Application of Intuitionistic Type-2 Fuzzy Set on Flat EEG Image

Suzelawati Zenian(✉)

Faculty of Science and Natural Resources, Universiti Malaysia Sabah, Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia
suzela@ums.edu.my

Abstract. In this paper, the Flat EEG images are enhanced based on intuitionistic type-2 fuzzy set. The non-membership function is defined by Sugeno type intuitionistic fuzzy generator. Moreover, a new membership function is defined by using Hamacher t-conorm. Experimental results show that the method provides better results compared to classical and fuzzy methods.

Keywords: Intuitionistic Fuzzy Set · Type-2 Fuzzy Set · Image Enhancement · Fuzzy Generator · Flat EEG

1 Introduction

Epilepsy is a symptom that cause disturbances in electrical signaling in the brain. The electrical activities of the brain are measured and recorded by a system namely electroencephalography (EEG). In 2008, the transformation of EEG signal into Flat EEG via flattening method has been done by Zakaria [1]. The method extracts information from EEG signal whereby it transforms the EEG signal that originated from a patient's head into two-dimensional plane. Furthermore, Flat EEG is transformed into digital Flat EEG and finally into Flat EEG image. This transformation is based on fuzzy approach whereby the Flat EEG was initially undergone the digitization process. After that, each of the pixel value of Flat EEG is assigned, and finally it is transformed into image data The Flat EEG image is displayed in gray-scale which consists of cluster centres during epileptic seizure for certain time. The location of the current sources which generate the corresponding magnetic fields is known as epileptic foci.

In the process of imaging and transformation of Flat EEG, uncertainties may arise within every transformation and in determining the membership values. The pixel values of Flat EEG image may not be precise as uncertainties inherent within the gray values such as grayness ambiguity and spatial ambiguity. Therefore, in modeling the uncertainty, Zadeh introduced the concept of fuzzy set in 1965 [2]. The ordinary (Type-1) fuzzy set is a generalization of the classical set and characterized by the membership function. Moreover, ordinary fuzzy set is extended to the advanced fuzzy set such as Type-2 fuzzy set, Intuitionistic fuzzy set (IFS), Interval-Valued fuzzy set, and so on. The advanced fuzzy set considers more uncertainties compared to the ordinary fuzzy approach.

N. Wahi et al. (Eds.): ICMSS 2022, ACSR 98, pp. 390–397, 2023.
https://doi.org/10.2991/978-94-6463-014-5_34

Type-2 fuzzy set was introduced by Zadeh in 1975 whereby the membership function is fuzzy. IFS was introduced by Atanassov in 1983 which considers uncertainties in terms of membership and non-membership functions [3]. The sum of membership and non-membership is not necessarily equal to one. Thus, there exists hesitancy in deciding the degree to which an element satisfies a particular property.

In this paper, an image enhancement method by using a combination of IFS and Type-2 fuzzy set is proposed for Flat EEG image. For IFS, the non-membership degree is calculated by using Sugeno type intuitionistic fuzzy generator. Hence, the hesitation degree is determined. For Type-2, the upper and lower membership degrees are computed. Moreover, Hamacher t-conorm is implemented to modify the membership degree. The aim of this work is to handle the uncertainty and improve the visibility of the clusters centres by using Intuitionistic Type-2 fuzzy set (IT2FS). The findings are compared with classical and fuzzy methods based on previous work by [4–6].

2 Preliminaries

In this section, the basic concepts of classical set. Ordinary fuzzy set, IFS, Type-2 fuzzy set, and Flat EEG are briefly discussed.

2.1 Classical Set

Let X be a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. A subset A of X has a membership function [2, 3]

$$\mu_A(x) : X \to \{0, 1\} \text{ where } \mu_A(x) = \begin{cases} 0 \text{ if } x \notin A \\ 1 \text{ if } x \in A \end{cases}$$

2.2 Ordinary (Type-1) Fuzzy Set

Let X be a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy set A in X has a membership function [2]

$$\mu_A(x) : X \to [0, 1]$$

where $\mu_A(x) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \\ 0 \text{ if } x \text{ is partly in } A \end{cases}$.

2.3 Intuitionistic Fuzzy Set (IFS)

Let A be an IFS in a finite set $X = \{x_1, x_2, \ldots, x_n\}$ which is defined as $A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}$ whereby $\mu_A(x), \nu_A(x) : X \to [0, 1]$ represent the membership and non-membership respectively. The necessary conditions that must be fulfilled are [3] $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ such that $0 \leq \pi_A(x) \leq 1$.

