Distributed Terminal Iterative Learning Strategy for a Convex Optimization with Application to Resource Allocation

Hongyu Yang[1] and Zijian Luo[2(✉)]

[1] School of Mathematics, Southwestern University of Finance and Economics, Chengdu, People's Republic of China

[2] Department of Computing and Artificial Intelligence, Southwestern University of Finance and Economics, Chengdu, People's Republic of China

zjluomath@126.com

Abstract. In the real world, network systems are ubiquitous such as supply chain inventory systems. In addition, resource allocation is an important research direction in the inventory control. Also, it inspired us to study the inventory resource optimization problem from the view of the network system. Thus, this paper investigates a class of resource allocation problem by applying terminal iterative learning control (ILC) strategy. According to the terminal ILC approach, the lowest cost can be obtained for a certain amount of inventory, i.e., the resource allocation problem is effectively solved. The main results are proposed with the help of consensus theory and iterative learning method. Different with the existing distributed optimization algorithms, our scheme provides another effective method of resolution. Finally, an example is given to verify the effectiveness of the main results.

Keywords: Terminal consensus · Distributed learning strategy · Distributed convex optimization · Multiagent systems · Inventory resource allocation

1 Introduction

Over the past decades, many works have been devoted to studying the iterative learning theory and its applications ([1, 14, 18] and references therein). Different with most conventional control methods, iterative learning control (ILC) approach provides an effective way to achieve the target tracking without a strict mathematical model. And ILC algorithm has strong robustness and adaptability at the same time. As a result, this control technology is widely applied in the filed of engineering, for example, robot manipulator [2], network systems [8, 9, 17], the robust tracking [10, 15] and so on. Undoubtedly, ILC approach provide an effective control scheme to solve the tracking problems. And until now, the ILC researches still attract the interest of engineering and science communities.

It is well known that distributed coordination problems have been widely studied because of the broad application scenarios. These problems also attract attention on the consensus [12], cooperative tracking [3], etc. Especially, distributed optimization is a

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hot topic in the applications of distributed cooperation [11]. And various distributed algorithms such as consensus approaches [16, 22], zero-gradient-sum algorithms [7] are designed to solve the optimization problems. Also, these distributed algorithms are applied in economic dispatch [19] and resource allocation [5] and so on. The above methods provide feasible algorithms for solving distributed optimization problems, but they are a little complicated. Therefore, it inspires us to study a class of distributed convex optimization problems with other effective strategies.

In the field of management and information, the modelling and analysis of supply chain management, particularly the research on optimization and control in supply chain inventory, has received a lot of attention. The research methods include operations research, optimization and control theory, information technology, etc. And many scholars conduct research with the help of control theory to solve dynamic problems in the supply chain system. For example, control strategy based on inventory fluctuation [4], fuzzy control [21] and H_∞ control scheme [6], etc. In addition, inventory optimization is another hot topic in supply chain management [13]. Then it is worthy to study the optimization issue in supply chain system. Modern inventory system has a typical distributed network structure. Thus, it is practical to study inventory optimization issue from the perspective of the network. Through coordination among multiagent, the distributed technique can simplify the calculation and reduce the difficulty of solving optimization problems. Therefore, it is meaningful to study the inventory optimization problem by means of the distributed optimization strategy.

According to the above discussion, a convex optimization problem is investigated by using the distributed terminal ILC strategy. And the contributions of this paper are as follows: 1) From the perspective of network systems, a terminal ILC strategy is used to analyze and solve the convex optimization problems. Moreover, different with existing algorithms, the strategy in this paper provides another effective method of resolution. 2) The optimization target is obtained without the global information of the multiagent network. It is a fully-distributed scheme. Further, the main results show that our learning strategy is effective to tackle the resource allocation problem.

The rest part of this paper is organized as follows: Necessary notations and the problem formulation are proposed in Sect. 2. And the terminal consensus optimization problem is considered, then the main results are obtained in Sect. 3. In Sect. 4, an example is given to verify the theoretical results. At last, the conclusion of this paper is shown in Sect. 5.

