



Research on Pricing of Duopoly Competition Platform and Quality Strategy of Free Product in Bilateral Market

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Abstract. In different market data environment, enterprises collect market data and process the market information reflected in the data, and improve the competitive advantage through pricing and version division, which is a problem faced by enterprise managers. On the basis of product pricing and version decision-making, data are collected and market data are analyzed to construct a utility equation, and the factors affecting platform pricing are obtained by comparing the bilateral optimal pricing, the dynamic game is verified by numerical simulation, and the bilateral optimal pricing, bilateral market share and platform profit are obtained. The results show that with the improvement of free product quality, the downstream market share decreases, and the pricing of the downstream consumer platform is only affected by the difference between the paid version and the free version. Upstream market pricing and market share by market data, enterprise free version of the higher quality, the higher the price. Finally, the best quality of free product in different market data environment is summarized by numerical simulation. For managers in different market data under the decision-making reference.

Keywords: Bilateral Market Data · Enterprise Information Management · Duopoly Competition Strategy

1 Introduction

With the development of social informatization, people's clothing, food, housing, and transportation are gradually infused with information and information technology, and information products stand out. Information products are products used to disseminate information in daily life and contain major elements such as software products, advertisements and media products. When a platform provides products and services to users on both sides, it is called a bilateral market. Most software products have the characteristics of a bilateral market, and the issue of pricing and product quality of software products has been the focus of enterprise decisions. The information product itself has network externalities, high cost and low marginal cost characteristics, as well as the cross-network externalities of the bilateral market, etc. all affecting the profitability of

enterprises. This paper discusses and studies the decision of platform-based firms to adopt price and quality competition of freemium services in the case of a duopoly by combining the bilateral market theory of software products and the theory of information product versioning.

2 Current Status of Domestic and International Research

Domestic and international scholars mainly analyze the choice of software product pricing and its quality in terms of network externalities, freemium models, bilateral markets, product quality and pricing decisions.

Network externality was first formulated explicitly by Katz and Shapiro scholars in 1985, where the utility gained by consumers increases or decreases with the number of people using a product or service. Network externalities have both positive and negative effects, and considering the positive network externality of consumers to advertisers along with the negative network externality of advertisers to consumers and the level of effort of the platform, the degree of impact of platform profit is obtained to be proportional to the level of effort and cost of the platform [2]. The existence of network externalities allows the freemium model to exist, and the high network externalities can compensate for the impact of firms' investment in free products on profits. Studied the process of selling products based on network externalities for newly developed products and already existing products in the market analyzing a two-stage double oligopoly cross-cycle pricing strategy [13]. Proposed a new learning framework to explore the design of freemium model strategies and related pricing strategies [5].

Considering bilateral markets is more realistic than considering unilateral markets, constructed a decision model for sellers, platforms and consumers in a double oligopoly market, and found that when firms implement exclusive deals, their network externalities increase and their market shares increase [10]. An asymmetric oligopoly demand model was developed to analyze the relationship between market share and platform profit, and it was obtained that in a bilateral market, the profit of the platform shows a U-shape with the increase of the market share proportion [8]. Argued that two products with different quality differences gained greater profits [7]. Studied that different types of platforms should adopt differentiated competitive strategies under the joint influence of self-network externalities, cross-network externalities and platform pricing on upstream and downstream users' demand in that platform [11].

For the optimal design of free product quality, investigated the effects of free version quality, network externalities, and incubation rate ratio on firm pricing and profits during the competition for firms with incubation and restriction strategies [3]. A differentiated product competition strategy under delayed network effects, and the presence of free versions allowed consumer surplus and social profit maximization [4]. Investigated the effect of network externality strength on the best quality of free products in a unilateral market and numerically simulated the feasible domain of free product quality [1]. Analyzed the impact of the quality of platform products on profit and user size [6]. Considered the information asymmetry in terms of information access, cited probability in whether consumers access the platform or not when performing model building, and

solved the optimal strategy with a backtracking algorithm [9]. The free strategy for low-quality products will be adopted only when the high-quality products can bring greater benefits to the firm [12].