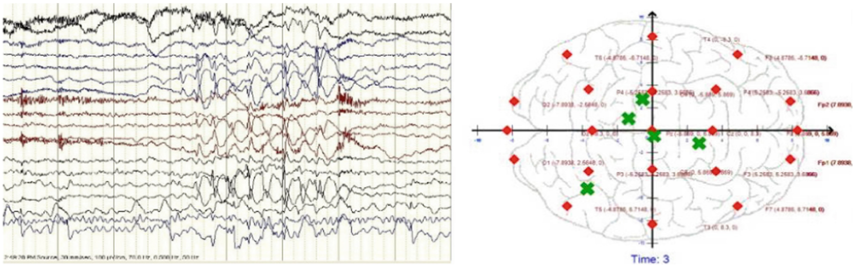


Fig. 1. Left: EEG signal, right: Analyzed EEG signal.

2.4 Type-2 Fuzzy Set

A Type-2 fuzzy set may be written as $A = \left\{ \left(x, \widehat{\mu}_A(x) \right) \middle| x \in X \right\}$, where $\widehat{\mu}_A(x)$ is a Type-2 membership function. The membership function is in an interval range as $\mu_{upper} = [\mu(x)]^{\alpha}$ and $\mu_{lower} = [\mu(x)]^{\frac{1}{\alpha}}$ whereby $0 < \alpha \leq 1$ [3].

2.5 Flat EEG

Flat EEEG was developed in 1999 by Fuzzy Research Group of UTM to visualize and preserve information recorded during seizure. It is a method for mapping high dimensional signal, namely EEG into a low dimensional space. In the process, the EEG signal Fig. 1(a) during seizure is compressed and analyzed second by second as in Fig. 1(b) [1]. As illustrated in Fig. 1(b), the clusters centres during epileptic seizures for a certain time are shown in green. Meanwhile, the location of sensors on the surface of the patient's head is represented as red.

3 Methodology

In this section, the proposed method is described in detail as follows:

Step 1: Fuzzification

The Flat EEG image of dimension $M \times N$ is initially fuzzified using Eq. (1)

$$\mu_A\left(g_{ij}\right) = \frac{g_{ij} - g_{\min}}{g_{\max} - g_{\min}}. \tag{1}$$

where g_{ij} is the $(i, j)^{th}$ gray level of the image in the range $[0, L - 1]$. The maximum and minimum values of the gray level are represented by $g_{\max}$ and $g_{\min}$, respectively.

Step 2: Construction of IFS

The non-membership function is computed by using Sugeno type intuitionistic fuzzy generator as in Eq. (2)

$$\nu_A\left(g_{ij}\right) = \frac{1 - \mu_A\left(g_{ij}\right)}{1 + \lambda\mu_A\left(g_{ij}\right)}, \quad \lambda > 0. \tag{2}$$

The hesitation degree is given by Eq. (3)

$$\pi_A(g_{ij}) = 1 - \mu_A(g_{ij}) - \frac{1 - \mu_A(g_{ij})}{1 + \lambda\mu_A(g_{ij})}. \tag{3}$$

Moreover, the average value of Eq. (1) is calculated as mean2($\mu_A(g_{ij})$). It computes the mean of all values in array $\mu_A(g_{ij})$. The modified membership value is given by

$$\mu_A^{mod}(g_{ij}) = \mu_A(g_{ij}) - \text{average} \times \pi_A(g_{ij}). \tag{4}$$

Step 3: Construction of IT2FS

In order to calculate the Hamacher t-conorm, the upper and lower membership are first calculated as in Eq. (5) and Eq. (6)

$$\mu_{upper}(g_{ij}) = \left[\mu_A^{mod}(g_{ij})\right]^{\alpha} \tag{5}$$

$$\mu_{lower}(g_{ij}) = \left[\mu_A^{mod}(g_{ij})\right]^{\frac{1}{\alpha}} \tag{6}$$

where $\mu_{upper}(g_{ij})$ and $\mu_{lower}(g_{ij})$ are the upper and lower membership function of type-2 fuzzy set. Furthermore, Eq. (5) and Eq. (6) are substituted into Hamacher t-conorm to obtain modified membership function as in Eq. (7)

$$\mu^{enh}(g_{ij}) = \frac{\mu_{upper}(g_{ij}) + \mu_{lower}(g_{ij}) + (\lambda - 2).\mu_{upper}(g_{ij}).\mu_{lower}(g_{ij})}{1 - (1 - \lambda).\mu_{upper}(g_{ij}).\mu_{lower}(g_{ij})} \tag{7}$$

