$ is a neutrosophic δβ−closed set disjoint from A. Since X is neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets V and W containing A and B respectively. Then $\mathrm{N}\,\delta\beta Cl(V)$ is disjoint from B. Since if $y_{(r,t,s)} \in B$, the set W is a neutrosophic δβ−open set containing $y_{(r,t,s)} \in B$ disjoint from V. Hence $\mathrm{N}\,\delta\beta Cl(V) \subseteq U$.

(b) ⇒ (c) : Let A and B be disjoint neutrosophic δβ−closed sets in X. Then B^C is a neutrosophic δβ−open set containing A. By (b), there exists a neutrosophic δβ−open set U containing A such that $\mathrm{N}\,\delta\beta Cl(U) \subseteq B^C$. Hence $\mathrm{N}\,\delta\beta Cl(U) \cap B = 0_N$. This proves (c).

(c) ⇒ (d) : Let A and B be disjoint neutrosophic δβ−closed sets in X. Then by (c), there exists a neutrosophic δβ−open set U containing A such that $\mathrm{N}\delta\beta Cl(U) \cap B = 0_N$. Since $\mathrm{N}\,\delta\beta Cl(U)$ is neutrosophic δβ−closed, B and $\mathrm{N}\,\delta\beta Cl(U)$ are disjoint neutrosophic δβ−closed sets in X. Again by (c), there exists a neutrosophic δβ−open set V containing B such that $\mathrm{N}\,\delta\beta Cl(U) \cap \mathrm{N}\,\delta\beta Cl(V) = 0_N$. This proves (d).

(d) ⇒ (a) : Let A and B be disjoint neutrosophic δβ−closed sets in X. By (d), there exist neutrosophic δβ−open sets U and V containing A and B respectively such that $\mathrm{N}\,\delta\beta Cl(U) \cap \mathrm{N}\,\delta\beta Cl(V) = 0_N$. Since $U \cap V \subseteq \mathrm{N}\,\delta\beta Cl(U) \cap \mathrm{N}\,\delta\beta Cl(V)$, U and V are disjoint neutrosophic δβ−open sets containing A and B respectively. Hence the result of (a) follows.

Theorem 3.3. A neutrosophic topological space (X, T_N) is neutrosophic δβ−normal if and only if for every neutrosophic δβ−closed set F and a neutrosophic δβ−open set W containing F, there exists a neutrosophic δβ−open set U such that $F \subseteq U \subseteq \mathrm{N}\,\delta\beta Cl(U) \subseteq W$.

Proof. Let (X, T_N) be neutrosophic δβ−normal. Let F be a neutrosophic δβ−closed set and let W be a neutrosophic δβ−open set containing F. Then F and W^C are disjoint neutrosophic δβ−closed sets. Since X is neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets U and V such that $F \subseteq U$ and $W^C \subseteq V$. Thus $F \subseteq U \subseteq V^C \subseteq W$. Since V^C is neutrosophic δβ−closed, so $\mathrm{N}\,\delta\beta Cl(U) \subseteq \mathrm{N}\,\delta\beta Cl\left(V^C\right) = V^C \subseteq W$. This implies that $F \subseteq U \subseteq \mathrm{N}\,\delta\beta Cl(U) \subseteq W$.

Conversely, suppose the condition holds. Let G and H be two disjoint neutrosophic δβ−closed sets in X. Then H^C is a neutrosophic δβ−open set containing G. By assumption, there exists a neutrosophic δβ−open set U such that $G \subseteq U \subseteq \mathrm{N}\,\delta\beta Cl(U) \subseteq H^C$. Since U is neutrosophic δβ−open and $\mathrm{N}\,\delta\beta Cl(U)$ is neutrosophic δβ−closed. Then

$(\mathrm{N}\,\delta\beta Cl(U))^C$ is neutrosophic δβ−open. Now $\mathrm{N}\,\delta\beta Cl(U) \subseteq H^C$ implies that $H \subseteq (\mathrm{N}\,\delta\beta Cl(U))^C$. Also $U \cap (\mathrm{N}\,\delta\beta Cl(U))^C \subseteq \mathrm{N}\,\delta\beta Cl(U) \cap (\mathrm{N}\,\delta\beta Cl(U))^C = 0_N$. That is U and $(\mathrm{N}\,\delta\beta Cl(U))^C$ are disjoint neutrosophic δβ−open sets containing G and H respectively. This shows that (X, T_N) is neutrosophic δβ−normal.

Theorem 3.4. Let (X, T_N) be a neutrosophic topological space. Then the following statements are equivalent:

(a) X is neutrosophic δβ−normal.
(b) For any two neutrosophic δβ−open sets U and V whose union is 1_N, there exist neutrosophic δβ−closed subsets A of U and B of V such that $A \cup B = 1_N$.

Proof. (a) ⇒ (b) : Let U and V be two neutrosophic δβ−open sets in a neutrosophic δβ−normal space X such that $U \cup V = 1_N$. Then U^C and V^C are disjoint neutrosophic δβ−closed sets. Since X is neutrosophic δβ−normal, then there exist disjoint neutrosophic δβ−open sets G and H such that $U^C \subseteq G$ and $V^C \subseteq H$. Let $A = G^C$ and $B = H^C$. Then A and B are neutrosophic δβ−closed subsets of U and V respectively such that $A \cup B = 1_N$. This proves (b).

