The Misconception and Risk of Using MTBF as a Reliability Indicator for Industry Management

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Abstract. MTBF (Mean Time between Failure) is perhaps the most common indicator for reliability level used in the industries. This paper examines the statistical foundation of this indicator and investigates the limitation and risk of using it. Examples are provided to demonstrate the errors when misusing the MTBF. The examination is not trivial if the error is found significant. Some misconceptions on the MTBF are also discussed to raise the awareness of using this indicator correctly in the paper. Furthermore, the paper points out some results from the demonstration in the paper might be opposite to our intuition. Industrial engineers should avoid to make decision according to those unverified intuition. Upon the limitation and problems, the paper furthermore proposes some approaches to mitigate the limitation and the problem of the use of the MTBF.

Keywords: MTBF · MTTF · Reliability Indicator · Misconception

1 Introduction

Purposing to numerically characterize the reliability level of a product, using the arithmetic mean of product lives as the reliability level is intuitive. The mean life is termed as mean time to failure (MTTF) for unrepairable system. For repairable system, MTTF corresponds to another definition: mean time between failures (MTBF). In industrial practises, the MTBF is used both for repairable and unrepairable units without differentiating those two. In this paper, we also call both the MTBF and MTTF as simply as MTBF. MTBF has been shown used in the aviation, electronic & electric, nuclear, oil and gas, railway industries etc. Take the aviation as an example. Both for military and civil purpose, it has seen using MTBF as reliability indicator for the communication system, flight control, navigation, landing gear, fire protection etc., and MTBF is used as a major indicator for aviation maintenance [1]. In electronic industry, MTBF is one of the specifications that presents in the manual for electronic & electric units, such as capacitor, resistor, transistor etc. [3]. In industry, plenty of engineering designers refer the MTBF to select unit supplier. Moreover, to address the perspective of reliability, safety, or quality, in industry, it is common to set the MTBF values as a target for

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development in the commercial contract. Using the simple numerical value MTBF as the indication is simple and easy to be understood by the manager, top level decision maker, and the end-users. However, the risk of using MTBF has been ignored. There are some premise conditions under which the MTBF is reasonable to serve as the correct reliability indicator. This paper is to figure out the misconceptions, shortcomings, and risk of the MTBF to raise the awareness of the risk of using it. The Sect. 2 discusses the major problem of using MTBF, the Sect. 3 discusses some misconceptions. The Sect. 4 presents mitigation approaches to reduce the risk of use MTBF. The Sect. 5 presents conclusion.

2 Problems of Using MTBF

The major implicit hypothesis of using MTBF is the mean of the product lives contains the reliability level information. The longer MTBF suggests a higher reliability level, and verse vice. It makes sense from our intuition. However, it could be misleading. We can illustrate by a simple example. Suppose the lives of products are.

$$1, 3, 5, 7, 9 \tag{1}$$

The mean of the sequences is 5. But the max and min is 9 and 1 respectively, which are far from the mean. If we use the 5 as the product life, it will not be able represent the max 9 and min 1, and the misleading is easy to occur. Mean is meaningless in this case since the sequence data are too scattered. We can further discuss the data from statistics perspective. The above sequence is a perfect uniform distribution. The mean for the uniform distribution does not contain much reliable information about the level of the production life as the data of the life are too dispersed.

Moreover, MTBF is defined as the arithmetic average of time to failure, as follows

$$MTBF = \frac{\sum_i t_i}{n} \tag{2}$$

which is highly sensitive to outlier. Contaminated data will induce a significant change in the value of MTBF, resulting to false MTBF. For example, for the above sequence, if the 9 is mistyped as 29, the mean of the sequence then becomes 9, which changed significantly from the original MTBF 5. The outlier is serious when the data sets is small. This is one problem of using MTBF.

It is well-known that in the reliability most products are a function of time. An older machine tends to fail more frequently than a newer one. In reliability engineering, the failure rate is used to represent the time-dependent information. Let us denote the failure rate as $r(t)$. For a machine with old age, the $r(t)$ is higher when the t is bigger, so that the aging information are represented. We can use a power form to represent the failure as

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \tag{3}$$

where the α and β are parameters. As shown in Fig. 1, when $\beta > 1$, the failure rate is increasing with time; while when $\beta < 1$, the failure rate decreases. When $\beta = 1$, the failure rate is constant.