Step 4: Defuzzification

Finally, the required enhanced image is obtained by the defuzzification process as in Eq. (8)

$$x_A^{enh}(g_{ij}) = \mu^{enh}(g_{ij}) \cdot (x_{\max} - x_{\min}) + x_{\min} \tag{8}$$

4 Results and Discussion

The IT2FS is implemented on Flat EEG input image at time $t = 1$ with dimension 201×201 as given in Fig. 2. Two clusters of electrical current sources are observed where the brightness represents the strength of the electrical potential. The frequency of occurrence of each gray level is represented by histogram of the input image as given in Fig. 3. The histogram of the input image shows that the pixels are mostly in the range between [30, 50]. The data of the cluster centres that are transformed into image form

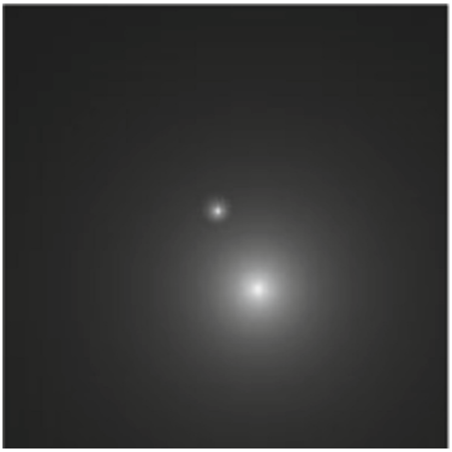


Fig. 2. Flat EEG input image.

is shown in Table 1. Figure 4 and 5 are the output images by implementing classical and fuzzy methods. Figure 4 and Fig. 5(a) that are used as a comparison are based on previous work by Suzelawati et al. [4]. The output image by applying IT2FS is given in Fig. 5(b). It shows that IT2FS has a better performance compared to the classical and Type-1 fuzzy methods.

Furthermore, the similarity between input and output images are measured by using mean of SSIM (i.e. MSSIM). The SSIM is given in Eq. (9) as follows [7]

$$SSIM\,(x, y) = \frac{\left(2\mu_x\mu_y + c_1\right)\left(2\sigma_{xy} + c_2\right)}{\left(\mu_x^2 + \mu_y^2 + c_1\right)\left(\sigma_x^2 + \sigma_y^2 + c_2\right)} \tag{9}$$

where μ_x and μ_y are the means of x and y respectively. Meanwhile, σ_x and σ_y are the standard deviations of x and y respectively. The standard deviations estimate the contrast between the images such that it measures how similar the contrast are. The range value of SSIM is in the interval [0, 1] with the best value of 1 if and only if the input and output images are the same. The MSSIM which represents the overall image quality is given in Eq. (10) as follows

$$MSSIM\,(x, y) = \frac{1}{M}\sum_{j=1}^{M} SSIM\,(x, y). \tag{10}$$

The MSSIM comparisons and SSIM index maps between input and output images for Flat EEG are shown in Table 2 and Fig. 6, respectively. Based on Table 2, IT2FS has the highest MSSIM value followed by Local Histogram Equalization, Global Histogram Equalization, and the lowest is Type-1 fuzzy.

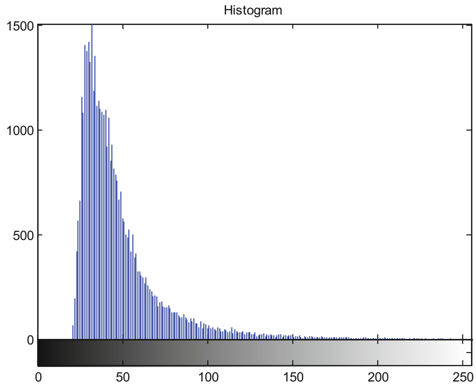
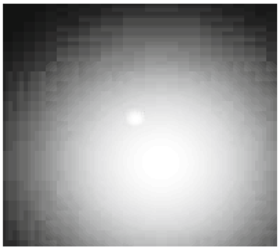


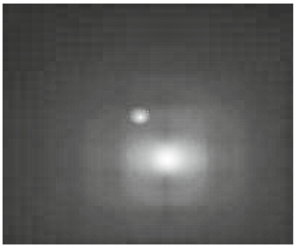
Fig. 3. Histogram of Flat EEG input image.

Table 1. Position and electrical potential of the cluster centres at $t = 1$.