2 Preliminaries

2.1 Notations

A collection of nodes in a network is denoted as $S_n = \{1, 2, \cdots, n\}$. $\mathcal{I}$ represents the identity matrix with appropriate dimension. For a vector $x = [x_1, x_2, \cdots x_n]^T \in R^n$, $||x||_\infty = max_{1\leq i\leq n}|x_i|$ is a vector norm. And $||A|_\infty = max_{1\leq i\leq n}\sum_{j=1}^{n}|a_{ij}|$ is a matrix norm of $A \in R^{n\times n}$. $[0, T]$ denotes the time interval, and T is the terminal time.

2.2 Graph Theory

The network model is described as $\mathcal{G} = (V, E, A)$, where $V = \{v_i : i \in S_n\}$ denotes the set of nodes. $E \subseteq V \times V$ denotes the set of edges. A directed edge from v_i to v_j is denoted by an ordered pair $(i, j) \in E$, which means v_j can receive the information from v_i. The neighborhood of v_i is denoted as $\mathcal{N}_i = \{v_j \in V | (j, i) \in E\}$. The weighted adjacency matrix of network graph $\mathcal{G}$ is represented as $= (a_{ij})_{n\times n}$. Specially, $a_{ii} = 0$, $a_{ij} > 0$ if $(j, i) \in E$, $a_{ij} = 0$ otherwise. The Laplace matrix of $\mathcal{G}$ is defined as $\mathcal{L} = (l_{ij})_{n\times n}$, where $l_{ij} = -a_{ij}$ if $i \neq j$, and $l_{ii} = \sum_{i\neq j} a_{ij}$. A spanning tree is a directed graph, which has exactly one root vertex. Other vertexes are the child nodes of the root vertex. And a graph has a spanning tree if V and a subset of E can form a tree. Furthermore, graph $\mathcal{G}$ is called balanced if $\sum_{j=1,i\neq j}^{n} a_{ij} = \sum_{j=1,i\neq j}^{n} a_{ji}$ holds for all $i \in S_n$.

2.3 Problem Formulation

Similar as [22], a class of convex optimization problem in this paper is considered as follows:

$$min_{x_1,\ldots,x_n} \sum_{i=1}^{n} f_i(x_i(t)) \tag{1}$$

$$\text{subject to } \sum_{i=1}^{n} x_i(t) = X_D, \tag{2}$$

where $f_i(x_i(t)) = a_i x_i^2(t) + b_i x_i(t) + c_i$ is the subobjective function. $x_i(t) \in R$, and a_i, b_i, c_i are the coefficients of the quadratic function $f_i(x_i(t))$. And X_D is a constant and provides a constrain to the sum of $x_i(t)$. In inventory management, X_D also represents the total resource or total inventory quantity, which is needed to be allocated and stored. The convex quadratic optimization problem (1) with equality constraint (2) appears in the economic dispatch, the resources allocation, etc. Thus, the research on this problem has theoretical and practical significance.

Remark 1. The problem (1) with constraints (2) can be tackled by traditional optimization algorithms. However, the centralized optimization method needs an information center to obtain the information of all nodes and integrate and handle them. And the centralized schemes are a little inefficient and uneconomical. Hence, the distributed coordination strategy is used to allocate the resources of each node, which helps to quickly minimize the target function. Based on the Lagrange multiplier method, one knows that the solution of (1) with (2) is equal to the solution of following issue.

$$\frac{\partial f_1(x_1(t))}{\partial x_1(t)} = \cdots = \frac{\partial f_n(x_n(t))}{\partial x_n(t)} = \lambda^* \tag{3}$$

where λ^* is the optimal Lagrange multiplier.

Further, (3) is equivalent to

$$2a_1 x_1(t) + b_1 = \cdots = 2a_n x_n(t) + b_n = \lambda^* \tag{4}$$

According to (2) and (4), it is not difficult to obtain $\lambda^* = (X_D + \sum_{i=1}^{n} \frac{b_i}{2a_i})/(\sum_{i=1}^{n} \frac{1}{2a_i})$ and $x^* = \frac{\lambda^* - b_i}{2a_i}$.

