# Forecasting the Yield Curve with Nelson-Siegel Model: Chinese Evidence

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**Abstract.** The Dynamic Nelson-Siegel model is commonly used to forecast interest rate curves, but the way its parameters are estimated has so far been an issue worth investigating. In this paper, the performance of state space model is compared with that of two-step method in estimating parameters. Based on Chinese government bond data, this paper explores whether state space model has higher forecasting progress and better forecasting performance.

**Keywords:** interest rate term structure · Dynamic Nelson-Siegel model · state space model

## 1 Introduction

Interest rates play an important role in finance, both to discount the present value of assets and to reflect macroeconomic conditions. The term structure of interest rates refers to the relationship between the interest rate and maturity for different maturities, which can usually be represented by a curve in three-dimensional space. Therefore, it is crucial to forecast forward interest rates. One of the most widely used is the Nelson-Siegel model proposed by Nelson and Siegel in 1987, which is a three-factor model with each factor having more significant macroscopic significance and term significance of interest rates. To better predict long-term forward interest rates, the Dynamic Nelson-Siegel model treats these three factors as time series for iteration [1] (Diebold and Li, 2006). The DNS model allows factor loadings to vary according to a VAR structure. Various statistical methods associated with state space models are used to estimate this time series [2] (Durbin and Koopman, 2001).

The classical DNS model is considered as a constant parameter and can be derived by picking a suitable medium-term interest rate inverse. However, treating it as a time-varying parameter may achieve better forecasting results [3] (Koopman, Mallee and Wel, 2008). To make volatility a time-varying parameter, the GARCH process is introduced into the DNS model [4] (Koopman et al., 2014). There are also models that directly model the stochastic volatility of the yield curve factors. In this case, the time-varying volatility of individual yields is captured by the volatility of the yield factors. These volatilities are naturally interpreted as the volatility of the underlying bond portfolio associated with short, medium and long maturities. [5] (Hautsch and Ou, 2008). A Monte Carlo algorithm based on Kalman filtering was applied to extract latent factors


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and time-varying volatilities, achieving better results than element-by-element sampling [6] (Hautsch and Yang, 2012).

Based on the empirical analysis of bond yields of Chinese treasury bonds, this paper analyzes the comparison of the effectiveness of the state-space model and the two-step method when the DNS model is used for parameter estimation, and numerical analysis for the effect is given in the paper.

Financial institutions such as banks will undergo a negative change in their stability due to the vicious rise in real estate price [7] (Koetter and Poghosyan, 2010). This will inhibit economic recovery in the long run.

## 2 The Nelson-Siegel Class of Models

### 2.1 Nelson-Siegel Model

First, the discount curve can be defined:

$$p(\tau) = e^{-\tau y(\tau)} \tag{1}$$

where $p(\tau)$ is the present value of the discounted bond in period $\tau$, both the present value of \$1 after period $\tau$ in the future; and $y(\tau)$ is the continuously compounded yield to maturity.

The forward rate curve is defined as:

$$f(\tau) = \frac{-p'(\tau)}{p(\tau)} \tag{2}$$

From $p(\tau)$ and $f(\tau)$, the yield curve can be derived as follows:

$$y(\tau) = \frac{1}{\tau} \int_0^\tau f(u)du \tag{3}$$

The model thus constructed is theoretically sound, but lacks a link to real economic activity. However, "the cross-correlation of bond yields is well described by a low-dimensional factor model, i.e., the first three principal components of bond yields...... Explains more than 95% of their variation..... A very similar three-factor representation emerges from an arbitrage free dynamic term structure model...... For a wide range of maturities." [8] (Joslin et al., 2010) This points out that the yield curve $y(\tau)$ has a three-factor structure and that these three factors are related to the level, slope, and curvature of the curve, respectively. More importantly, these three factors can be related to real-world economic activity - the level factor is usually highly correlated with inflation; the slope factor is highly correlated with real activity; and the curvature factor appears to be uncorrelated with any major macroeconomic variable.

### 2.2 Dynamic Nelson-Siegel Model

The equation for fitting the yield curve is given in Nelson and Siegel's 1987 paper:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{4}$$

Dynamic Nelson-Sigel introduces a time parameter t based on the Nelson-Siegel model, i.e.

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{5}$$

In the DNS model, the loading of $\beta_{1t}$ is constant 1, so it does not converge to 0 even after taking the limit ($\tau \to \infty$). Therefore, unlike the other two loadings, it affects long-term returns and is called the long-term factor; the loading of $\beta_{2t}$ is $\frac{1-e^{-\lambda\tau}}{\lambda\tau}$, which starts at 1 but decreases monotonically to 0 quickly. Therefore, it is called the short-term factor and affects short-term returns; the loading of $\beta_{3t}$ loadings are $\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}$, which starts at 0 and decreases to 0. Therefore, it is called the medium-term factor and affects the medium-term return.

