Research on Optimal Pricing of Time-Sharing Car Rental Service Under Unbalanced Demand

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Abstract. With the rapid development of the country and social economy, people's demand for vehicles is increasing, but the purchase costs and maintenance costs of vehicles are generally high, so now time-sharing car rental is more and more popular. In order to improve the utilization rate of vehicles and the profits of car rental companies, this paper takes the profit maximization of car rental companies as the objective function, and builds an optimal pricing model for car rental services under the condition of unbalanced demand. The price compensation is used to adjust user needs, to achieve a "balance" between vehicle distribution and user needs, and to solve the model to obtain an optimal pricing/compensation scheme.

Keywords: optimal pricing · time-sharing car rental service · unbalanced demand

1 Introduction

With the development of mobile internet technology at home and abroad, time-sharing rental has become a new type of car rental model. Compared with the traditional car rental service, consumers can pay according to the using time, ranging from a few minutes to several hours. The car rental place is also very flexible, bringing great convenience to consumers, meeting consumers' short distances, short time and travel needs, and providing users with a convenient way to travel. For the society, the time-sharing car rental model optimizes resource allocation, effectively integrates vehicle resources, realizes the effective matching of users' vehicle demand and vehicle supply, reduces the number of vehicles owned by individuals, and plays a certain role in environmental protection. For users, the time-sharing car rental model improves the flexibility of users' car sources, reduces travel costs, and improves travel efficiency.

For time-sharing car rental, due to the differences between urban regions and the heterogeneity of residents' travel, the demands and flows of vehicles between regions are often uneven. When consumers pick up and return cars at different places, the distribution of vehicles and the geographical distribution of demand will be mismatched. On the one hand, there will be idle vehicles waiting in some areas, while in other places with less distribution of vehicles, there will be unsatisfied residents' travel needs. In this case, idle vehicle resources of time-sharing rental companies will be wasted, as well as the losses

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caused by the lack of vehicles in areas with user demand. The waste of resources means the reduction of income and thus affects the company's goal of maximizing profits.

Therefore, the decision of vehicle allocation and vehicle pricing is very important. In decision making based on internet technology, time-sharing car rental company needs to consider requirements of users in different areas, the willingness to pay and a vehicle pricing in the platform. By influencing users' willingness to pay for car prices in different regions, car rental companies can adjust vehicle distribution indirectly, realize the balance of the vehicles demand and distribution, so as to maximize the company's operating profit.

The study on time-sharing pricing abroad is more in-depth. Kostas Bimpikis studies the pricing of space on shared mobile platforms [3], discusses the significant impacts of price discrimination on the operation of shared travel place network and shared travel platform, put forward that platform profit and total consumer surplus reached the maximum when the network location demand pattern is balanced, profit from the platform can be generated by optimizing pricing/compensation, and the induced surplus consumers increase with the balance of potential demand patterns. Siddhartha Banerjee studies pricing and optimization of shared vehicle systems with different goals (Bimpikis, 2017), they designed a general approximation framework in the shared vehicle system to provide approximate ratios for any shared vehicle system with n stations and m vehicles. For the pricing strategy model of 1 + (n–1) / m, this method provides an efficient algorithm, a strict approximation of a broad objective function (throughput, revenue, welfare), and extends to a more complex situation, such as the rebalancing an empty car, directing passengers to nearby vehicles and multi-objective setting (such as Ramsey pricing).

2 Optimal Pricing Model

This model is based on the research of Kostas Bimpikis, Ozan Candogan and Daniela Saban on shared rides. Taking the ride-sharing platform of taxi service network as the research object, they found that drivers on the platform decide when and where to provide services according to compensation and price to maximize their expected benefits, and explored the effect of price discrimination.

This model is based on the spatial pricing research of ride-sharing network, in the case of time-sharing car rental without drivers (users as drivers), setting different prices or compensation in different areas will affect users' demands and choices of vehicle use, by adjusting users' vehicle use behavior through different pricing methods to achieve the "balance" between vehicle distribution and users' demands, the profits of car rental companies can be maximized.

In the original models, the profits of the car rental companies was equal to the price paid by the passengers minus the drivers' profits, this model does not need to consider the expected incomes of the drivers, but only needs to consider the users' rental costs of using the vehicles, the idle costs of the vehicle and the compensation costs given to the users by different locations to adjusting the users' demands, so as to establish this model.