Let $z_i(t) = 2a_i x_i(t) + b_i$, then (4) is rewritten as

$$z_1(t) = \cdots = z_n(t) = \lambda^* \tag{5}$$

Therefore, the problem (1) with constraints (2) is transformed as the consensus problem (5). And the original optimization problem is solved as $z_1(t) \to \lambda^*$. In the next section, the terminal ILC algorithm is proposed to solve the problem (5).

3 Theoretical Analysis

In this section, the main purpose is applying terminal iterative learning algorithm to obtain the distributed optimization target.

3.1 Consensus Terminal ILC Strategy

Inspired by (Meng, Jia and Du 2014, Zhang, Luo and Xiong 2022), the dynamic of each agent in multiagent systems is as follows:

$$\frac{d}{dt} z_{k,i}(t) = u_{k,i}, \forall t \in [0, T], i \in S_n \tag{6}$$

where k denotes the k th iteration. $u_{k,i}$ denotes the learning control input, and it is a constant at the k th iteration. Then, the purpose is applying terminal ILC strategy to obtain the consensus state z^c, i.e.

$$\lim_{k\to\infty} z_{k,i}(T) = z^c, i \in S_n. \tag{7}$$

In addition, the consensus state z^c is generally not equal to the optimal Lagrange multiplier λ^*. Thus, making z^c equals to λ^* is another target.

The terminal consensus ILC strategy is as follows:

$$u_{k+1,i} = u_{k,i} + r_i \sum\nolimits_{j\in\mathcal{N}_i} a_{ij}[x_{k,j}(T) - x_{k,i}(T)], \tag{8}$$

where a_{ij} is the (j, i) entre in the adjacency matrix A. $r_i > 0$ denotes the learning gain parameter. And the initial input $u_{0,i}$ is arbitrarily.

Next, the following assumption is necessary in the theoretical analysis.

Assumption 1. There is no offset for the initial state of agent, i.e., $x_{k,i}(0) = x_{k+1,i}(0) \equiv x_i(0)$. Moreover, $x_i(0)$ always satisfies the constraint (2).

Since $z_i(0) = 2a_i x_i(0) + b_i$, it is obviously that there is no offset for $z_i(0)$.

Remark 2. The purpose of this paper is different with previous research work [20]. We try to solve the optimization problem (1) with constraint (2) on the basis of achieving terminal consensus.

3.2 Related Lemmas

The following two lemmas are necessary, which play an important role in the analysis of the main results. More details can be seen in (Meng, Jia and Du 2014). And these two results imply that the consensus states can be achieved through the information interaction between adjacent agents.

Lemma 1. Consider (6) and (8) with a directed graph $\mathcal{G}$, and let the positive learning gain satisfy the inequality $Tr_i \sum_{j \in \mathcal{N}_i} a_{ij} < 1,\ i \in S_n$. Then, the consensus objective (7) can be achieved as $k \to \infty$ if and only if $\mathcal{G}$ has a spanning tree.

Lemma 2. Consider (6) and (8) with a balanced directed graph $\mathcal{G}$, the positive learning gain satisfies the inequality $Tr_i \sum_{j \in \mathcal{N}_i} a_{ij} < 1,\ i \in S_n$. Meanwhile, the initial input is zero. If $\mathcal{G}$ has a spanning tree, the consensus objective (7) can be achieved as $k \to \infty$ with the consensus state z^c, which is given by $z^c = \sum_{i=1}^{n} r_i^{-1} z_i(0) / \sum_{i=1}^{n} r_i^{-1}$.

3.3 Consensus Analysis

Theorem 1. With assumption 1, consider (6) and (8) with the directed and balanced graph $\mathcal{G}$. let the learning gain $r_i = 2a_i\tau$, τ is a constant, and the initial input $u_{0,i} = 0$. Then, the agent's state at each iteration satisfies the constraint (2).

