Research on the Application of Minimum Variance Model and Utility Maximization Model in Stock Market Portfolio

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Abstract. In financial market investing, returns always come with risk. The theoretical research and practice of portfolio selection have yielded quite rich results. Based on Markowitz's mean-variance theory, this paper analyses the returns and risks of stock portfolios, and uses the minimum variance and maximum utility models to find the optimal stock portfolios under different risk preferences. The research results show that in stock investment, when the correlation coefficient of two stocks is positive, the investment ratio of two stocks, one is greater than 1, and the other is less than 0. Investors can avoid risk by shorting one of the stocks in the following ways. When the correlation coefficient between two stocks is negative, the investment ratio of the least risky investment is greater than 0. This paper provides a favorable demonstration that investors can use utility-maximizing portfolio models to obtain optimal stock portfolios. The conclusion of this paper has important practical significance for investors to effectively avoid risks in the financial market.

Keywords: Minimum Model · Stock Portfolio · Utility Maximization · Comparison

1 Introduction

1.1 Research Background and Motivation

The occurrence of the subprime mortgage crisis in the United States in 2008 had an impact on global economic development, leading to a significant decline in the scale of foreign trade and weakened economic growth momentum. After the report of the 19th Party Congress put forward that "China's economy has shifted from the stage of high-speed growth to the stage of high-quality development", high-quality development has rapidly become a research hotspot in the academic field, and its related research literature has been increasing, and many scholars have put forward their own opinions on the connotation of high-quality development [1]. Economy to suffer an unprecedented

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blow, and according to the International Monetary Fund (IMF), the World Bank, and the OECD (Organization for Economic Cooperation and Development), the global economy has experienced the first negative growth in nearly a decade. Meanwhile, governments have repeatedly lowered their economic growth expectations for 2020 and 2021. The World Bank's Global Economic Outlook, released in June 2020, predicts that the global economy will decline by 5.2% in 2020, the most severe recession since World War II [1]. Many countries are still under the shadow of the epidemic, international financial markets are unstable, and economic and trade activities are hampered. It is estimated that world economic growth will continue to decline in the first half of 2021 and throughout the year but will rebound significantly if the epidemic is effectively controlled in the second half of the year. Governments are currently adopting extraordinary fiscal and monetary policies to stimulate economic recovery, however, the endogenous drivers of economic growth - business investment and personal consumption demand - are clearly insufficient, and there is a lack of new growth points and endogenous power. In the medium to long term, the global economy will continue to be weak, and demand is insufficient, and it will be difficult to recover to the level before the outbreak. On October 6, 2020, the WTO released an updated report on "Global Trade Data and Outlook". The report predicts that the volume of global trade in goods will contract by 9.2% in 2020, better than the previous forecast of a 13% to 32% decline. The report also forecasts global trade in goods to grow by 7.2% in 2021, significantly lower than the previous forecast of over 20% growth. The development and optimization of foreign trade has become an issue of great importance to every country, and the development of foreign trade can, to a certain extent, indicate a country's international status and international influence, which can bring benefits or cause damage to the country.

In the international environment, how to better expand the self-interest side of foreign trade has become a problematic goal that every country is studying and hoping to achieve. Foreign trade is an important part of our national economy and an important driving force. In recent years, the international foreign trade situation is complex and severe, the instability factors increase, and the downward pressure is also increasing. Promoting the stabilization and improvement of foreign trade and promoting the innovative development of foreign trade are of great significance to the stable operation and upgrading development of China's economy.

1.2 Literature Review

Portfolio theory and practice have been one of the hot issues in academia and industry in recent years, which plays an important role in asset preservation and risk management. According to the statistics of Boston Consulting Group, the global asset management scale has reached 103 trillion USD by the end of 2020; as of the first half of 2021, the asset management scale of China is about 11 billion USD, making it the second-largest asset management market in the world after the United States. Therefore, the study of investment strategies can help serve the needs of national and social development and has positive practical significance for promoting the healthy development of financial markets and economic development. For individual stocks, empirical evidence from Ang et al. shows that stocks with higher volatility or heterogeneous volatility have lower future returns. They refer to this negative relationship as a volatility anomaly [2, 3]. To reduce portfolio turnover, Blitz and Vliet use a long-term volatility measure instead of a short-term volatility measure and find that not only high-volatility stocks have lower future returns, but also low-volatility stocks have exceptionally high future returns, which they refer to as the low-volatility effect [4].