Time (second)	Position		Electrical potential (μV)
	x	y	
1	−0.31297	−0.63736	8.5622
	1.46707	2.93728	45.7730

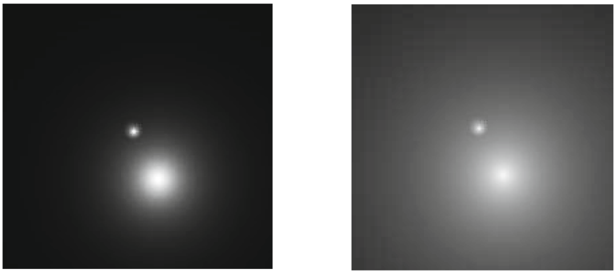


a) Global Histogram Equalization



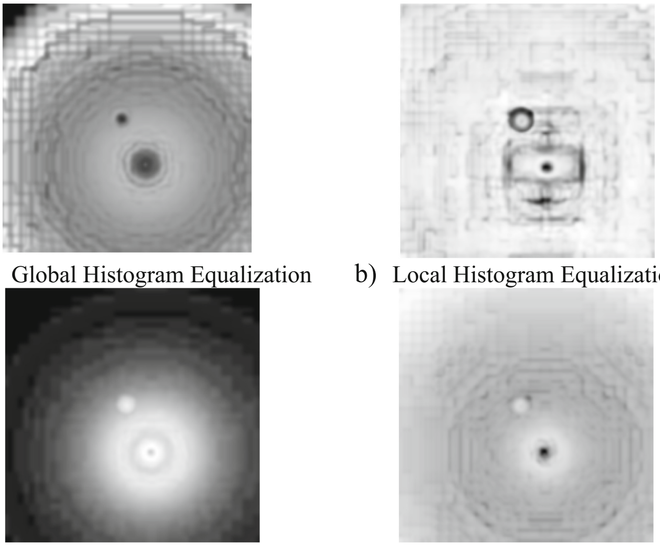
b) Local Histogram Equalization

Fig. 4. Classical method



a) Type-1 Fuzzy b) IT2FS

Fig. 5. Fuzzy method



a) Global Histogram Equalization b) Local Histogram Equalization

c) Type-1 Fuzzy d) IT2FS

Fig. 6. SSIM index map

Table 2. MSSIM performance comparisons.

Method	MSSIM
Global Histogram Equalization	0.5414
Local Histogram Equalization	0.9272
Type-1 Fuzzy	0.2966
IT2FS	0.9425

5 Conclusions

This paper introduced an enhancement method based on IT2FS for Flat EEG image. The results demonstrate that the proposed method shows better performance compared to the other methods. The method is capable in reducing the spread of the vague boundaries and preserving the cluster centres. Moreover, MSSIM is carried out to measure the similarity between the input and output images.

Acknowledgments. The author would like to express her appreciation and gratitude to the Research Management Centre, Universiti Malaysia Sabah for granting this research study under Skim Geran Acculturation (SGA0055-2019).

Authors' Contributions. The author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

References

1. Zakaria, F.: Dynamic profiling of EEG data during seizure using fuzzy information. Ph.D Thesis, Universiti Teknologi Malaysia (2008)
2. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**(3), 338–353 (1965)
3. Atanassov, K.T.: Intuitionistic fuzzy sets. J. Fuzzy Sets Syst. **20**, 87–96 (1986)
4. Zenian, S., Ahmad, T., Idris, A.: Contrast comparison of flat electroencephalography image: classical, fuzzy, and intuitionistic fuzzy set. In: Berry, M.W., Mohamed, A.H., Wah, Y.B. (eds.) SCDS 2015. CCIS, vol. 545, pp. 97–105. Springer, Singapore (2015). https://doi.org/10.1007/978-981-287-936-3_10
5. Zenian, S., Ahmad, T., Idris, A.: Intuitionistic fuzzy set: fEEG image representation. In: AIP Conference Proceedings (2019). https://doi.org/10.1063/1.5136486
6. Zenian, S., Ahmad, T., Idris, A.: Advanced fuzzy set: an application to flat electroencephalography image. In: Alfred, R., Lim, Y., Haviluddin, H., On, C.K. (eds.) Computational Science and Technology. LNEE, vol. 603, pp. 649–657. Springer, Singapore (2020). https://doi.org/10.1007/978-981-15-0058-9_62
7. Wang, Z., Alan, C.B.: A universal image quality index. IEEE Signal Process. Lett. **9**(3), 81–84 (2002)