Summarizing the above four areas of literature, it can be seen that there have been various aspects of research on information product pricing, but for bilateral markets under the consideration of the impact of network externalities, most of the research is on the impact of pricing and obtaining profits for enterprises, and profits are composed of the profits of both upstream enterprises and downstream consumers, and few of them take into account the size of upstream enterprises on the platform when building the function. The impact of pricing on consumers is rarely considered. The free model is becoming more and more common in the current market, and most of the determination of the quality of the free version has chosen to analyze the quality difference or the trend between the quality of the freemium product and other variables, and few of them have studied the quality of the free version separately. This paper is based on the competition between the two oligopolistic firms in the bilateral market, and considers the influence of cross-network externalities to study the optimal strategies of platform firms for pricing and free version quality of products in the bilateral market, which is the innovation of this paper.

3 Problem Description and Model Building

Assume that there are two competing platform firms 1 and 2 in a bilateral market, and that both platform firms offer free and paid versions of their products to consumers, assuming that the quality of the free product is v , the quality of the paid product is s , and $0 < v < s$. List the utility equation: Consumers use the complete and free product utility of firms 1, 2. The symbols used in the article and their representations are given in Table 1.

$$u_{1b} = s - \beta\lambda x + rQ_1 + \theta Q_{s1} - p_1 \tag{1}$$

$$u_{2b} = s - \beta\lambda(1 - x) + rQ_2 + \theta Q_{s2} - p_2 \tag{2}$$

$$u_{1f} = v_1 - \lambda x + rQ_1 + \theta Q_{s1} \tag{3}$$

$$u_{2f} = s - \beta\lambda(1 - x) + rQ_2 + \theta Q_{s2} \tag{4}$$

The utility gained by sellers accessing platforms 1 and 2.

$$u_1 = z + tQ_1 - p_{s1} \tag{5}$$

$$u_2 = (1 - z) + tQ_2 - p_{s2} \tag{6}$$

Based on the principle of utility equality, we can find the utility non-difference point.

$$x_{11} = \frac{s - v_1}{4(\beta - 1)\lambda} \tag{7}$$

Table 1. Meaning of symbols used in the model

Symbols	Description
β	(seller-to-consumer) network externalities
t	(Consumer to seller) network externalities
θ	Network externalities within product groups
$\lambda/\beta\lambda$	Unit mismatch using free/complete product
u_i	The intrinsic value of enterprise i free products
s_i	The intrinsic value of the enterprise i complete product
Q_i/Q_{si}	Size of users/sellers of Enterprise i
p_i/p_{si}	Fees levied by Enterprise i for users/sellers

$$x_{12} = \frac{1}{2} + \frac{v_1 - v_2[3(\lambda - \gamma) - 2\theta t]}{2(\lambda - \gamma - \theta t)} \tag{8}$$

$$z = \frac{1}{2} + \frac{(v_1 - v_2)t}{6(\lambda - \gamma - \theta t)} \tag{9}$$

Build profit models for platforms 1 and 2.

$$\pi_1 = p_1x_{11} + p_{s1}(1 - z_1) \tag{10}$$

$$\pi_2 = p_2(1 - x_{22}) + p_{s2}z_1 \tag{11}$$

The optimal profit expression is finally found as follows:

$$\pi_1^* = \frac{(s - v_1)^2}{4(\beta - 1)\lambda} + \frac{\lambda - \gamma - \theta t}{2(\lambda - \gamma)} - \frac{(v_1 - v_2)^2 t^2}{18(\lambda - \gamma)(\lambda - \gamma - \theta t)} \tag{12}$$

$$\pi_2^* = \frac{(s - v_2)^2}{4(\beta - 1)\lambda} + \frac{\lambda - \gamma - \theta t}{2(\lambda - \gamma)} - \frac{(v_1 - v_2)^2 t^2}{18(\lambda - \gamma)(\lambda - \gamma - \theta t)} \tag{13}$$

4 Analysis of Equilibrium Results

4.1 Market Share Analysis

Proposition 1. If the two firms have the same quality of the free product, the upstream and downstream markets are equally divided in equilibrium. When the quality is not the same, depending on market conditions, for the upstream market, when $\lambda - \gamma < \theta t$, the larger the quality of the free version the smaller the market share of the product. When $\lambda - \gamma > \theta t$, the larger the quality of the free version, the larger the market share of the product. For the downstream market, when $\lambda < \gamma$ or $\lambda - \gamma > \theta t$, the larger the free version quality the larger the market share of the product. The larger the free version quality, the smaller the market share of the product when $0 < \lambda - \gamma < \theta t$ is used.