(b) ⇒ (a) : Let A and B be disjoint neutrosophic δβ−closed sets in X. Then A^C and B^C are neutrosophic δβ−open sets whose union is 1_N. By (b), there exist neutrosophic δβ−closed sets E and F such that $E \subseteq A^C$, $F \subseteq B^C$ and $E \cup F = 1_N$. Then E^C and F^C are disjoint neutrosophic δβ−open sets containing A and B respectively. Therefore X is neutrosophic δβ−normal.

Theorem 3.5. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a function.

(a) If f is injective, neutrosophic δβ−irresolute neutrosophic δβ−open and X is neutrosophic δβ−normal, then Y is neutrosophic δβ−normal.
(b) If f is neutrosophic δβ−irresolute, neutrosophic δβ−closed and Y is neutrosophic δβ−normal, then X is neutrosophic δβ−normal.

Proof. (a) Suppose X is neutrosophic δβ−normal. Let A and B be disjoint neutrosophic δβ−closed sets in Y. Since f is neutrosophic δβ−irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint neutrosophic δβ−closed sets in X. Since X is neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Now $f^{-1}(A) \subseteq U$ implies that $A \subseteq f(U)$ and $f^{-1}(B) \subseteq V$ implies that $B \subseteq f(V)$. Since f is a neutrosophic δβ−open map, $f(U)$ and $f(V)$ are neutrosophic δβ−open in Y. Also $U \cap V = 0_N$ implies that $f(U \cap V) = 0_N$ and f is injective, then $f(U) \cap f(V) = 0_N$. Thus $f(U)$ and $f(V)$ are disjoint neutrosophic δβ−open sets in Y containing A and B respectively. Thus, Y is neutrosophic δβ−normal.

(b) Suppose Y is neutrosophic δβ−normal. Let A and B be disjoint neutrosophic δβ−closed sets in X. Since f is neutrosophic δβ−closed map, $f(A)$ and $f(B)$ are neutrosophic δβ−closed sets in Y. Since Y is neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets U and V in Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. That is $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Since f is neutrosophic δβ−irresolute, $f^{-1}(U)$ and f^{-1}

(V) are disjoint neutrosophic δβ−open sets such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Thus, X is neutrosophic δβ−normal.

Theorem 3.6. Let $f : (X, T_N) \to (Y, \sigma_N)$ be a neutrosophic continuous, neutrosophic δβ−open bijection of a neutrosophic normal space X onto a neutrosophic space Y and if every neutrosophic δβ−closed set in Y is neutrosophic closed, then Y is neutrosophic δβ−normal.

Proof. Let A and B be disjoint neutrosophic δβ−closed sets in Y. Then by assumption, A and B are neutrosophic closed in Y. Since f is a neutrosophic continuous bijection, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint neutrosophic closed sets in X. Since X is neutrosophic normal space, there exist disjoint neutrosophic open sets G and H in X such that $f^{-1}(A) \subseteq G$ and $f^{-1}(B) \subseteq H$. Since f is neutrosophic δβ−open bijection, $f(G)$ and $f(H)$ are disjoint neutrosophic δβ−open sets in Y containing A and B respectively. Hence Y is neutrosophic δβ−normal.

4 Strongly Neutrosophic $\delta\beta-$ Normal Spaces

In this section, we introduce strongly neutrosophic δβ−normal space and study its properties.

Definition 4.1. A neutrosophic topological space (X, T_N) is said to be strongly neutrosophic δβ−normal if for every pair of disjoint neutrosophic closed sets A and B in X, there are disjoint neutrosophic δβ−open sets U and V in X containing A and B respectively.

Theorem 4.2. Every neutrosophic δβ−normal space is strongly neutrosophic δβ−normal.

Proof. Suppose X is neutrosophic δβ−normal. Let A and B be disjoint neutrosophic closed sets in X. Then A and B are disjoint neutrosophic δβ−closed sets in X. Since X is neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets U and V containing A and B respectively. This implies that X is strongly neutrosophic δβ−normal.

Theorem 4.3. Let (X, T_N) be a neutrosophic topological space. Then the following are equivalent:

(a) X is strongly neutrosophic δβ−normal.
(b) For every neutrosophic closed set F in X and every neutrosophic open set U containing F, there exists a neutrosophic δβ−open set V containing F such that $N\,\delta\beta Cl(V) \subseteq U$.
(c) For each pair of disjoint neutrosophic closed sets A and B in X, there exists a neutrosophic δβ−open set U containing A such that $N\,\delta\beta Cl(U) \cap B = 0_N$.

Proof. (a) $\Rightarrow$ (b) : Let U be a neutrosophic open set containing the neutrosophic closed set F. Then $H = U^C$ is a neutrosophic closed set disjoint from F. Since X is strongly neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets V and W containing F and H respectively. Then $\mathrm{N}\,\delta\beta Cl(V)$ is disjoint from H, since if $y_{(r,t,s)} \in H$, the set W is a neutrosophic *semi−α−open* set containing $y_{(r,t,s)}$ disjoint from V. Hence $\mathrm{N}\,\delta\beta Cl(V) \subseteq U$.