2.1 Basic Assumptions of The Model

In the case of unbalanced demands, the optimal pricing model of car rental service considers a discrete-time model of car rental network with n locations. Each time period has a large-scale continuous potential users θ_i arriving at the location i. The probability of the user who wants to go to location j at location i is determined by the elements in matrix A, represented by α_{ij}. For all locations i, $\sum_j \alpha_{ij} = 1$. At the same time, A can also be regarded as a weighted adjacency matrix associated with car rental network where α_{ij} is greater than zero, which can be called the demand mode of network (A, θ) (Bimpikis, 2016).

1) The distribution of users' willingness to use vehicles and their total destination preferences are known.
2) The goal of time-sharing car rental platform is to maximize the total profit.
3) This model studies how to maximize its profit through a location network. Therefore, the model focuses on the spatial dimension of platform pricing, assuming that the demand model is static.
4) When the demand model reaches "balance", the profit is the largest. If the demand model is balanced, the potential demand of each location is roughly the same as the number of passengers as the destination. We believe that when the potential demand model becomes more balanced, the profits generated by the platform will increase.
5) There are two assumptions about the demand model of the time-sharing car rental network (A, θ):

a) For all locations i, those who want to rent cars are all very positive, which is $\theta_i > 0$.
b) Each component of a directed graph defined by adjacency matrix A is connected.

Users are heterogeneous in their willingness to spend money. Specifically, if the price paid by the user is set to p, the induced demand between locations i and j is $\theta_i \alpha_{ij}(1 - F(p))$, where the function can be regarded as the cumulative distribution of (empirical) vehicles.

2.2 Definition of Model Parameters and Variables

- β: β ∈ (0,1), the probability that the user chooses to use the vehicle, and the value of β is affected by the price.
- θ_i: the number of users at location i.
- x_i: the number of available vehicles at location i.
- δ_i: the vehicles idle at location i.
- c_i: the compensation received by users who arrive at location i.
- p_i: the price that users have to pay for reaching location i.
- d_i: the number of demands for service received by the car-sharing platform at location i.
- w: the production cost of each idle vehicle.
- F(·): cumulative distribution of (empirical) vehicles.

2.3 Model Objective Function

If the price that users are willing to pay is greater than or equal to the price set by the platform, users will select vehicles in their location. At this time, the effective demand for car rental is $\theta_i(1 - F(p_i))$. x_i represents the number of available vehicles at location i, which satisfies the following equation:

$$x_i = \beta\left[\sum_j \alpha_{ji}\min\{x_j, \theta_j(1 - F(p_j))\}\right] + \delta_i \tag{1}$$

The first sum in formula (1) is equal to the number of vehicles $\min\{x_j, \theta_j(1 - F(p_j))\}$ arriving at i after completing the journey from place j, which is equal to the total demands for platform service at position j. δ_i represents idle vehicles at location i.

The car rental platform determines the tuple $\{p_i, c_i\}_{i=1}^n$, that is, the prices paid by users from each of the n locations and the compensation for riding in the region. Its goal is to maximize the total profit margin of the n locations. Specifically, the problem to be optimized takes the form as follow:

$$\max \sum_{i=1}^{n} min\{x_i, \theta_i(1 - F(p_i))\} \cdot (p_i - c_i) \tag{2}$$

$(p_i - c_i)$ represents the price actually paid by each user at location i. $\min\{x_i, \theta_i(1 - F(p_i))\}$ is equal to the total demands of users served by the platform.

For ease of analysis, an alternative optimization formula will be used below, in which the constraints are relaxed. The optimization formula can be simplified as:

$$\max \sum_i p_i d_i - w\sum_i \delta_i \tag{3}$$

For all i,

$$d_i = (1 - F(p_i))\theta_i$$

$$x_i = \beta\left[\sum_j \alpha_{ji}\min\{x_j, \theta_j(1 - F(p_j))\}\right] + \delta_i$$

For all i, j have $p_{i,}\delta_i \geq 0$.

The objective function in formula (3) consists of two parts: the first is equal to the total revenue flow of the platform $\sum_i p_i d_i$, where *di* represents the number of users actually served by the time-sharing car rental platform at location i. It can be easily concluded that $d_i = min\{x_i, \theta_i(1 - F(p_i))\}$.