Proof. In case of unequal need to look at the market environment, when $\lambda < \gamma$ or $\lambda - \gamma > \theta t$ at this time, v_1 and v_2 the larger the difference, the smaller the market share of platform 1. The opposite is true. $\lambda - \gamma > \theta t$ indicates that the market is dominated by information product characteristics, and $\lambda - \gamma < \theta t$ indicates that the market is dominated by bilateral market characteristics.

4.2 Price Analysis of Upstream and Downstream Companies

It can be seen that the price for the downstream firms is determined by the difference between the quality of the paid and free versions. Platform for the upstream firms, we can see that the upstream price is related by the market structure and the difference in the free product quality between the two firms.

Proposition 2. For the upstream market, when $\lambda < \gamma$ or $\lambda - \gamma > \theta t$, the difference between the free product quality of two firms is less than $\frac{3(\lambda - \gamma - \theta t)}{t}$ and both coexist in the upstream market. When $0 < \lambda - \gamma < \theta t$, at most one firm can own the upstream firm.

Proof. When $\lambda < \gamma$: When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} < v_1 < v_2 - \frac{3(\lambda - \gamma - \theta t)}{t}$, then $p_{s1}^* > 0, p_{s2}^* > 0$.

When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} > v_1$, then $p_{s1}^* > 0, p_{s2}^* < 0$. When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} < v_1$, then $p_{s1}^* < 0, p_{s2}^* > 0$.

When $0 < \lambda - \gamma < \theta t$, When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} < v_1 < v_2 - \frac{3(\lambda - \gamma - \theta t)}{t}$, then $p_{s1}^* < 0, p_{s2}^* < 0$.

When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} > v_1$, then $p_{s1}^* < 0, p_{s2}^* > 0$. When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} < v_1$, then $p_{s1}^* > 0, p_{s2}^* < 0$.

When $\lambda - \gamma > \theta t$, When $v_2 - \frac{3(\lambda - \gamma - \theta t)}{t} < v_1 < v_2 + \frac{3(\lambda - \gamma - \theta t)}{t}$, then $p_{s1}^* > 0, p_{s2}^* > 0$.

When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} > v_1$, then $p_{s1}^* > 0, p_{s2}^* < 0$. When $v_2 + \frac{3(\lambda - \gamma - \theta t)}{t} < v_1$, then $p_{s1}^* < 0, p_{s2}^* > 0$.

4.3 The Relationship Between Platform Profit and Quality

4.3.1 Theoretical Analysis of the Relationship Between Platform Profit and Quality

In the optimal profit expression, we can see that the quality of the publication is determined by the enterprise, and the other factors are exogenous variables. Enterprises can determine the price, by adjusting the quality of their product version to affect their profits.

Proposition 3. When the total product quality is increased, the firm's profit increases accordingly. Platform 1 The trend of total profit with increasing free version quality is related to the relationship between exogenous variables.

Proof. Here we take Platform 1 as an example and Platform 2 as the same.

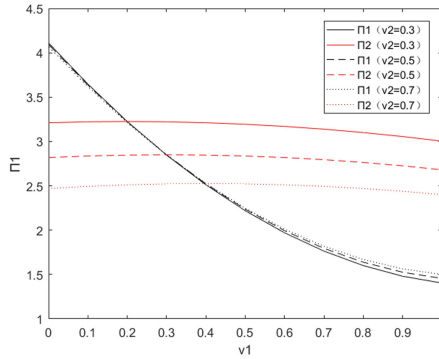


Fig. 1. $\lambda < \gamma$ the relationship between free mass and profit for $M > 0$.