(b) $\Rightarrow$ (c) : Let A and B be disjoint neutrosophic closed sets in X. Then B^C is a neutrosophic open set containing A. By (b), there exists a neutrosophic δβ−open set U containing A such that $\mathrm{N}\,\delta\beta Cl(U) \subseteq B^C$. Hence $\mathrm{N}\,\delta\beta Cl(U) \cap B = 0_N$. This proves (c).

(c) $\Rightarrow$ (a) : Let A and B be disjoint neutrosophic δβ−closed sets in X. By (c), there exists a neutrosophic δβ−open set U containing A such that $\mathrm{N}\,\delta\beta Cl(U) \cap B = 0_N$. Take $V = (\mathrm{N}\,\delta\beta Cl(U))^C$. Then U and V are disjoint neutrosophic δβ−open sets containing A and B respectively. Thus X is strongly neutrosophic δβ−normal.

Theorem 4.4. Let (X, T_N) be a neutrosophic topological space. Then the following are equivalent:

(a) X is strongly neutrosophic δβ−normal.
(b) For any two neutrosophic open sets U and V whose union is 1_N, there exist neutrosophic δβ−closed subsets A of U and B of V such that $A \cup B = 1_N$.

Proof. (a) $\Rightarrow$ (b) : Let U and V be two neutrosophic open sets in a strongly neutrosophic δβ−normal space X such that $U \cup V = 1_N$. Then U^C and V^C are disjoint neutrosophic closed sets. Since X is strongly neutrosophic δβ−normal, then there exist disjoint neutrosophic δβ−open sets G and H such that $U^C \subseteq G$ and $V^C \subseteq H$. Let $A = G^C$ and $B = H^C$. Then A and B are neutrosophic δβ−closed subsets of U and V respectively such that $A \cup B = 1_N$.

(b) $\Rightarrow$ (a) : Let A and B be disjoint neutrosophic closed sets in X. Then A^C and B^C are neutrosophic open sets such that $A^C \cup B^C = 1_N$. By (b), there exists neutrosophic δβ−closed sets G and H such that $G \subseteq A^C$, $H \subseteq B^C$ and $G \cup H = 1_N$. Then G^C and H^C are disjoint neutrosophic δβ−open sets containing A and B respectively. Therefore X is strongly neutrosophic δβ−normal.

Theorem 4.5. Let $f : (X, \mathrm{T}_N) \to (Y, \sigma_N)$ be a function.

a) If f is injective, neutrosophic continuous, neutrosophic δβ−open and X is strongly neutrosophic δβ−normal, then Y is strongly neutrosophic δβ−normal.
b) If f is neutrosophic δβ−irresolute, neutrosophic δβ−closed map and Y is strongly neutrosophic δβ−normal, then X is strongly neutrosophic δβ−normal.

Proof. (a) Suppose X is strongly neutrosophic δβ−normal. Let A and B be disjoint neutrosophic closed sets in Y. Since f is neutrosophic continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are neutrosophic closed in X. Since X is strongly neutrosophic δβ−normal, there exist disjoint neutrosophic δβ−open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Now $f^{-1}(A) \subseteq U$ implies that $A \subseteq f(U)$ and $f^{-1}(B) \subseteq V$ implies that $B \subseteq f$

(V). Since f is a neutrosophic $\delta\beta-$open map, $f(U)$ and $f(V)$ are neutrosophic $\delta\beta-$open sets in Y. Also $U \cap V = 0_N$ implies that $f(U \cap V) = 0_N$ and f is injective, then $f(U) \cap f(V) = 0_N$. Thus $f(U)$ and $f(V)$ are disjoint neutrosophic $\delta\beta-$open sets in Y containing A and B respectively. Thus, Y is strongly neutrosophic $\delta\beta-$normal.

(b) Suppose Y is strongly neutrosophic $\delta\beta-$normal. Let A and B be disjoint neutrosophic closed sets in X. Since f is neutrosophic $\delta\beta-$closed map, $f(A)$ and $f(B)$ are neutrosophic $\delta\beta-$closed in Y. Since Y is neutrosophic $\delta\beta-$normal, there exist disjoint neutrosophic $\delta\beta-$open sets U and V in Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. That is $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Since f is neutrosophic $\delta\beta-$irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint neutrosophic $\delta\beta-$open sets such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Thus, X is strongly neutrosophic $\delta\beta-$normal.

5 Conclusions

We introduced neutrosophic $\delta\beta-$normal space and strongly $\delta\beta-$normal space using neutrosophic $\delta\beta-$open sets and $\delta\beta-$closed sets in neutrosophic topological spaces. We investigated their several fundamental properties and characterizations in neutrosophic topological spaces.

Acknowledgments. The author is highly and gratefully indebted to Prince Mohammad Bin Fahd University Al Khobar Saudi Arabia, for providing excellent research facilities during the preparation of this research paper.

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