In the case of stock portfolios, since the portfolio can provide diversification of the risk of a single stock, i.e., the volatility of a single stock cannot constitute the risk of a portfolio through a linear combination, more studies have used the covariance matrix as a measure of risk for further analysis. According to Markowitz's portfolio theory, when the expected return of a portfolio is given, the risk can be minimized by adjusting the weights of individual assets within the portfolio. Still, when this method is introduced to the study of portfolio-specific volatility and portfolio return, there are problems that the expected return is difficult to estimate accurately. The optimal weights are sensitive to the expected return. To solve this problem, the focus of the study is shifted to the minimum variance portfolio with portfolio weights independent of expected return. The results show that volatility anomalies exist in the minimum variance portfolios. For the Chinese stock market, most of the findings suggest that there is a negative correlation between stock returns and their volatilities, i.e., volatility anomalies exist in the Chinese stock market. However, these studies mainly focus on the relationship between individual stock returns and their volatilities, and the construction of minimum variance portfolios is limited to theoretical derivation, without providing the construction method of minimum variance portfolios and the analysis of portfolio returns and risks. In view of this, this paper attempts to investigate the existence of volatility anomalies and the reasons for the anomalies by constructing a minimum variance portfolio in the Chinese stock market.

In the context of economic globalization, investors are increasingly using their idle funds to invest in financial products such as stocks and funds to obtain higher returns [5]. With the increase in investment frequency and knowledge, investors realize that they cannot focus on returns without considering risks in the securities market, and rational investors should reasonably avoid risks and maximize expected returns [6]. A common risk-averse tool used by investors in investment management is portfolio investment, which increases the diversity of risky assets in a portfolio by selecting stocks of companies in different industries to reduce the unsystematic risk associated with holding highly correlated stocks [7]. The modern portfolio theory was proposed by Markowitz in 1952 and aims at diversifying risk through portfolios, avoiding the unsystematic risk in investments, and pursuing return maximization and risk minimization. The investor's utility is determined by the risk aversion of the investor, the expected return, and the risk of the project, which is a function of the expected return and the standard deviation of the portfolio [8]. Therefore, research-based on portfolio theory is relevant for guiding investors' investment behaviour. The purpose of this study is to quantify the return/risk correlation of four companies' stocks and the investment decisions of different investors in different investment markets by calculating the expected return and standard deviation of stocks using the Markowitz model, to provide a reference for investors' investment management strategies.

Markowitz proposed the mean-variance model, an optimization model that considers both the return and risk characteristics of a portfolio [9]. Subsequently, a large body of literature has focused on improving the optimization criteria of portfolio optimization models, such as the mean-absolute variance criterion, the mean-semi variance criterion, the mean-variance CVaR criterion, and so on [10–13]. Sharpe proposed a Sharpe ratio maximization criterion, where the Sharpe ratio is defined as the ratio of the excess return of a portfolio to its standard deviation [14]. By maximizing the Sharpe ratio, the investor can consider both portfolio return and risk in the objective function of the optimization problem with an economic interpretation. Song Hongyu analysed the application of the Sharpe ratio in investment strategies under non-short selling conditions [15]. Ming Zhou et al. studied the optimal reinsurance strategy of insurance companies based on maximizing the Sharpe ratio criterion [16]. Sahamkhadam et al. combined GARCH-EVT- Copula model for portfolio optimization under maximizing Sharpe ratio criterion [17]. Zhao et al. studied the optimal insurance investment strategy based on the spectral risk measure and maximizing the RAROC criterion [18].

1.3 Research Contents and Framework

This paper will consider maximizing the Sharpe ratio as the optimization criterion and constructing an asset dependence network by choosing four different correlation coefficients to characterize the inter-asset dependence relationships to study the optimal investment strategy and compare it with the strategy under the global risk minimization criterion. The empirical analysis is based on eight stocks and fund data from Investing.com, and the dynamically adjusted optimal portfolio is obtained using the rolling window method (the inner sample window widths are set to one year and two years, respectively), and the inner sample weight structure and outer sample performance of the optimal portfolio are further analysed. As China's economy continues to grow, people have more and more money at their disposal, and many people are looking for stocks, expecting to earn high returns through dividend income and stock trading spreads. The securities investment market is a market with both benefits and risks, and although it can bring high returns to investors, it also carries high risks. Therefore, it is important for investors to optimize the portfolio of securities investments to obtain greater returns with minimum risk.