In terms of the geometric meaning of the interest rate curve, $\beta_{1t}$ affects the level, flattening the yield curve and increasing both short- and medium-term and long-term yields equally; $\beta_{2t}$ affects the slope, increasing short-term yields significantly but leaving long-term yields unchanged; and an increase in $\beta_{3t}$ does not change short-term and long-term yields much, but it does change medium-term yields.

## 2.3 State Space Model

For the purpose of introducing state space model, emphasizing the geometric significance of the three factors, the DNS model is rewritten as

$$y_t(\tau) = l_t + s_t\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + c_t\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{6}$$

where $t = 1, .., T, \tau = 1, .., N$

The state space model can be given by this equation.

$$y_t = \Lambda f_t + \varepsilon_t \tag{7}$$

where $y_t = \begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \cdots \\ y_t(\tau_n) \end{pmatrix}$, $f_t = \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix}$, $\varepsilon_t = \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \cdots \\ \varepsilon_t(\tau_n) \end{pmatrix}$, $\varepsilon_t$ sometimes is called idiosyncratic factor.

$$\Lambda = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \cdots & \cdots & \cdots \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix} \tag{8}$$

The transition equation is: $(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t$

Where $f_t = \begin{pmatrix} l_t \\ s_t \\ c_t \end{pmatrix}$, $\eta_t = \begin{pmatrix} \eta_t^l \\ \eta_t^s \\ \eta_t^c \end{pmatrix}$, both $f_t$ and $\eta_t$ are variables.

$$= \begin{pmatrix} \mu^l \\ \mu^s \\ \mu^c \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

, both $\mu$ and $A$ are parameters. $\mu$ and $A$ can be estimated by Kalman Filter method (Hautsch and Yang, 2010).

Here we assuming that the white noise transitions and measurement disturbances are orthogonal and orthogonal to the initial state:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right) \tag{9}$$

$$E\left(f_0\eta_t'\right) = 0 \tag{10}$$

$$E\left(f_0\varepsilon_t'\right) = 0 \tag{11}$$

## 3 Empirical Analysis

### 3.1 Data

In this paper, the yield to maturity of China's government bonds from 2017–2022 is used as the target to use the DNS model, with data as of 2022.5.11. Data are obtained from the Choice database and the China Bond.

In the dataset, a total of 13 maturities are used in this paper, namely (in months) 3, 6, 9, 12, 24, 36, 60, 84, 120, 180, 240, 360, 600.

### 3.2 Results

The data set was analyzed using MATLAB and the following results were obtained:

From the Fig. 1, 2 and 3, it can be seen that the difference between the parameters obtained from the state-space model and the two-step method estimation is not very large in both the level factor and the slope factor obtained, but there is significant difference in the results of the curvature factor estimation (Fig. 4).