On the other hand, the second item $w\sum_i \delta_i$ is equal to the loss cost of idle vehicles on the time-sharing car rental platform. It is emphasized here that these two formulas use different ways to express the cost of service platform induced demand. Specifically, a cost (compensation) c_i is allocated to each user in formula (2), while the cost of each idle vehicle is w in formula (3). Considering the incentive compatibility constraint of users, the latter is a lower bound of the cost of time-sharing car rental platform at the equilibrium point, which means that it is assumed that the induced demand under the two expressions is the same, and the objective function of formula (3) is the upper bound of formula (2). In addition, the constraints in formula (3) correspond to the equilibrium flow constraints previously described.

2.4 Constraints of the Model

1) At each time period, there are successive large-scale potential users θ_i arriving at location i. The probability of a user at location i who wants to go to location j is determined by the elements in matrix A, denoted by α_{ij}. For all positions i, $\sum_j \alpha_{ij} = 1$.
2. $x_i = \beta\left[\sum_j \alpha_{ji}\min\left\{x_j, \theta_j\left(1 - F\left(p_j\right)\right)\right\}\right] + \delta_i$
   For formula (3), the number of idle vehicles δi at position i is greater than 0, which corresponds to the incentive compatibility constraint of formula (2), that is, the compensation price ci.
3. For all i,

$$di = (1 - F(pi))\theta i$$

$$xi = \beta\left[\sum_j \alpha ji \min\{xj, \theta j(1 - F(pj))\}\right] + \delta i.$$

For all i, j, have $pi, \delta i \geq 0$

3 Solution of the Optimal Pricing Model for Car Rental Service

By establishing the objective function, the objective of this section is to determine how the demand model of the destination location affects the optimal pricing strategy and profit of the platform, and how to determine the optimal solution of the model.

3.1 Solution of the Optimal Pricing Model for Car Rental Service Under Balanced Demand

The solution under the condition of balanced demand provides us with ideas to solve the unbalanced situation. The first step to achieve this goal is to determine the concept of “balance” demand model [1].

Balanced demand model, if demand model (A, θ) is balanced, then:

$$(\beta A^T - \mathrm{I}) \leq 0 \tag{4}$$

Furthermore, if inequality (4) works for every β, it follows that the demand pattern is well balanced. The formula succinctly describes the set of demand patterns where the platform can profit the most from all demand patterns, that is, the users in each location are the same.

The optimal price and compensation of the platform meet the following properties:

1) If $w(1-\beta) \geq 1$, the time-sharing car rental platform will not meet any location demand of any network.
2) If $w(1-\beta) < 1$, we can get the optimal price and compensation of the platform.

a) The profit corresponding to the balanced demand mode is the highest under any demand mode. At this time, the potential user vector θ is the same at this time.
b) In a balanced demand model, the platform maximizes its profits by setting the same price at all locations. The optimal price is given by the following formula:

$$p_i^* = \frac{1}{2} + \frac{(1-\beta)w}{2}$$

In addition, the best solution can be supported by the same compensation for users in all locations, that is:

$$c_i^* 1 - (\beta)w$$

c) In a balanced demand model, the vehicles are never idle.

3.2 Solution of the Optimal Pricing Model for Car Rental Service Under Unbalanced Demand

Although the enterprise profit is maximized when the demand mode is balanced by setting the same price in all locations, this is usually not the case for the demand mode where inequality (4) is not applicable. Next, we discuss that the time-sharing car rental platform sets different prices according to the regional location and the heterogeneous needs of users, so as to achieve the target profit [2].

Assuming that all locations have the same number of potential users, we normalize it to 1, that is equal to 1. In addition, the idle vehicle cost w is equal to 1.

Assuming that the number of potential users in all locations is equal, the imbalance condition between supply and demand caused by user destination preference can be directly analyzed without considering other factors affecting demand caused by the number of users. We first provide the optimal price and compensation characteristics of the platform.