$\beta > 1, \frac{\partial \pi_1}{\partial s} = \frac{s-v_1}{2(\beta-1)\lambda} > 0, \frac{\partial \pi_2}{\partial s} = \frac{s-v_2}{2(\beta-1)\lambda} > 0$, the profit of the platform increases with the complete value of the product, so in any case, the platform can choose to increase the total quality of its own product to increase its profit. $\frac{\partial \pi_1}{\partial v_1} = \frac{9(s-v_1)(\lambda-\gamma)(\lambda-\gamma-\theta t)+2(v_1-v_2)\lambda t^2(\beta-1)}{18(\beta-1)\lambda(\lambda-\gamma)(\lambda-\gamma-\theta t)}$

Set $K = \frac{9s(\lambda-\gamma)(\lambda-\gamma-\theta t)+2v_2\lambda t^2(\beta-1)}{9(\lambda-\gamma)(\lambda-\gamma-\theta t)-2\lambda t^2(\beta-1)}, M = 9(\lambda-\gamma)(\lambda-\gamma-\theta t) - 2\lambda t^2(\beta-1)$.

When $M > 0, K > 1$, then $v_1 < K$.

When $\lambda < \gamma, \frac{\partial \pi_1}{\partial v_1} > 0$. When $\lambda - \gamma > \theta t, \frac{\partial \pi_1}{\partial v_1} < 0$.

When $M < 0$, the $\lambda < \gamma$ or $\lambda - \gamma > \theta t$.

When $v_1 < K, \frac{\partial \pi_1}{\partial v_1} > 0, v_1 > K, \frac{\partial \pi_1}{\partial v_1} < 0, v_1 = K, \pi_{1max}$.

$0 < \lambda - \gamma < \theta t$, When $v_1 < K, \frac{\partial \pi_1}{\partial v_1} < 0, v_1 > K, \frac{\partial \pi_1}{\partial v_1} > 0, v_1 = K, \pi_{1min}$.

When $M > 0$, the unilateral market characteristic is significant or the market with product quality characteristic is significant, when the free product quality increases. Platform profits will decrease as the quality of the free version increases.

When $M < 0$, when the bilateral market characteristics are significant, the profit gained by platform 1 will first decrease and then increase as the quality of the free version increases. When the bilateral market characteristics are not significant, the total profit of platform 1 is maximum when the free version quality of platform 1 is K . When the bilateral market characteristics are significant, the total profit of platform 1 is the smallest when the free version quality of platform 1 is K .

4.3.2 Numerical Simulation of the Relationship Between Platform Profit and Quality When $M < 0, K > 1$

Taking $\beta = 2, \lambda = 0.1, \theta = 0.5, \gamma = 0.3, s = 1, t = 0.9$.

And taking $\beta = 2, \lambda = 0.7, \theta = 0.5, \gamma = 0.1, s = 1, t = 0.1$. Take $v_2 = 0.3, 0.5$, and 0.7 , respectively, and let v_1 take values on 0 to 1. Plot the relationship curve between profit about the quality of the free version, as shown in Fig. 1 and 2.

In this case, the optimal strategy for platform 1 should be to maintain the quality of its own free product $v_1 < v_2$.

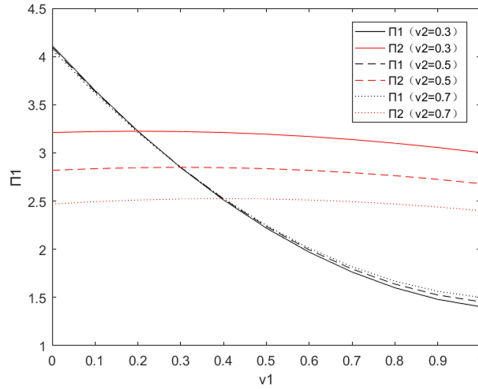


Fig. 2. $\lambda - \gamma > \theta t$, the relationship between free mass and profit for $M > 0$