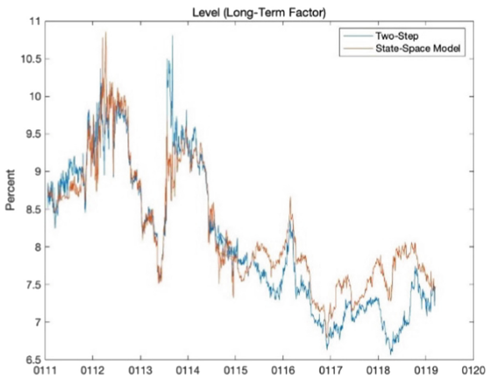


**Fig. 1.** Comparison of level factor


**Fig. 2.** Comparison of slope factor

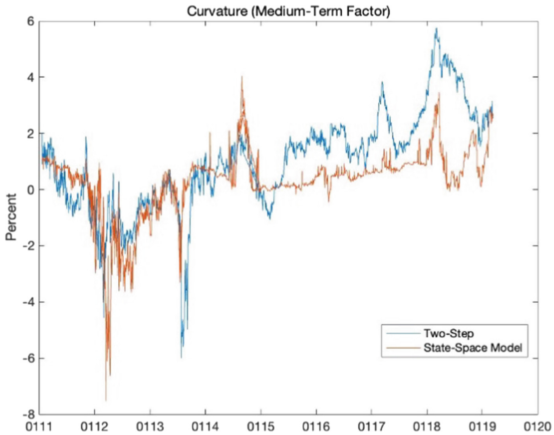


**Fig. 3.** Comparison of curvature factor

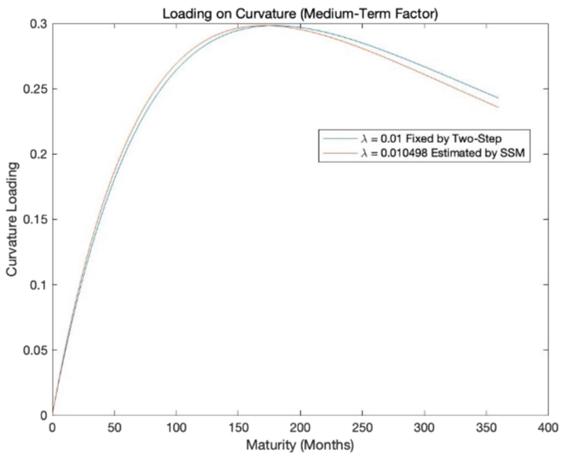


**Fig. 4.** Comparison of loading on curvature

As can be seen from Fig. 5, the 1–10 year immediate interest rate increases faster with increasing maturity, is essentially flat above 16 years, and has a slight downward bias above 24 years. In fact, the term structure of Treasury rates tends to increase with increasing maturity, a finding consistent with the liquidity preference theory of traditional interest rate term structure theory. As maturity increases, the necessary yield demanded by investors increases to compensate for liquidity (Fig. 6).

Figure 7 gives the probability distribution functions for forward rates at 1, 6, and 12 months, respectively. It can be seen that the probability distribution approximately follows a normal distribution. This is of great importance for interest rate forecasting. We can use this probability distribution function to calculate the function of the bond price

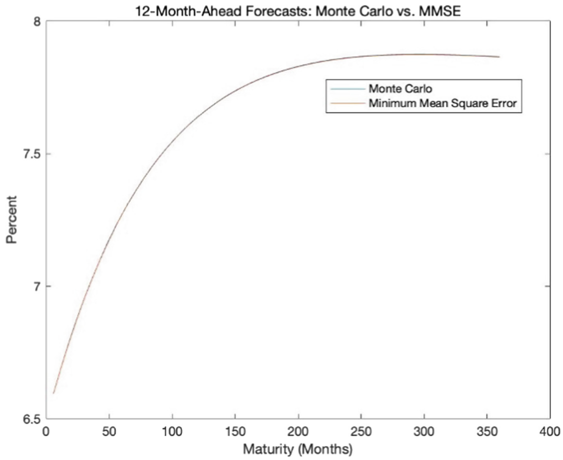


**Fig. 5.** Comparison of forecast about interest rate after 12 months

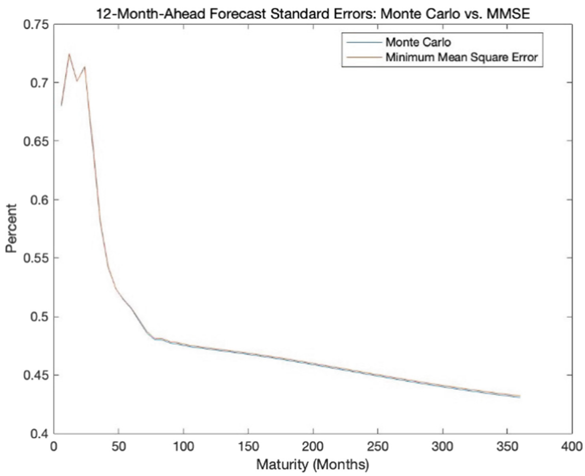


**Fig. 6.** Comparison of S.E. of the forecast

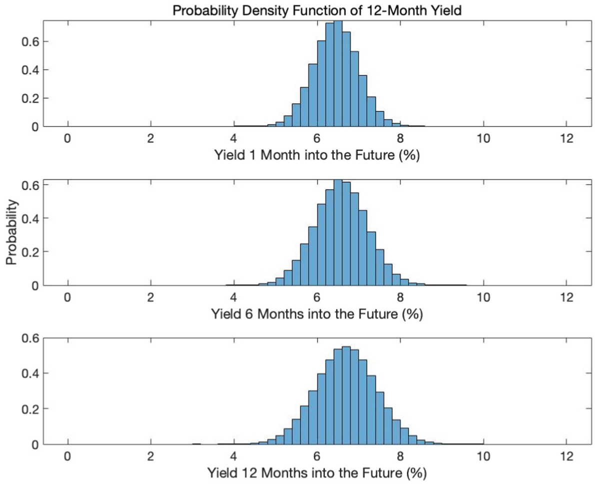


**Fig. 7.** Possibility density function of the result

distribution in a future period. Confidence intervals are then calculated for a given level of significance to construct a trading strategy that makes it possible to short Treasury futures at overvaluations and long Treasury futures at troughs. As long as there is no black swan event, stable profits can be made.

## 4 Conclusion

This paper mainly describes the principles of DNS model and state space model. And the prediction performance is compared for the state space model and the two-step method. It can be concluded that the state-space model can achieve better prediction results. In addition, the empirical analysis in this paper is based on Chinese treasury data, and we can analyze the trend of interest rate changes from the forecasting results to determine the future economic cycle and thus know the approximate time period of economic recovery.

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