2 Methodology

2.1 Model Selection

Stock market risk is the possibility of loss caused by the rise or fall of stock price in the stock market. It mainly arises from the change of stock market price and is the main risk of investment. This kind of risk can be avoided by diversifying investment and forming an appropriate portfolio. Stock investors invest all their funds in a single stock. The investment risk is too high. Investors usually choose the appropriate portfolio to reduce the risk. Stock portfolio investment is a method to study the optimal equilibrium relationship between overall return and risk in the case of uncertainty, in which investors allocate their funds to different stocks according to a certain proportion.

2.2 The Rationale for the Model

2.2.1 Profit Model

Assuming that there are two stocks, S_1 and S_2, whose return rates are R_1 and R_2 respectively, and the investor constructs the stock portfolio according to the proportion of w_1 and w_2, the return rate r of the stock portfolio can be expressed as:

$$R = W_1R_1 + W_2R_2 \tag{1}$$

where $W_1 + W_2 = 1$, w_1 and w_2 can be negative numbers, indicating short-selling of the stock.

When there are N stocks, the expression for stock returns is.

$$R = W_1R_1 + W_2R_2 + \ldots + W_nR_n \tag{2}$$

2.2.2 Risk Model

The stock risk is usually measured by the variance of the return rate. The variance is usually expressed in. The variance of the two stock combinations is

$$\sigma^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2Cov(R_1, R_2) \tag{3}$$

where Cov (R_1, R_2) is the covariance between the returns of securities S_1 and S_2. In order to better measure the correlation between the two securities, we introduce the correlation coefficient to overcome the problem of inconsistent data orders. The calculation is as follows:

$$\rho_{12} = \frac{Cov(R_1, R_2)}{\sigma_1\sigma_2} \tag{4}$$

Portfolio risk can also be calculated as:

$$\sigma^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho_{12}\sigma_1\sigma_2 \tag{5}$$

The correlation coefficient can better measure the correlation between the changes of returns of the two securities. When the correlation coefficient is positive, the changes of returns of securities S_1 and S_2 are positively correlated, and the returns of the two stocks change in the same direction; The correlation coefficient is negative, the change of securities S_1 and S_2 yield is negatively correlated, and the two stocks change in the opposite direction; The correlation coefficient is zero, and the changes of securities S_1 and S_2 yield are irrelevant.

When there are N stocks, the expression for stock risk is:

$$\sigma^2 = W_1^2\sigma_1^2 + \ldots + W_n^2\sigma_n^2 + 2W_1W_2\rho_{12}\sigma_1\sigma_2 + 2W_1W_3\rho_{13} + 2W_{n-1}W_n\rho_{23}\sigma_{n-1}\sigma_n \tag{6}$$

2.2.3 Portfolio Model

This paper aims to minimize the risk, that is, the variance. Given $W_1 + W_2 = 1$, bring in and minimize the following formula:

$$\sigma^2 = W_1^2\sigma_1^2 + (1 - W_1)^2\sigma_2^2 + 2W_1(1 - W_1)\rho_{12}\sigma_1\sigma_2 \tag{7}$$

Let $\frac{d\sigma^2}{dw_1} = 0$, we can get the proportion of the portfolio invested.

$$W_1 = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \tag{8}$$

$$W_2 = \frac{\sigma_1^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \tag{9}$$

Utility U is the satisfaction of investors in investment activities, and the utility function is defined as:

$$U = E(R) - 0.5A\sigma^2 \tag{10}$$

where A is the risk aversion coefficient. A larger value of A indicates a higher degree of risk aversion.

In this model, this paper aims to maximize utility. Known w1 + w2 = 1, bring in the utility function to get

$$U = W_1R_1 + W_2R_2 - 0.5A(W_1^2\sigma_1^2 + (1 - W_1)^2\sigma_2^2 + 2W_1(1 - W_1)\rho_{12}\sigma_1\sigma_2 \tag{11}$$

Let $\frac{d\sigma^2}{dw_1} = 0$, we can get the proportion of the portfolio invested.

$$W_1 = \frac{R_1 - R_2 + A\left(\sigma_2^2 - \rho_{12}\sigma_1\sigma_2\right)}{A\left(\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2\right)} \tag{12}$$

3 Empirical Results Analysis

3.1 Verification of Binary Portfolio Model

This paper selects the closing price data of eight stocks in Investing. After checking the stock data for missing values and outliers, the stock returns, variances and standard deviations are calculated. The results are shown in Table 1.