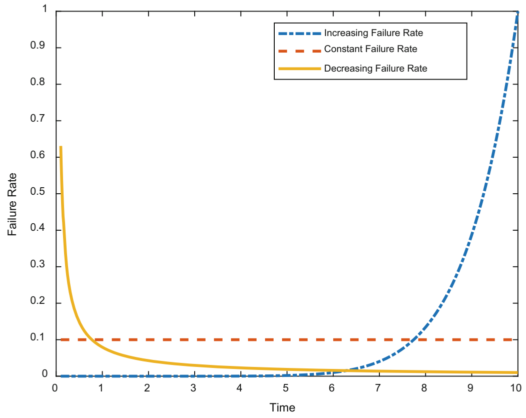


Fig. 1. Failure Rate Plot

If the $r(t)$ is constant, implying the failure occurrence will be independent from the operating time t. In statistics, when the failure rate is constant, which is a very special circumstance, the lifetime distribution corresponds to the one-parameter Exponential distribution. The sole parameter in the distribution is failure rate λ. The maximum likelihood estimate of the failure rate is

$$\lambda = \frac{n}{\sum_i t_i} \tag{4}$$

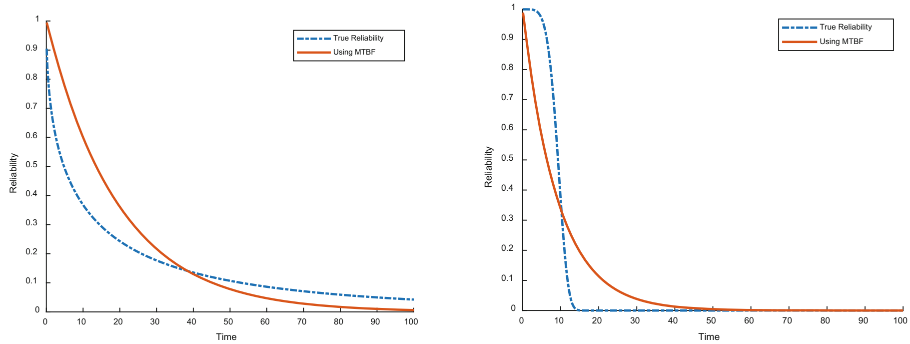
The t_i denotes the time between failure. Then we can easily find the MTBF is the inverse of the failure rate

$$MTBF = \frac{1}{\lambda} \tag{5}$$

which is constant and it means the MTBF gains the mathematical ground only when the failure rate is constant. Conclusively, we can summarize the use of MTBF implicitly assumes the product life follows Exponential distribution with constant failure rate. However, it is well-known, the product life can follow other distribution such as Normal distribution, Weibull distribution, Beta distribution etc., where the failure rate is not constant, but time dependent. Exponential is just one of the distributions, albeit the simplest one. This is a pre-condition of using MTBF.

However, regardless of the underlying distribution, in practise, we can anyway use the Formula (1) to calculate a MTBF value. Is then there any error will arouse? An example can be illustrated to check the degree of error. Suppose the failure rate is not constant. We demonstrate two distributions without constant failure rates, which are considered as true distribution of the life data. One is with $\beta = 5$ and the other is with $\beta = 0.5$ in the Eq. (3). In both case, the reliability function considers time. The true reliability as

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \tag{6}$$



a. $\beta = 0.5$ b. $\beta = 5$

Fig. 2. Reliability Comparison using MTBF with Real Reliability Values

Table 1. Numerical Values of Reliability

$\beta = 0.5$				
$t = 1$	$t = 5$	$t = 8$	$t = 10$	$t = 20$
0.7289	0.4931	0.4088	0.3679	0.2431
0.9521	0.7824	0.6752	0.6121	0.3747
$t = 30$	$t = 40$	$t = 90$	$t = 100$	
0.1769	0.1353	0.0490	0.0423	
0.2293	0.1404	0.0115	0.0074	
$\beta = 5$				
$t = 1$	$t = 5$	$t = 8$	$t = 10$	$t = 20$
1	0.9692	0.7206	0.3679	0
0.8963	0.5784	0.4164	0.3345	0.1119
$t = 30$	$t = 40$	$t = 90$	$t = 100$	
0	0	0	0	
0.0374	0.0125	0	0	

To compare this reliability with that from constant failure rate, i.e., from using the MTBF as constant failure rate, we can calculate another reliability value. According to the reliability function of Exponential distribution, the reliability function with time is

$$R(t) = e^{-t/MTBF} \tag{7}$$

We can examine the error by using simulation. We simulated 5,000 data sets, which is a large sample size, as the life data for each β. The outlier problem is not serious in this case since the data set is large. The plot of reliability for each β are shown in Fig. 2.