Proof. According to (6), one has $z_{k,i}(t) = z_{k,i}(0) + tu_{k,i}$. Then, one has $2a_i x_{k,i}(t) + b_i = 2a_i x_i(0) + b_i + tu_{k,i}$, i.e., $x_{k,i}(t) = x_i(0) + (t/2a_i)u_{k,i}$. Then, one obtains

$$\sum_{i=1}^{n} x_{k,i}(t) = \sum_{i=1}^{n} x_i(0) + t \sum_{i=1}^{n} \frac{1}{2a_i} u_{k,i}. \tag{9}$$

Combing with (8) and $r_i = 2a_i\tau$, one has

$$\begin{aligned} \sum_{i=1}^{n} \frac{1}{2a_i} u_{k+1,i} &= \sum_{i=1}^{n} \frac{1}{2a_i} u_{k,i} \\ &+ \sum_{i=1}^{n} \tau \sum_{j \in \mathcal{N}_i} a_{ij} \left[x_{k,j}(T) - x_{k,i}(T) \right]. \end{aligned}$$

The above equality is rewritten as the compact form:

$$\begin{aligned} &\left[\tfrac{1}{2a_1} \cdots \tfrac{1}{2a_n} \right] \begin{bmatrix} u_{k+1,1} \\ \vdots \\ u_{k+1,n} \end{bmatrix} \\ &= \left[\tfrac{1}{2a_1} \cdots \tfrac{1}{2a_n} \right] \begin{bmatrix} u_{k,1} \\ \vdots \\ u_{k,n} \end{bmatrix} - \left[\tau \cdots \tau \right] \mathcal{L} \begin{bmatrix} x_{k,1}(T) \\ \vdots \\ x_{k,n}(T) \end{bmatrix}. \end{aligned}$$

Since graph $\mathcal{G}$ is balanced, it is not hard to obtain $[\tau \cdots \tau]\mathcal{L}$=$[0 \cdots 0]$. It means

$$\begin{bmatrix} \frac{1}{2a_1} & \cdots & \frac{1}{2a_n} \end{bmatrix} \begin{bmatrix} u_{k,1} \\ \vdots \\ u_{k,n} \end{bmatrix} = \cdots = \begin{bmatrix} \frac{1}{2a_1} & \cdots & \frac{1}{2a_n} \end{bmatrix} \begin{bmatrix} u_{0,1} \\ \vdots \\ u_{0,n} \end{bmatrix}.$$

Note that the initial input $u_{0,i} = 0$, one obtains $\left[\frac{1}{2a_1}, \cdots \frac{1}{2a_n}\right]\left[u_{0,1}, \cdots u_{0,n}\right]^T = 0$, i.e., $\sum_{i=1}^{n} \frac{1}{2a_i} u_{k,i} = 0$. Therefore, according to (9), it is easy to get

$$\sum\nolimits_{i=1}^{\mathrm{n}} x_{k,i}(t) = \sum\nolimits_{i=1}^{\mathrm{n}} x_i(0) = \mathrm{X}_D \tag{10}$$

The equality (10) indicates that the constraint (2) holds. Theorem 1 shows that ILC strategy (8) does not change constraints. □

Theorem 2. With assumption 1, consider (6) and (8) with the directed and balanced graph $\mathcal{G}$. let the initial input $u_{0,i} = 0$. Then, one has the following equality $\sum_{i=1}^{n} r_i^{-1} z_{k,i}(t) / \sum_{i=1}^{n} r_i^{-1} z_{k,i}(0)$, $\forall t \in [0, T]$.

Proof. According to (6), one has $z_{k,i}(t) = z_i(0) + t u_{k,i}$. Thus, one has

$$\sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{r_i} z_{k,i}(t) = \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{r_i} z_i(0) + t \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{r_i} u_{k,i}. \tag{11}$$

Similar as the derivation of Theorem 1, it is not hard to obtain $\sum_{i=1}^{n} \frac{1}{r_i} u_{k,i} = 0$. Therefore, from (11), one immediately gets the result. □

Remark 3. The result of Lemma 2 can be directly obtained by combining Theorems 1 and 2. And Lemma 2 demonstrates that, when the graph $\mathcal{G}$ is balanced, selecting appropriate learning gain parameters can achieve the consensus state z^c under equality constraints. It implies us to choose appropriate parameters and initial values to achieve the optimal state λ^*.