Assuming that the optimal solution $\{\delta_i, x_i\}_{i,j=1}^n$ is maintained and considered for the optimization problem, let $\{\lambda_i^*\}_{i=1}^n$ considered represent a set of optimal dual variables corresponding to the equality constraints in formula (3). Then,the optimal price takes the following form:

$$p_i^* = \frac{1 + \lambda_i - \beta \sum_j \alpha_{ij} \lambda_j^*}{2} \tag{5}$$

In addition, it is assumed that k is the place where a large number of users enter, and is the oversupply location of the best solution. So,

$$\frac{1}{2} + \frac{(1-\beta)w}{2} \leq p_i^* \tag{6}$$

Finally, tuple $\{\delta, x_i\}_{i,j=1}^n$ form a balance with the following formula under the condition of $\{p_i, c_i\}_{i=1}^n$:

$$c_i^* = \lambda_i^* - \beta \sum_j \alpha_{ij} \lambda_j^* \tag{7}$$

As can be seen from (5) and (7), the price and compensation clearly reflect the marginal value of idle vehicles utilized by the platform at each time-sharing car rental network location. Therefore, it can be seen from (6) that the platform is the best solution to compensate users at the oversupply position, which in turn will increase the demand of users of the time-sharing car rental platform in these places, so as to improve the utilization of vehicles.

3.3 Pricing and Compensation Scheme

Based on the establishment and solution of the above model, we propose some pricing schemes and compensation schemes to achieve the goal of maximizing profits for time-sharing car rental companies [4].

First of all, in order to solve the optimal pricing problem under unbalanced demand, we have formulated the pricing scheme of the platform: 1). Set the same unit price in the entire network; 2). Set the starting price of different departure places according to users' needs and the number of vehicles distributed; 3). Set the price compensation for the destination reached by the user.

The main objective of this paper is to illustrate the benefits of spatial price discrimination in practice, that is, through the different settings of the initial price/compensation to affect the user's vehicle demand, optimize and adjust the vehicle layout, so as to achieve the balance between vehicle distribution and user demand.

The following will compare the profits generated by the time-sharing car rental platform under the following three pricing schemes:

1) Single pricing. The time-sharing car rental platform sets the same price p for all locations, regardless of their departure or destination (the platform can set different driving compensation according to the starting point of the car). By solving the optimization problem (3) and the additional constraints p_i= p for all i, the optimal price p and compensation vector $\{c_i\}_{i=1}^n$ can be obtained.
2) Starting point pricing. Without considering the destination, the price is determined by the starting place and optimized $\{p_i, c_i\}_{i=1}^n$ by solving formula (3), where *pi* and *ci* represent the price and compensation of the journey generated at position i.
3) Starting point-destination pricing. The time-sharing car rental platform is optimized on $\{p_{ij}, c_{ij}\}_{i,j=1}^n$. The price and compensation used are functions of its starting point and destination.

It can be seen that single pricing scheme is a special case of origin-destination pricing, and origin-destination pricing is also a special case of starting point-destination pricing. Thus, starting point-destination pricing is most profitable for time-share platforms, and least profitable when time-share platforms use the same price across the network.

Obviously, the starting point-destination pricing takes full account of the whole journey of the users' use of the vehicle. Instead of adopting a single price and only considering the user's pick up point or return point, it is based on the whole journey of the user's use of the vehicle from the start point to the destination return point, so it is the optimal pricing scheme. [5].

4 Conclusion

This paper establishes the time-sharing car rental car rental demand imbalance situation, the optimal pricing model, puts forward the basic assumption of the model, sets up the constraint conditions of the model, establishes with prices as the decision variables of the objective function of profit maximization, provides a method of solving the model, solves the "equilibrium" optimal pricing problem under demand patterns, gets the optimal form of price and compensation, which provides a solution to the more complex pricing problem of imbalanced demand car rental service.

According to this optimal form, it can deduce by analogy the form of pricing and compensation in the case of unbalanced demand. According to the solution of the model, this paper puts forward some pricing and compensation scheme. In essence, the compensation scheme used is to exploit spatial price discrimination, that is, by setting different starting place prices or compensation, affecting users' car demand, optimizing vehicle layout, it achieves a balance between vehicle distribution and user needs, and ultimately maximize the profits of time-sharing car rental companies.

Finally, this paper also explores three pricing schemes, and determines the optimal full-process pricing or compensation scheme that takes into account the vehicle return from the starting point to the destination.

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