For the data simulated from $\beta = 0.5$, the λ as the inverse of MTBF is estimated around 0.05. For data sets simulated from $\beta = 5$, the estimated λ is around 0.11. The numerical values of the reliability for various time corresponding to the Fig. 2 is shown in Table 1. For each time horizon, the upper values are the true reliability, the lower values are the reliability estimated from using MTBF.

As shown from the Table 2, for some time horizons, the error of using MTBF can have around 40% of error, which is significantly different. Wrong decision making could be resulted from the reliability calculated from the MTBF. For example, for

$\beta = 5, t = 5$, a real reliability with 0.9692 will be estimated as only 0.5784 when using MTBF. The simulation example demonstrates that the risk of using the MTBF is very high when the failure is essentially time dependent.

3 Other Misconceptions in MTBF

It is naturally to consider the MTBF as the product life in practise. However, opposite to our simple intuition, mean life does not mean the real life equals to the MTBF. In the specification of some hard driver disks, manufacturers mention the MTBF can reach 200 years. However, the life observed from online-storage company shows the life is actually ranged between 1 to 5 years for most HDD when it works continuously. The huge difference between them reveals the weakness of use the MTBF. Actually, it is nocorrect if the value of MTBF is considered as the product life for an individual. MTBF is a statistic. According to the large number theory, the arithmetic mean of samples, which is MTBF in our case tends to expectation of its distribution [2] when the sample size tends to infinite.

$$\frac{\sum_i t_i}{n} \to E(t). \tag{8}$$

In another word, the Eq. (8) means the MTBF for the continuous distribution with an infinite number of data sets, is the mean of the distribution. If the product life follows Exponential distribution, meaning the MTBF is eligible as we discussed in the previous section. If the MTBF is the life, then the reliability of the product at the MTBF ($MTBF$=1/ λ) time is

$$R(t) = e^{-\lambda t} = e^{-\lambda .MTBF} = e^{-1} = 0.3679 \tag{9}$$

It means only about 36.79% of the population can reach the MTBF life, which is unexpectedly low. If the product does not have constant failure rate, for example, the product life follows Normal distribution, the percentage of the population can reach MTBF is only 50%. The 50% from the Normal distribution is straightforward since the Normal distribution is symmetric in its probability density function. For the Weibull distribution which is popular in reliability engineering, the MTBF is a little bit complicated, the mean of the Weibull distribution is $\alpha\Gamma(1 + 1/\beta)$, the reliability at the MTBF is [4]

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = e^{-(\Gamma(1+1/\beta))^{\beta}} \tag{10}$$

The reliability at MTBF for various β are shown in Table 2.

The percentage of the population that can reach the MTBF life is also low for all β. It can demonstrate that the MTBF does mean the produce life can reach this value. On

Table 2. Reliability at MTBF for various β

$\beta = 0.5$	$\beta = 0.8$	$\beta = 1$	$\beta = 2$	$\beta = 5$
0.2431	0.3312	0.3679	0.4559	0.5207

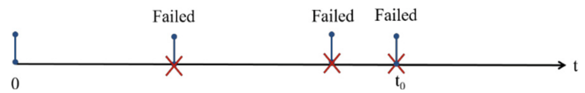


Fig. 3. Failure Process

the opposite, for most types of life data distribution, less than 50% of the product can reach the life of MTBF. This percentage should be aware when we make decision for example on the selection of spare parts from suppliers.

MTTF and MTBF

The above discussed scenario is on the product life from a population, these products are one-shot product, once it failed, it is discarded, meaning the product is an unrepairable system. For the case of unrepairable units, the distribution works. However, strictly speaking, in reliability engineering, as we have mentioned in the introduction of this paper, the product life for unrepairable unit should not be called MTBF, but MTTF (Mean Time To Failure). The real MTBF is for the repairable system. Mean time between failures, implies the life between two failures for this repairable system. Once the system failed, it is repaired, and the system has been restored to work. If the system can be restored to as good as new, the whole process is then a homogenous Poisson process (HPP), as shown in Fig. 3.

In the HPP, the time between the failures follows Exponential distribution. Corresponding to the failure rate for the unrepairable system, the HPP relates to a concept named intensity rate. Similar to the MTTF for unrepairable system, the intensity rate is calculated as

$$\lambda = \frac{n}{\sum \Delta t_i} \tag{11}$$

The MTBF is the inverse of the intensity rate. However, the Formula (11) only applies to the homogenous Poisson process with a constant intensity rate. For Non-homogenous Poisson process, for example, the repair effectiveness is not as good as new, but the same as old, or imperfect repair, the Formula (11) does not apply, which is similar to the unrepairable system. The MTBF shows obvious limitation for the case of Non-homogenous Poisson process.