Theorem 3. With assumption 1, consider (6) and (8) with the directed and balanced graph $\mathcal{G}$. let the learning parameter $r_i = 2a_i\tau$, τ is a constant, and r_i satisfies $Tr_i \sum_{j \in \mathcal{N}_i} a_{ij} < 1$. The initial input $u_{0,i} = 0$. Then, the agent's state at each iteration satisfies the constraint (2). If $\mathcal{G}$ has a spanning tree, the consensus objective (7) can be achieved as $k \to \infty$ with the consensus state $z^c = \lambda^*$. And the original problem (1) with constraint (2) is solved.

Proof. Since $r_i = 2a_i\tau$ and $u_{0,i} = 0$, according to Theorem 1, one knows the agent's state at each iteration satisfies the constraint (2). Linking with Lemma 2, one can see the consensus state z^c can be achieved. Thus, we only show z^c is equal to λ^*.

$$\begin{aligned} z^c &= \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{r_i} z_i(0) / \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{r_i} \\ &= \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{2a_i} (2a_i x_i(0) + b_i) / \sum\nolimits_{i=1}^{\mathrm{n}} \frac{1}{2a_i} \end{aligned}$$

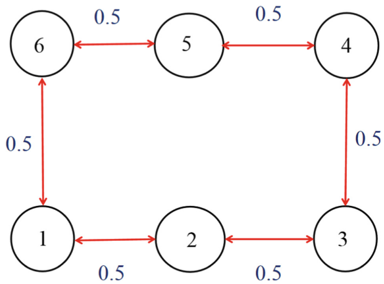

Fig. 1. Distributed network communication graph.

$$= \left(\sum\nolimits_{i=1}^{n} x_i(0) + \sum\nolimits_{i=1}^{n} \frac{b_i}{2a_i} \right) / \sum\nolimits_{i=1}^{n} \frac{1}{2a_i}$$

$$= \left(\mathrm{X}_D + \sum\nolimits_{i=1}^{n} \frac{b_i}{2a_i} \right) / \sum\nolimits_{i=1}^{n} \frac{1}{2a_i}. \tag{12}$$

Combining with previous analysis, one can see $z^c = \lambda^*$. Further, one can immediately get the solution of the original problem from $x^* = \frac{\lambda^* - b_i}{2a_i}$. □

Remark 4. Theorem 3 shows that the quadratic convex optimization problem (1) with (2) can be solved by applying ILC algorithm (8). Thus, ILC strategy (8) is effectively. Compared with the event-triggered algorithm [22] and consensus-based ILC approach [16], the global information of the network is not required in this paper. It reduces the conservatism of the theorem conditions to some extent.

4 Numerical Results

In this section, a numerical simulation is presented to demonstrate the effectiveness of the consensus terminal ILC algorithm. And the similar example appears in [22].

The network is composed of six nodes, it is shown in Fig. 1. It can be viewed as the distributed inventory system. Nodes 1–6 represent inventory warehouses in supply chains. And the direction of information communication is represented by the red lines. The values above represent the corresponding weights.

From Fig. 1, the communication network is a balanced graph with a spanning tree, as can be seen from the graph, which meets the conditions of the theorem in this paper. Further, the parameters of objective function $f_i(x_i(t))$ is shown in Table 1.

From Table 1 and by calculation, one knows $X_D = 4.1$ and $\lambda^* = 0.482$.Then the solution of the original optimization problem (1) can be obtained, it is shown in Table 2.

Let $t \in [0, T]$, $T = 1$, and $x_{k,i}(0) = x_i(0)$ in Table 1. Set $\tau = 0.1$ and $u_{0,i} = 0$, then $r_i = 2a_i\tau$ satisfies the inequality $Tr_i \sum_{j \in \mathcal{N}_i} a_{ij} < 1$. And the conditions of Theorems 1–3 hold. Hence, the equality constraint (2) holds, consensus state can be achieved, and $z^c = \lambda^*$. The simulation results are shown in the following.