Next, this paper calculates the stock correlation coefficient matrix, as shown in Table 2.

Table 1. Eigenvalue Analysis of Stocks

Variables	ZJ	JI	TGR	BYD	Moutai	JP	AHM	PA
Average Value	0.00252	0.00540	-0.00335	-0.00289	-0.00310	-0.00091	-0.00162	-0.00090
Variance	0.50869	5.21194	0.10164	140.37874	11071.05797	11.78386	3.19193	8.14110
Standard Deviation	0.71322	2.28297	0.31881	11.84815	105.21910	3.43276	1.78660	2.85326

Table 2. Correlation and covariance matrices for eight stocks

Covariance	ZJ	JI	TGR	BYD	MOUTAI	JP	AHM	PA
ZJ	0.00071							
JI	0.000505	0.003885						
TGR	0.000258	0.000253	0.000382					
BYD	0.000391	0.000525	0.000325	0.001115				
MOUTAI	0.000156	0.000296	0.00019	0.00038	0.000496			
JP	-1.38E-05	-0.000491	0.000185	0.000445	0.000133	0.001496		
AHM	0.000153	-0.000219	0.000275	0.000648	0.000304	0.0006	0.001612	
PA	0.000298	0.000585	0.000234	0.000216	0.000196	-0.000105	5.85E-05	0.000471
Correlation	ZJ	JI	TGR	BYD	MOUTAI	JP	AHM	PA
ZJ	1							
JI	0.303701	1						
TGR	0.495209	0.207204	1					
BYD	0.438886	0.252325	0.497391	1				
MOUTAI	0.262116	0.213222	0.437007	0.510727	1			
JP	-0.013345	-0.203605	0.245246	0.344306	0.154512	1		
AHM	0.143374	-0.087678	0.350892	0.483236	0.339915	0.386339	1	
PA	0.516263	0.432424	0.551698	0.298572	0.406484	-0.124669	0.067115	1

3.1.1 When the Correlation Coefficient of Two Stocks is Positive

Considering that the correlation coefficient of ZJ and JI is positive, this paper takes ZJ and JI as examples to calculate the difference in their portfolio weights when the variance is minimal and the utility is maximizing, respectively. The calculation results are shown in Table 3. As shown in Table 3, the results of the minimum variance model show that when two assets, ZJ and JI, are selected as investment portfolios, their weights are W_{zj} equal to 0.87 and W_{ji} equal to 0.13. The results of the maximum utility model show different results depending on the value of A. When A is 0.1, the weights of ZJ and JI are -5.84 and 6.84, respectively, and when A is 0.5, the weights of ZJ and JI are-0.47 and -1.49, respectively.

Table 3. Portfolio weights under different conditions

Model		W_{zj}	Wji
Minimum variance model		0.87	0.13
Maximum utility model	A = 0.1	-5.84	6.84
	A = 0.5	-0.47	1.47

Table 4. Portfolio weights under different conditions

Model		Wzj	W_{ji}
Minimum variance model		0.68	0.32
Maximum utility model	A = 0.1	1.62	-0.62
	A = 0.5	3.79	-2.79

3.1.2 When the Correlation Coefficient of Two Stocks is Negative

Considering that the correlation coefficient of ZJ and JI is negative, this paper takes ZJ and JP as examples to calculate the difference of their portfolio weights when the variance is minimum and the utility is maximum, respectively. The calculation results are shown in Table 4. The results of the minimum variance model show that when two assets ZJ and JP are selected as the portfolio, their weights W_{zj} are equal to 0.68 and W_{JP} are equal to 0.32. The results of the maximum utility model show different results depending on the value of A. When A is 0.1, the weights of ZJ and JP are 1.62 and -0.62, respectively, and when A is 0.5, the weights of ZJ and JP are 3.79 and -2.79, respectively.

4 Conclusion

When the correlation coefficient between the two stocks ZJ and JI is positive, the investment ratio of one of the two stocks is greater than 1 and the other is less than 0, and one needs to be shorted to avoid risks; and when the correlation coefficient between the two stocks ZJ and JP is negative, the investment ratio is greater than 0 to minimize the risk. Further considering investors' consideration of the balance between return and risk, due to the different risk preferences of different investors, there are obvious differences in the optimal investment portfolio of different investors. The larger the value of the risk preference coefficient A, the more investors tend to invest in less risky investment schemes, and the proportion of investment in high-risk stocks will decrease. Empirical analysis shows that the utility-maximizing portfolio model can more scientifically reflect the relationship between risk and return and provide a reference for investment.