4 Mitigation Approach of Using MTBF

The MTBF is practically obtained as a statistic from the life data. As we discussed above, the MTBF is essentially the mean of a distribution. A further problem is: as an indicator of reliability level, this indicator does not contain information about the uncertainty of the data. To mitigate it, when we evaluate the MTBF from the field data, the variance or standard deviation should be assessed and provided together with the MTBF. The variance can capture the uncertainty of the evaluated MTBF to some extent. For example, the data sets of (1) is with mean of 5 and the standard deviation is 3.16. Given another data sets such as

$$5, 5, 6, 4, 5 \tag{12}$$

is also with mean 5. But we can see it is obvious the mean of 5 for (11) is more credible as the data is more concentrated. The standard deviation of the data sets (12) is 0.7071, much lower than the standard deviation of data sets (1). This means the evaluated MTBF from data sets (12) is more reliable than from data sets (1). Aware of this problem, industries such as oil&gas like the OREDA handbook provides both the MTBF and standard deviation values, which make more sense than only using MTBF [6].

Another point regards the data size problem. Even the product has a constant failure rate, the size of the data sets is a concern. In the practise of reliability, the data size is very normal around 20–50. In statistics, the data set at this level is very small. The statistics such as the MTBF obtained from such small size is with relatively high uncertainty. To mitigate it, it is recommended to calculate the confidence interval using the Fisher Information method or moment method or any other method for the evaluated MTBF. A shorter confidence interval is preferred.

As we have described in Sect. 2, the MTBF has the solid mathematical foundation when the failure rate of the product is constant. For non-constant failure rate, the use of MTBF would result to a large error, as demonstrated in Fig. 2. When we evaluate the MTBF, it is strongly recommended to conduct statistical verification to check if the collected life data follows Exponential distribution or check the failure rate is constant. Goodness fitting test such as using the Q-Q plot approach or K-S test can be used.

Alternatively or additionally, the verification can be done from the failure mechanism perspective [5]. However, in most circumstances, investigating from failure mechanism perspective is infeasible and with high cost, thus verification from it is hard to conduct. In reliability engineering, most engineer judges it from the previous experiences. For example, it considers the failure rate of electronic system, complex system as constant, according to previous experiences. When the data sets strongly do not agree with the constant failure rate, as we have demonstrated in the Example in Sect. 2, it is strongly not recommended to use the MTBF as the reliability level.

Furthermore, even the MTBF is a strict homogeneous Poisson process or the MTTF follows confidently the Exponential distribution, in practise, however, it is still highly recommended to combine the knowledge obtained from the empirical data with the experience we already gained before or from some experts. This approach gains some success in the electronic industry, where for some basic units, from the real filed experience and extensive lab test, some of them are well-known with constant failure rate. For those unit, we can use the MTBF with less doubt.

5 Conclusion

This paper examines the risk of using Mean Time to Failure (MTBF) as an indicator of reliability level. The statistical background of the MTBF is discussed. The MTBF is eligible only when the failure rate of the product is constant. For non-constant failure rate, the demonstrated example shows the reliability could lead as high as around 40% error. It is then recommended to verify the time-dependency of the failure rate before to assess the MTBF level.

Common misconceptions such as considering the MTBF value as the life of the product is also discussed, the MTBF is a statistic from the life data, even for the constant failure rate unit/product, only 36.79% of the population can reach the MTBF life, which is opposite to our intuitions. Therefore, it would not recommend expecting the product life can really reach the MTBF life when we make decision.

The future research will focus on the investigating on the limitations of using MTBF when the failure process is non-homogenous process. The degree of the error of using MTBF for the non-homogenous process will be also further assessed. And data from industry will be collected to demonstrate the limitations under real industrial background.

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References

1. AIRBUS 2012. Maintenance Planning Document: Airbus 320. Airbus.
2. BECKMAN, R. J. 1973. Introductory Engineering Statistics, 2nd Edition - Guttman,I, Wilks,Ss and Hunter,S. *Journal of the American Statistical Association,* 68**,** 247–247.
3. DEPARTMENT OF DEFENSE, U. 1991. MIL-HDBK-217: Reliability Prediction of Electronic Equipment. Department of defense, USA.
4. LEWIS, E. E. 1996. *Introduction to reliability engineering,* New York ; Chichester, Wiley.
5. MCPHERSON, J. W. 2013. Reliability physics and engineering time-to-failure modeling. 2nd ed. Cham ; New York: Springer,.
6. SINTEF&NTNU 2015. *OREDA Handbook*, OREDA Participants.