Table 1. Parameters of objective function of each node.

f_i	a_i	b_i	c_i	$x_i(0)$
Node 1	1.2	−1.2	5.1	0.4
Node 2	2	−3.01	3.1	0.2
Node 3	3	−2.53	7.8	0.5
Node 4	2.4	−4.02	4.2	1.2
Node 5	2.5	−2.9	5.7	0.8
Node 6	4	−2.72	4.9	1

Table 2. The optimal state of Optimization problem (1)

Nodes	$x_i *$
Node 1	0.7096
Node 2	0.8732
Node 3	0.5022
Node 4	0.9066
Node 5	0.6766
Node 6	0.4004

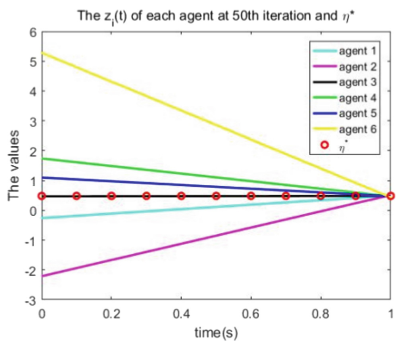

Fig. 2. The value of $z_{k,i}(t)$ at 50th iteration.

According to Figs. 2 and 3, one can see the consensus state z^c is basically achieved at the terminal time. The result also implies that our algorithm has good convergence after the 20th iteration. Figure 4 shows the terminal value of $x_{k,i}(T)$ of each agent at 1–100th iteration. Figure 5 shows the sum of $x_{k,i}(t)$ at each time of 1–100th iteration. It means the constraint (2) always holds. Figures 4 and 5 also illustrate the original quadratic optimization problem with equality constraint can be solved quickly by means of the terminal ILC scheme. Furthermore, appropriate learning parameters can improve

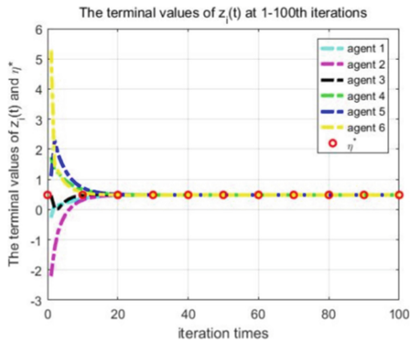

Fig. 3. The terminal value of $z_{k,i}(T)$ of each agent at 1–100th iteration.

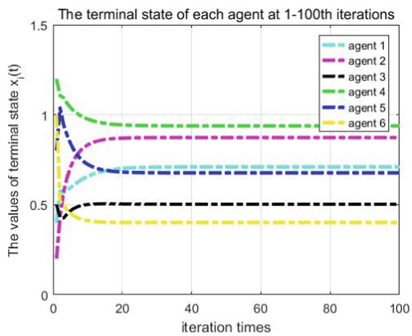

Fig. 4. The terminal value of $x_{k,i}(T)$ of each agent at 1–100th iteration.

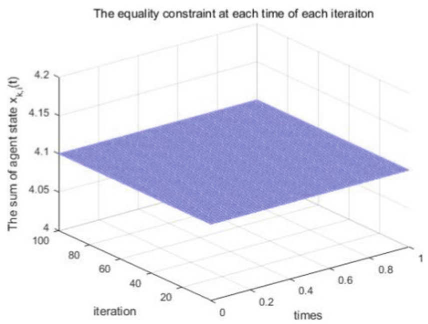

Fig. 5. The sum of $x_{k,i}(t)$ at each time of 1–100th iteration.

learning efficiency and convergence rate. Thus, appropriate learning control parameters need to be selected to improve the applicability of the algorithm in practical application.

5 Conclusions

This paper has studied a class of distributed convex optimization problem. Based on the Lagrange multiplier method, the original optimization problem has been transformed as the consensus problem. By using the terminal ILC strategy, the terminal consensus problem has been studied through the information interaction between agents. Further, the optimization target has been obtained by choosing the appropriate parameters. And the terminal iterative learning strategy has been applied to solve the resource allocation problem. The effectiveness of the proposed scheme has been verified by a numerical example. The main results demonstrate our strategy has provided another effective method to solve the optimization problems. And it might provide a theoretical reference for optimizing inventory in supply chains.

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