References

1. L. A., Rong et al. Investor attention and cryptocurrency: Evidence from wavelet-based quantile Granger causality analysis - ScienceDirect. Research in International Business and Finance, 2021. Doi: https://doi.org/10.1016/j.ribaf.2021.101389
2. A., Ang, et al. "The Cross-Section of Volatility and Expected Returns." NBER Working Papers 61.1, 2004:259–299. Doi: https://doi.org/10.1111/j.1540-6261.2006.00836.x
3. A., Ang, et al. "High idiosyncratic volatility and low returns: International and further U.S. evidence." Journal of Financial Economics 91, 2009. Doi: https://doi.org/10.1016/j.jfineco.2007.12.005
4. D. C., Blitz, and P. Van Vliet. "The Volatility Effect: Lower Risk without Lower Return." Erim Report 34.1, 2007. Doi: https://doi.org/10.3905/jpm.2007.698039
5. K. D., Liu. "Investment Portfolio of Markowitz Theory". Modern commerce, 2018, (36):44–45. Doi: CNKI: SUN: XDBY.0.2018–36–022
6. S. L., Li. "Research on Markowitz Portfolio Model with Mean and Variance Changes". Economic outlook around Bohai Sea, 2020, (2):185–186. Doi: CNKI: SUN: HBHJ.0.2020–02–138
7. C. H., Rong. and N. N., Yao. "Research On Stock Portfolio Management Strategy". Science and Education Literature Collection, 2020, (2):189-190. Doi: CNKI: SUN: KJWH.0.2020-02-089
8. J. Y., Tao. "Research On Stock Portfolio Based on Risk Assessment". Modern trade industry, 2019,40(6):115–118. Doi: CNKI: SUN: XDSM.0.2019–06–056
9. H. M., Markowitz. "Portfolio Selection". Journal of Finance, 1952, 7(1):77–91. Doi: https://doi.org/10.2307/2975974
10. H., Konno, and H., Yamazaki. "Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market". Managemant Science, 1991, 37(5):519–531. Doi: http://www.jstor.org/stable/2632458
11. W., Ogryczak and A., Ruszczynski. "From Stochastic Dominance to Mean-Risk Models: Semideviations as Risk Measures". European Journal of Operation Research, 1999, 116(1):33–50. Doi:https://doi.org/10.1016/S0377-2217(98)00167-2
12. J., Gao, et al. Dynamic mean-risk portfolio selection with multiple risk measures in continuous-time. European Journal of Operational Research, 2016, 249(2):647–656. Doi: https://doi.org/10.2139/ssrn.2382343
13. X., Zhao and Y., Shi Optimal asset allocation and reinsurance strategy for insurance companies based on Mean-Variance-CVaR criterion (in English). Applied Probability Statistics, 2020, 36 (5):536–550. Doi: https://doi.org/10.3969/j.issn.1001-4268.2020.05.008
14. Sharpe, and F. William. The sharpe ratio. the Journal of Portfolio Management, 1994, 21(1):49–58. Doi: https://doi.org/10.3905/jpm.1994.409501
15. H.Y., Songu. Exploring the application of Sharpe ratio in investment management. Statistics and Decision Making, 2006, 12 (24):107–109. Doi: https://doi.org/10.3969/j.issn.1002-6487.2006.24.040
16. M., Zhou, W., Kou, H.J., Li. Optimal reinsurance strategy based on Sharpe ratio. Mathematical Statistics and Management, 2013, 32(5):910-922.Doi: CNKI: SUN: SLTJ.0.2013-05-017
17. M., Sahamkhadam, et al. Portfolio optimization based on GARCH-EVT-Copula forecasting models. International Journal of Forecasting, 2018, 34(3):497–506.Doi: https://doi.org/10.1016/j.ijforecast.2018.02.004
18. X., Zhang, et al. Optimization Problem of Insurance Investment Based on Spectral Risk Measure and RAROC Criterion. Mathematical Problems in Engineering, 2018, 2018(5):1–7. Doi: https://doi.org/10.1155/2018/9838437