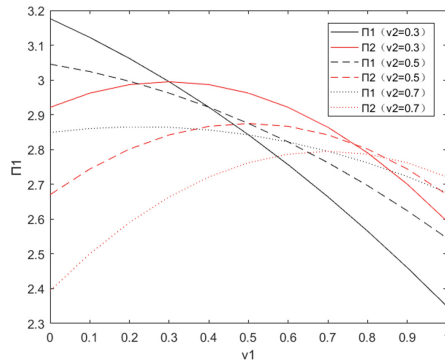


Fig. 3. $\lambda < \gamma$, the relationship between free mass and profit for $M < 0$

Taking $\beta = 2, \lambda = 0.1, \theta = 0.5, \gamma = 0.2, s = 1, t = 0.9$. And taking $\beta = 2, \lambda = 0.7, \theta = 0.5, \gamma = 0.3, s = 1, t = 0.9$ Take $v_2 = 0.3, 0.5$, and 0.7 , respectively, and let v_1 take values on 0 to 1. Plot the relationship curve between profit about the quality of the free version, as shown in Fig. 3 and 4. In this case, the optimal strategy for platform 1 should be to reduce the quality of their free products.

Simulation experiments and results analysis show that: 1) the platform profits with their own free version of the increase in quality, in different circumstances, presents a different state. 2) for both platforms, the relationship between the quality of a free product and its profit depends on exogenous variables. 3) When $\lambda < \gamma$ or $\lambda - \gamma > \theta t$ with $M > 0$, to increase profits, you need to keep the quality of your free version smaller than that of the free product on another platform. 4) When $\lambda < \gamma$ or $\lambda - \gamma > \theta t$ with $M < 0$, if you want to increase your profit, if K is greater than 0, choose K . If K is less than 0, keep your free version quality smaller than the free version quality of another platform; 5) When $0 < \lambda - \gamma < \theta t$ with $M < 0$, v_1 needs to stay as far away from K as possible if it wants to increase profits.

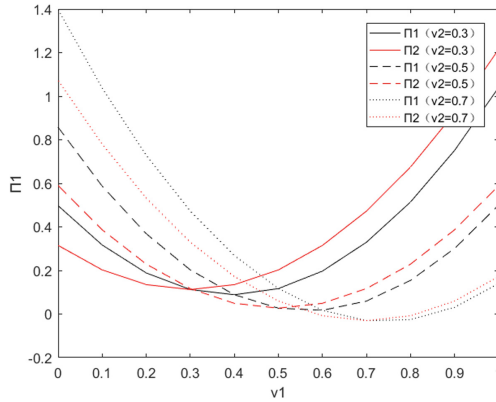


Fig. 4. $\lambda - \gamma > \theta t$, the relationship between free mass and profit for $M < 0$

5 Conclusions

This paper addresses the issue of price and quality version division strategy selection for products of duopoly firms in bilateral markets, and investigates bilateral pricing of products and optimal free version strategy selection of products from a new perspective. The influence of product network externalities and cross-network externalities on market share, optimal pricing, product quality and revenue, and the dependence of optimal values on the cost structure and network externality structure of the bilateral market are considered. The conclusion shows that, firstly, in both upstream and downstream markets, enterprises can increase their market share by improving the quality of the free version when the total quality is certain. Secondly, the price of the downstream enterprise is determined by the difference between the quality of the paid product and the free version, and the price of the upstream enterprise is not only determined by the poor quality, but also the dominant factor of the market. For upstream firms, the greater the quality of the free version, the smaller the market share, and vice versa, when the bilateral market characteristics are significant. For downstream firms, the greater the quality of the free version, the greater the market share, and vice versa, when the bilateral market characteristics are not significant. Then, for the upstream market, when the characteristics of the two markets are not significant, the difference between the quality of the two enterprise free versions is less than a certain value, and the two coexist in the upstream market. Under certain circumstances, when the characteristics of the two-sided market are obvious, the upstream firm will only choose one particular case, and when the characteristics of the two-sided market are not obvious, the profit of taking a certain value will reach the maximum. When the characteristics of the two-sided market are obvious, the profit of the two-sided market is the minimum, and when the two-sided market is remarkable, the quality of the free version is less than that of the free version of another enterprise. The research in this paper is conducted under certain assumptions and has certain limitations; future research can be discussed and studied in conjunction with the multi-attribution problem.

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